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ON THE COMPUTATIONAL IDENTIFICATION OF TEMPERATURE-VARIABLE CHARACTERISTICS OF HEAT TRANSFER

Jiří Vala

Brno University of Technology, Faculty of Civil Engineering,
Department of Mathematics and Descriptive Geometry
Veveří 95, 602 00 Brno, Czech Republic
vala.j@fce.vutbr.cz

Abstract

The mathematical analysis of a heat equation and its solutions is a standard part of most textbook of applied mathematics and computational mechanics. However, serious problems from engineering practice do not respect formal simplifications of such analysis, namely at high temperatures, for phase-change materials, etc. This paper, motivated by the material design and testing of a high-temperature thermal accumulator, as a substantial part of the Czech-Swedish project of an original equipment for exploiting solar energy using optical fibres, demonstrates the possibility of both direct and inverse analysis, physically transparent and mathematically correct, paying attention to the set of basic temperature-variable characteristics of thermal transfer.

1. Introduction

Most textbooks, both from applied mathematics and computational mechanics, present a heat transfer equation as a slightly modified Poisson equation, supplied by standard Dirichlet or Neumann boundary conditions, with a few constant material characteristics. Consequently, some general analytic results, as [2], p. 184, or at least semi-analytic ones, making use of the Fourier method by [2], p. 219, can be derived. Applying the variational approach, the existence and uniqueness of solution of a linear equation can be verified using the Lax-Milgram theorem together with some basic facts from the variational calculus; moreover, for the convergence of sequences of approximate solutions, the proper error analysis both for space and time discretization, applying various approaches by [17], is available. However, it is not easy to find such ideal closed simple systems in the nature. All engineering applications, especially in the design of advanced materials, structures and technologies (where sufficiently long experience with their behaviour is missing) work with materials of complicated micro-structure, including potential phase changes. Their effective material characteristics cannot be evaluated in a simple way and may not exist at

all in any reasonable sense, at least in that using some standard (e.g. two-scale) periodic homogenization, as discussed in [5], p. 204, or its generalization (including non-periodic phenomena and stochastic analysis) by [8]. Even in the homogeneous and isotropic case, at least from the macroscopic point of view, the determination of material parameters, making use of incomplete data from available experiments, can generate non-trivial inverse problems, not covered by [11], p. 255. To compensate the usual lack of input data, the formulation of such identification problems should avoid all multi-physical considerations, as the hygro-thermo-chemo-mechanical ones in [21] (and in a lot of papers referenced there), based on the complete set of conservation laws of continuum thermomechanics by [3], p. 4, i. e. for mass, (linear and angular) momentum and energy (or enthalpy), related to particular material components, including their phase changes. Even in the case of reflective insulation layers with air gaps or layers, reviewed in [12] and [9], most authors try to avoid (as much as possible) any methods of computational fluid dynamics, to obtain some simplified formulae for energy conservation only. However, the need of knowledge of results from various research areas justifies the extensive list of references even in this paper.

Figure 1: Experiments with the exploitations of solar energy (Hudiksvall, Sweden).

The principal motivation for the deeper analysis of heat transfer phenomena, sketched in this paper, comes from the Czech-Swedish project of the advanced exploitation of solar energy using optical fibres (cf. *Acknowledgements*). The left-hand part of Fig. 1 illustrates the development of the needed technological equipment, whereas its right-hand part shows one model (a representative from several alternatives) of the heat accumulator, whose effective functionality at high temperatures (up to 1000 °C) is a crucial part of the whole system; more information (without technical details) can be found in [18]. Fig. 2 shows a hot-wire measurement for the identification of material characteristics under standard laboratory conditions at the Faculty of Civil Engineering of Brno University of Technology. This method is open to its upgrade to high temperatures (more expensive components for a measurement device are necessary); another active cooperation exists with PD-Refractories CZ (former Moravian Fire and Schistous Clay Works) in Velké Opatovice. Nevertheless,

Figure 2: A simple equipment for the non-stationary hot-wire measurements (Brno University of Technology, Czech Republic).

the computational approach of [1], related to this method (much better than valid European technical standards), based on the simplifying physical and geometrical assumptions and on the properties of Bessel functions, needs substantial improvements just in the case of high temperatures.

