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INTERPLAY OF SIMPLE STOCHASTIC GAMES AS MODELS FOR THE ECONOMY

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Abstract

Using the interplay among three simple exchange games, one may give a satisfactory representation of a conservative economic system where total wealth and number of agents do not change in time. With these games it is possible to investigate the emergence of statistical equilibrium in a simple pure-exchange environment. The exchange dynamics is composed of three mechanisms: a decentralized interaction, which mimics the pair-wise exchange of wealth between two economic agents, a failure mechanism, which takes into account occasional failures of agents and includes wealth redistribution favoring richer agents, and a centralized mechanism, which describes the result of a redistributive effort. According to the interplay between these three mechanisms, their relative strength, as well as the details of redistribution, different outcomes are possible.

...But Mr. Lebeziatnikov who keeps up with modern ideas explained the other day that compassion is forbidden nowadays by science itself, and that that's what is done now in England, where there is political economy...

Crime and Punishment, Chapter 1, Fyodor Dostoevsky

1. Introductory considerations

In economics, distributional problems emerge in contexts of economic growth and allocation of resources, among others. Distributions of relevant economic variables are important for policy making purposes, however, there is a general policy problem that was emphasized by Federico Caffè [1]:

> ...when, in economic reasoning, the social wealth distribution is assumed 'given', this means that the existing distribution is accepted, without evaluating whether it is good or bad, acceptable or unacceptable... this must be explicitly done further clarifying that the conclusions are conditioned on the acceptability of the distributional set-up.

In the past, we have studied simple exchange mechanisms (games) based on exact probabilistic dynamics and leading to statistical equilibrium distributions [3, 4, 5, 7]. Our aim is to give a satisfactory representation of an economy and investigate the statistical equilibrium by means of interplay between these mechanisms. One of them describes centralized activities in terms of taxation and redistribution of wealth and it produces exponential tails. In order to include a process that gives power-law tails we involve a mechanism consisting of occasional failures of agents together with redistribution of their wealth. This mechanism is discussed in [3], Chapter 10, and it leads to the Yule distribution, which was originally proposed by Yule to account for the data on biological species [10]. It was the idea of Simon to use it in order to describe a class of distributions that appears in a wide range of empirical data, including economic phenomena [8]. The last but crucial mechanism represents pairwise exchange of wealth between agents and its interplay ensures that the system does not break down due to the failures of agents.

Since it is difficult to study the interplay of the three games analytically, we want to develop a statistical procedure based on statistical inference from the data to obtain relevant distributional properties of economic variables. In particular, we focus on the wealth distribution. However, here the emphasis will be on modelling and studying the aggregate wealth distribution in a conservative system where the number of agents and the total wealth do not change in time. For a simple trading rule in an active market, where number of agents and money is not conserved, Kusmartsev found that the wealth distribution has a general Bose-Einstein form, whose parameters depend on wealth exchange parameter, i.e. activity of agents [6]. Regarding conservation of wealth in the system, as pointed out in [9], ordinary agents in an economy can only exchange money with each other, so there is a local conservation of wealth. However, a government or a central bank can cause a change of wealth, but as long as it does not cause hyperinflation, the system can be close to statistical equilibrium, with slowly changing parameters.

1.1. Descriptions for the state of the system

The basic random variables for the description of the games are introduced in [3]. A general framework for agent-based models consists of the allocation of n objects among g agents (categories). Categories represent economic agents and objects may represent money or wealth. In this paper objects will be called coins.

The most complete description of the states is in terms of individual (coin) configurations $\mathbf{X}$. An *individual description* $\mathbf{X} = (X_1 = x_1, X_2 = x_2, \ldots, X_n = x_n)$, where $x_i \in \{1, \ldots, g\}$, is a list telling us, for each coin to which agent it belongs.

The total number of configurations for n coins distributed among g agents is g^n. A *statistical description* $\mathbf{Y} = (Y_1 = n_1, Y_2 = n_2, \ldots, Y_g = n_g)$ is a list giving us the number of coins for each agent, with the constraint $\sum_1^g n_i = n$. The total number of these agent descriptions for g agents sharing n coins is $\binom{n+g-1}{n}$. A *partition description* $\mathbf{Z} = (Z_0 = z_0, Z_1 = z_1, \ldots, Z_n = z_n)$ is the number of agents with zero coins, one coin, etc., with the constraints for $\mathbf{Z}$ $\sum_0^n z_i = g$, $\sum_0^n iz_i = n$. This is the less complete description, commonly referred to as *wealth distribution* and it will be mostly used throughout this paper. Figure 1 shows a state of a system and illustrates the meaning of the various descriptions.