To demonstrate a (nearly) realistic computational problem without complicated notations and technical difficulties, we shall consider, apart from its material microstructure, a homogeneous and isotropic material, whose thermal behaviour can be studied using the energy balance in the solid phase by [3], p. 7, without any changes in geometrical configuration, in the 3-dimensional Euclidean space R^3 and at the time interval $I = \langle 0, \tau \rangle$ for some positive τ. Usually such material is surrounded by other layers from the measurement system, whose properties should be a priori known, as explained in [20]; here we shall consider only a separate material specimen, located in some open set Ω in R^3, with all boundary conditions prescribed on the boundary $\partial\Omega$ of Ω in R^3. The heat conduction in the specimen will be conditioned by the heat convection and radiation from its environment. We shall study i) how the temperature-dependent material characteristics can be inserted both to the direct calculations of the time development of unknown temperature fields for a priori known values of such characteristics, solving standard initial and boundary value problems, ii) how these characteristics can be evaluated in the case of overdetermined boundary conditions.

2. Direct problems

Let us consider some system of Cartesian coordinates $x = (x_1, x_2, x_3)$ in R^3 and the time variable $t \in I$; upper dot symbols will be reserved for the derivatives with respect to t, prime symbols for the ordinary derivatives of functions of one real variable, ∇ for $(\partial/\partial x_1, \partial/\partial x_2, \partial/\partial x_3)$ and $\cdot$ for scalar products of vectors from R^3. The most frequently used heat transfer equation in the literature is

$$c(\theta)\dot{\theta} - \nabla \cdot (\kappa(\theta)\nabla\theta) + f = 0 \qquad \text{on } \Omega \times I, \tag{1}$$

where $\theta(x,t)$ [K] is the unknown temperature, $\kappa(\theta)$ [W/(m·K)] the heat conductivity

(crucial for the thermal insulation ability of a material) $c(\theta)$ [J/m^3·K] the heat capacity (important for the thermal accumulation property) and f [W/m^3] the volume heat source. The thermal diffusivity $\alpha(\theta) := \kappa(\theta)/c(\theta)$ [m^2/s] occurs frequently, too.

The obvious initial condition is

$$\theta(.,0) = \theta_0 \qquad \text{on } \Omega\,; \tag{2}$$

θ_0 has to be prescribed. The boundary $\partial\Omega$ is supposed to contain a set Γ, where (in general nonlinear) boundary conditions of the type

$$\kappa(\theta)\nabla\theta\cdot\nu + \varphi(\theta,\theta_e)(|\theta|^{n-1}\theta - \theta_e^n) + g = 0 \qquad \text{on } \Gamma\times I \tag{3}$$

are satisfied; here g [W/m^2] is the surface heat source, $\nu := (\nu_1,\nu_2,\nu_3)$ denotes the unit normal vector to Γ (with the outside orientation), θ_e means the temperature of the environment and $\varphi(\theta,\theta_e)$ [W/(m^2·K^n)]refers to some boundary characteristic, related to a real $n \geq 1$ (mostly integer in technical applications), namely to that for interface convection by [6], p. 37, with $n = 1$, or that for interface radiation by [6], p. 116, with $n = 4$, well-known as the Stefan-Boltzmann law; the natural and simple generalization is to combine a finite number of additive terms of such type on the left-hand side of (3). Here we can see that even in the case of constant c we cannot substitute, like (1), κ by α totally, because it cannot be removed from (3) except the case of (practically) empty Γ. We shall also assume that

$$\theta = \theta_e \qquad \text{on } \Theta\times I, \tag{4}$$

where Θ is some part of the boundary $\partial\Omega$; in this section we shall consider disjoint Θ and Γ, whose closure covers the whole boundary $\partial\Omega$.

Nevertheless, following [3], p. 8, for the energy balance the most important quantity is the internal energy $\varepsilon(x,t)$ [W/kg]; thus we have

$$(\rho(\varepsilon)\varepsilon)^{\cdot} - \nabla\cdot(\sigma(\varepsilon)\nabla\varepsilon) + f = 0 \qquad \text{on } \Omega\times I, \tag{5}$$

where $\rho(\varepsilon)$ is the material density [kg/m^3] and $\sigma(\varepsilon)$ [kg/m] is the new material characteristic, expected to be replaced using $\kappa(\theta)$ from (3); from the point of view of practical measurements the values of θ can be obtained much easier than those of ε. Frequently $\rho(\varepsilon)\dot{\varepsilon}$ occurs instead of the first additive term in (5), referring to the mass conservation; however, some applications, e. g. [16], studying the early-age behaviour of concrete mixtures, require variable ρ due to the change of material structure, thus we are only allowed to define $\overline{\rho}(\varepsilon) := \rho(\varepsilon) + \rho'(\varepsilon)\varepsilon$ and write $\overline{\rho}(\varepsilon)\dot{\varepsilon}$ instead of the first additive term in (5). Dividing (5) by $\overline{\rho}(\varepsilon)$, assumed to be non-zero, we receive

$$\dot{\varepsilon} - \nabla\cdot(a(\varepsilon)\nabla\varepsilon) + b(\varepsilon)\nabla\varepsilon\cdot\nabla\varepsilon + \overline{f}(\varepsilon) = 0 \qquad \text{on } \Omega\times I, \tag{6}$$

with $a(\varepsilon) := \sigma(\varepsilon)/\overline{\rho}(\varepsilon)$ [m^2], $b(\varepsilon) := -\sigma(\varepsilon)\overline{\rho}'(\varepsilon)/\rho^2(\varepsilon)$ [m^2] and $\overline{f}(\varepsilon) := f/\overline{\rho}(\varepsilon)$ [W/kg].

Let us now introduce the following simplified notation: let $\widehat{\psi}(.)$ be an arbitrary real function with its derivative identical with some given real function $\psi(.)$ (defined up to an additive constant). Using such notation, we are able to set $\varepsilon = \widehat{c}_m(\theta)$,

where $c_m(\theta)$ [J/(kg·K)] denotes the heat capacity related to the unit mass (unlike c related to the unit volume); consequently $\theta = \widehat{c}_m^{-1}(\varepsilon)$. Thus we obtain $\sigma(\varepsilon)\nabla\varepsilon = \sigma(\widehat{c}_m(\theta))\nabla(\widehat{c}_m(\theta)) = \sigma(\widehat{c}_m(\theta))c_m(\theta)\nabla\theta = \kappa(\theta)\nabla\theta$, which implies $\sigma(\varepsilon) = \kappa(\theta)/\overline{\rho}(\varepsilon)$. Similarly $(\rho(\varepsilon)\varepsilon)^{\cdot} = \overline{\rho}(\varepsilon)\dot{\varepsilon} = \overline{\rho}(\varepsilon)c_m(\theta)\dot{\theta} = c(\theta)\dot{\theta}$ gives $c(\theta) = \overline{\rho}(\varepsilon)c_m(\theta)$. Consequently we are able to evaluate the thermal diffusivity from the (not very simple) formula $\alpha(\theta) = \kappa(\theta)/(c_m(\theta)\overline{\rho}(\widehat{c}_m(\theta)))$. Another important information is that for positive values of κ and $\overline{\rho}$ and negative values of $\overline{\rho}'$ (which is the physically realistic setting) both factors a and b in (6) remain positive.

The initial and boundary conditions, as a simple analogy to (2), (3), and (4), are

$$\varepsilon(.,0) = \varepsilon(\theta_0) \qquad \text{on } \Omega\,, \tag{7}$$

$$\sigma(\varepsilon)\nabla\varepsilon\cdot\nu + \varphi(c_m^{-1}(\varepsilon),\theta_e)(|c_m^{-1}(\varepsilon)|^{n-1}c_m^{-1}(\varepsilon) - \theta_e^n) + g = 0 \quad \text{on } \Gamma\times I\,, \tag{8}$$

$$\varepsilon(\theta) = \varepsilon(\theta_e) \qquad \text{on } \Theta\times I\,. \tag{9}$$

To find the solution, i. e. the space- and time- variable temperature field ε (and consequently to express θ, too), of (6) with the initial conditions (7) and the boundary conditions (8) and (9) in a reasonable sense, admitting its numerical analysis, in some appropriate space of mappings from I to Lebesgue and Sobolev spaces defined on Ω and $\partial\Omega$ is not easy because of the presence of various type of nonlinearities in (5) and (9). Some interesting ideas and partial existence and uniqueness results can be found in [14], referring to the former analysis of [7]. However, the set of formal simplifying assumptions hidden there does not enable to handle realistic engineering problems, as needed in this paper.

Seemingly it could be useful to formulate a similar problem to the just discussed one for θ directly, without any transformation using ε. Indeed, dividing (1) by $c(\theta)$ (whose values are positive usually), we receive

$$\dot{\theta} - \nabla\cdot(a_*(\theta)(\kappa(\theta)\nabla\theta)) + b_*(\theta)\nabla\cdot\nabla\theta + f_*(\theta) = 0 \qquad \text{on } \Omega\times I \tag{10}$$

with $a_*(\theta) := \kappa(\theta)/c(\theta)$, $b_*(\theta) := -\kappa(\theta)c'(\theta)/c^2(\theta)$, and $f_*(\theta) := f/c(\theta)$, thus we should find the solution of (10) with the boundary conditions (3) and (4) and the initial condition (2). The arguments on the positive values of a_* and b_* (instead of those related to a and b) can be repeated, but at least (10) is even more complicated than (6) and difficulties similar to those in [14] can be expected.

Some difficulties of the above mentioned type can be removed using the Kirchhoff transformation $u = \widehat{c}(\theta)$ [W/m^3], seemingly the slight modification of the discussed $\varepsilon = \widehat{c}_m(\theta)$; consequently $\theta = c^{-1}(u)$. Now we have $\dot{u} = \overline{\rho}(\varepsilon)\dot{\varepsilon}$ and, introducing $\beta(u) := \widehat{\kappa}(c^{-1}(u))$, also $\nabla\beta(u) = \beta'(u)\nabla u$. Then (1) can be converted to the form

$$\dot{u} - \nabla\cdot\nabla\beta(u) + f = 0 \qquad \text{on } \Omega\times I \tag{11}$$

and supplied by the initial and boundary conditions

$$u(.,0) = u_0 \qquad \text{on } \Omega, \tag{12}$$

$$\nabla\beta(u)\cdot\nu + \psi_e(u)(|\gamma(u)|^{n-1}\gamma(u) - \theta_e^n) + g = 0 \qquad \text{on } \Gamma\times I\,, \tag{13}$$

$$u = u_e \qquad \text{on } \Theta\times I, \tag{14}$$

with $u_0 := u(\theta_0)$, $u_e := u(\theta_e)$, $\gamma(u) := \widehat{\kappa}^{-1}(u)$ and $\psi_e(u) := \varphi(\kappa^{-1}(u),\theta_e)$.

For the sake of simplicity, let us now assume that Ω is a domain in R^3 with a sufficiently smooth boundary to satisfy theorems on the Sobolev and Lebesgue spaces introduced on Ω and $\partial\Omega$, namely the Sobolev (compact) imbedding, the Poincaré-Friedrichs and the trace theorems by [15], p. 17; much more general geometrical configurations (bringing unpleasant technical difficulties) are discussed in [13], pp. 62, 222, and 385. Moreoever, let β', γ and ψ_e be continuous real functions and Θ be an empty set (this last assumption will be removed soon). One can see that, even for $n = 1$ in (13), the classical theory of monotone operators by [10], p. 243, is not applicable, because the monotonicity is violated for any non-constant β' or γ, thus more general results on pseudomonotone or weakly continuous operators are needed. Applying the standard notation of Sobolev, Lebesgue and Bochner spaces, let us choose $V = W^{1,2}(\Omega)$ with its dual space V^* and consider $u_0 \in V$, $f \in L^2(I, L^{6/5}(\Omega))$ and $g, \theta_e^n \in L^2(I, L^{4/3}(\Gamma))$. In all following considerations, δ will be some positive constant (a priori known, small in practice). Let us suppose that $\beta'(r) \geq \delta$, $1/\delta \geq \gamma(r) \geq \delta$ and $1/\delta \geq \psi_e(r) \geq \delta$ for any $r \in R$. Then by [15], p. 237 (after rather long verification of abstract assumptions), thanks to the properties of quasilinear pseudomonotone mappings, the problem formulated by (11), (12) and (13) has a weak solution $u \in W^{1,2,2}(I, V, V^*)$ in the sense

$$\begin{aligned}
&\dot{u}(t)v(t) + \int_\Omega \beta'(u(x,t))\nabla u(x,t) \cdot \nabla v(x)\,\mathrm{d}x \\
&\qquad + \int_\Gamma \psi_e(u(x,t))(|\gamma(u(x,t))|^{n-1}\gamma(u(x,t)) - \theta_e^n(x,t))v(x)\,\mathrm{d}s(x) \qquad (15)\\
&\qquad = \int_\Omega f(x,t)v(x)\,\mathrm{d}x - \int_\Omega g(x,t)v(x)\,\mathrm{d}s(x) \\
&\text{for all } v \in V \text{ and almost every } t \in I
\end{aligned}$$

if $\beta'(r) \leq 1/\delta$ for any $r \in R$ and $n = 1$. Moreover, by [15], p. 241, thanks to the properties of quasilinear weakly continuous mappings, the same problem has a very weak solution $u \in L^2(I, V)$ in the sense

$$\begin{aligned}
&\int_\Omega (u(x,\tau)v(x,\tau) - u_0(x)v(x,0))\,\mathrm{d}x \\
&+ \int_I \int_\Omega \beta'(u(x,t))\nabla(u(x,t)) \cdot \nabla(v(x,t))\,\mathrm{d}x\,\mathrm{d}t \\
&+ \int_I \int_\Gamma \psi_e(u(x,t))(|\gamma(u(x,t))|^{n-1}\gamma(u(x,t)) - \theta_e^n(x,t))v(x,t)\,\mathrm{d}s(x)\,\mathrm{d}t \qquad (16)\\
&= \int_I \int_\Omega f(x,t)v(x,t)\,\mathrm{d}x\,\mathrm{d}t - \int_I \int_\Omega g(x,t)v(x,t)\,\mathrm{d}s(x)\,\mathrm{d}t \\
&\text{for all } v \in W^{1,\infty,\infty}(I, W^{1,\infty}(\Omega), L^{6/5}(\Omega))
\end{aligned}$$

if $\beta'(r) \leq (1 + |r|^{5/3-\delta})/\delta$ for any $r \in R$ and $n \leq 2$.

We can see that the very weak solution, unlike the weak one, admits e. g. the linear growth of $\beta'(r)$, which is useful in practice. However, the requirement $n \leq 2$ is not realistic, namely in the analysis of radiation effects. The remedy is to choose V as the space of all v from $W^{1,2}(\Omega)$, whose traces belong to $L^n(\Gamma)$; the properties

of such spaces are discussed in [15], pp. 64 and 253. Another needed generalization is to remove the assumption $\Gamma = \partial\Omega$. This can be done using the transformation $\widetilde{u} = u - u_*$, where some u_* from the same space, as required for u, satisfies (14) instead of u. Consequently we have only $\widetilde{u}(.,0) = u_0 - u_*(.,0)$ on Ω instead of (12), in addition to (11) and (13) in their slightly modified forms containing $\widetilde{u}$; then it is sufficient to take the subspace of all functions from V with zero values on Θ instead of V. However, the practical construction of u_* may not be easy.

Since the derivation of solutions (15) and (16) is based on the Rothe sequences and Galerkin approximations, the numerical construction of sequences of approximate solutions is available, although the verification of their convergence is not trivial because of the presence of non-linear terms in (15) and (16). However, in any algorithm of discretization in time, based on the Euler implicit, Crank-Nicholson or similar schemes, it is natural to take arguments of $\beta'(.)$, $\gamma(.)$, $\psi_e(.)$ and $|.|$ from the preceding time step, thus we obtain only linear systems; the proper convergence analysis then relies on various compactness theorems. Let us also notice that some our assumptions can be weakened, e. g. it is possible to work with arbitrary $f \in L^1(\Omega \times I)$; however, the derivation of relevant results, using accretive mappings and nonlinear semigroups, by [15], p. 291, does not seem friendly to the construction of simple computational algorithms.

3. Inverse problems

Due to the limited extent of this paper, we shall refer to the notations and considerations of the previous section as much as possible. The first step in the inverse analysis then admits the intersection $\Gamma \cup \Theta$ with non-zero measure on $\partial\Omega$, compensating the imperfect knowledge of β', γ and ψ_e. It is then useful to introduce $\Xi := \Theta \setminus \Gamma$ and $\Psi := \Gamma \setminus \Theta$ (in direct problems clearly $\Xi = \Theta$ and $\Psi = \Gamma$). Let P be a set of admissible parameters; its simplest choice can be a closed set in R^N with an integer number N of unknown parameters. Now we can consider $\beta'(r,p)$, $\gamma(r,p)$ and $\psi_e(r,p)$ as functions of $(r,p) \in R \times P$, instead as functions defined on R only. We shall suppose that all these functions satisfy assumptions of (15) or (16), taking into account their above sketched generalizations, too, for arbitrary $p \in P$.

Following [4], pp. 123 and 368, it is natural to define

$$F(p) = \int_I \int_\Xi |u(x,t,p) - u_e(x,t)|^\omega \, \mathrm{d}s(x) \, \mathrm{d}t, \tag{17}$$

where $1 \leq \omega \leq \infty$ (the well-known choice is the classical least-squares one, i. e. $\omega = 2$); $\Theta \times I$ in (14) must be reduced to $\Xi \times I$ in all direct problems (with fixed p). Minimizing F, which can be interpreted as an error in our overdetermined problem where $u(.,.,p) \approx u_e(.,.)$ on $\Xi \times I$, is required in some reasonable sense (the equality here is not realistic because of the inexact measurements of u_e and other input data, our physical and geometrical simplifying assumptions, disturbing effects from other physical processes, etc.). Let us notice that F is only a function of N real variables here, with respect to p. The setting of p enables us to identify all material characteristics

completely (although the corresponding algebraic manipulations may not be quite easy).

Another access is seemingly available, too: to define

$$G(p) = \int_I \int_\Xi |\nabla\beta(u(x,t,p)\cdot\nu(x) + \psi_e(u(x,t,p))(|\gamma(u(x,t,p))|^{n-1}\gamma(u(x,t,p)) - \theta_e^n(x,t)) + g(x,t)|^\omega \,\mathrm{d}s(x)\,\mathrm{d}t \tag{18}$$

similarly to (17); $\Gamma \times I$ in (13) must be reduced to $\Psi \times I$ in all direct problems. Minimizing G, also interpretable as an error of the (exactly zero) term $|.|$ in (18), analogous to that of F, but formulated (from the physical point of view) for the interface heat fluxes instead of the interface temperature, is possible, but rarely used in practice because i) the evaluation of G (and its derivatives) in (18) is much more difficult that that of F in (17) and ii) the reliability of recorded values of g is usually much lower than that of θ_e in most engineering applications, including that mentioned in *Introduction*.

Let us pay attention to (17) only. Let us assume that P is a closed bounded set in R^N, thus (because N is finite) it must be compact. To verify the existence of some minimum of (17), by [10], p. 191, it is then sufficient to prove its continuity. However, it is not quite simple, even in the case (15) and $\omega = 2$, although it seems to be easy i) to consider a sequence of $p_k \in P$ with $k \in \{1, 2, \ldots\}$ with the limit $p \in P$, ii) to derive a corresponding $u_k(.,.,p_k)$ by (15) to p_k, as well as $u(.,.,p)$ to p, iii) to insert $v(.) = u_k(.,.,p_k) - u(.,.p)$ into (15) with p_k and into (15) with p and calculate their diference, iv) to integrate the result over I to try to get estimates of $u_k(.,.,p_k) - u(.,.,p)$ in appropriate norms following [10], p. 264. The lack of monotonicity, crucial for iv), has to be overcome by more advanced tricks, inspired by the sequence of exercises from [15], p. 66.

The sketched approach gives us only one rough information on the uncertainty of identified characteristics: the minimal value of F. The further step of the inverse analysis, motivated by [22], then should be to interpret P as a sample space of elementary events, supplied by the minimal σ-algebra and by certain probability measure $\mathcal{P}$. Then, instead of (17), we should minimize

$$\Phi(p) = \int_P \int_I \int_\Xi |u(x,t,p) - u_e(x,t)|^\omega \,\mathrm{d}s(x)\,\mathrm{d}t\,\mathrm{d}\mathcal{P}\,, \tag{19}$$

with respect to all other modified conditions, improved by $\mathcal{P}$. Some preparatory results of such type for a linearized heat transfer problem, including much more references, remarks to direct, sensitivity and adjoint problems and to the convergence analysis of nonlinear conjugate gradient algorithms, generalizing the Newton-type ones, applicable to (17) (although the exact values of derivatives cannot be computed easily), to minimize Φ, have been presented in [19]. Unfortunately, the general case contains still open problems because of the absence of such lemmas, as the (generalized) Aubin-Lions one by [15], p. 194, crucial for the compactness results in the deterministic case, and corresponding interpolation ones; this makes it difficult to replace I from (17) by $I \times P$ from (19) with some probabilistic measure.

4. Conclusion

We have shown that the proper analysis of the heat transfer equation with temperature-variable characteristics, including the inverse problem of identification of such characteristics, open to the uncertainty estimates, too, brings substantial difficulties in comparison with the linearized model problems. However, these difficulties can be overcome by means of recent functional and numerical analysis. More detailed considerations (including complete proofs) should be published in the near future.

The further research is motivated by the design of thermal accumulator, mentioned in *Introduction*, although the deep mathematical analysis does not seem to be its most important part. Some original experimental devices and MATLAB-based software packages have been prepared; the complete technical equipment must be functional until the end of 2014.

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