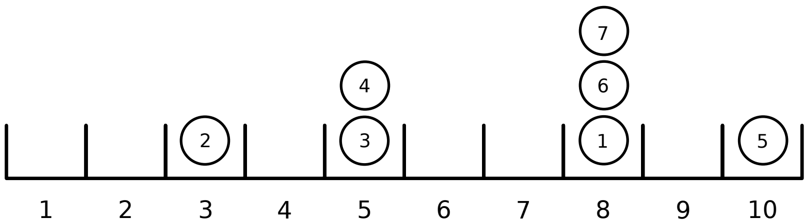


Figure 1: A state of a system with ten agents and seven coins. The individual description of the state is $x_1 = 8,\ x_2 = 3,\ x_3 = 5, \ldots, x_7 = 8$. The statistical description is $y_1 = y_2 = y_4 = y_6 = y_7 = y_9 = 0,\ y_3 = y_{10} = 1,\ y_5 = 2,\ y_8 = 3$ and the wealth distribution in this case is $z_0 = 6,\ z_1 = 2,\ z_2 = 1,\ z_3 = 1,\ z_4 = z_5 = z_6 = z_7 = 0$.

2. Simple exchange games

2.1. Random coin exchange (Bennati-Drăgulescu-Yakovenko)

Bennati-Drăgulescu-Yakovenko (BDY) model was introduced in Bennati's work (1988, 1993) and rediscovered in [2]. Later it was studied in [7]. It is a discrete model, where number of agents and wealth measured by coins are conserved. The BDY game is played as follows. In the system of g agents sharing n coins, at each time step, two agents are randomly selected. The selection is such that each pair of agents has equal probability to be chosen. One of the agents (randomly chosen) becomes the loser and gives one coin to the other, who becomes the winner. Indebtedness is not possible, i.e. if the loser has zero coins, the move is not taken into account and a new pair of players is selected. In order to avoid null moves, the game can be formulated in the following way – a loser is chosen randomly from the agents who have at least one coin and the winner is chosen among all agents, randomly as well. In case the loser and the winner coincide, there will be no change in the state of the system.

The appropriate description of the system is the statistical description, in terms of agents. Let assume that at a given time t, the agents are described by the state $\mathbf{Y}_t = (n_1, ..., n_g) := \mathbf{n}$ and at the next step by the $\mathbf{Y}_{t+1} = (n_1, ..., n_i - 1, ..., n_j + 1, .., n_g) := \mathbf{n}_i^j$, that corresponds to a loss of the agent i and a win of the agent j.

The transition between these states follows a homogenous Markov dynamics with transition probability:

$$\mathbb{P}(\mathbf{n}_i^j|\mathbf{n}) = \frac{1-\delta_{n_i,0}}{g - z_0(\mathbf{n})}\frac{1}{g}, \tag{1}$$

where $\delta_{n_i,0}$ is the usual Kronecker's delta equal to 1 for $n_i = 0$ and zero otherwise. The first part is the probability of selecting a loser (agent i) from all agents with at least one coin and the second part $(1/g)$ is the probability that the agent j is the winner. The sequence $\mathbf{Y}_0, \mathbf{Y}_1, \ldots$ is a finite Markov chain with irreducible set of states and no periodicity. Therefore, an invariant probability distribution exists and it coincides with the equilibrium distribution. Its form is the following:

$$\pi(\mathbf{n}) = C \cdot (g - z_0(\mathbf{n})), \quad C = \left[\sum_{k=1}^{g} \binom{g}{k}\binom{n-1}{n-k}\right]^{-1} \tag{2}$$

and it can be derived by means of detailed balance given that the chain is reversible. The exact solution of this problem is not as simple as it appears in [2]. One can see that the invariant probability distribution for this model depends on number of agents with zero coins (z_0); more precisely it is proportional to the number of agents with at least one coin in their pocket, and hence, it is not uniform.

At the beginning of each simulation in this paper n/g coins are given to the each agent, i.e. the initial wealth distribution is a Dirac delta, $\delta(n/g - i)$ for $i = 0, 1, \ldots,$. Results of simulations present expected wealth distributions, given in terms of partition vector $\mathbf{Z} = (Z_0, Z_1, \ldots, Z_n)$, namely time means of relative frequencies of agents with i coins, that approximate $\mathbb{E}(Z_i)/g$.

Figure 2 shows the expected wealth distribution in the system with dynamics given by the BDY game. Time means of relative frequencies of agents with $0, 1, \ldots, 500$ coins obtained from simulations are compared with theoretical expected wealth distribution given by exact formulas and with exponential distribution, which is the distribution in the limit of large density and large number of agents $(n \gg g \gg 1)$. A detailed derivation of the expected wealth distribution can be found in [7] and in general, it is not exponential – it becomes exponential only in the appropriate limit. Therefore, conclusions from [2] on exponential wealth distribution are not fully correct if one considers equation (2).

2.2. Taxation and redistribution

The second exchange game mimics taxation and redistribution in a simplified way. The taxation-redistribution model was introduced in [4]. There are still n coins to be allocated among g agents and n and g are conserved in time.

The simplest form of this game consists in taking a coin from one agent and redistributing it to another agent. Taxation is represented by a step where a coin is taken from an agent and temporarily removed from the population, so the state of the system is

$$\mathbf{n}_i := (n_1, ..., n_i - 1, ..., n_g).$$

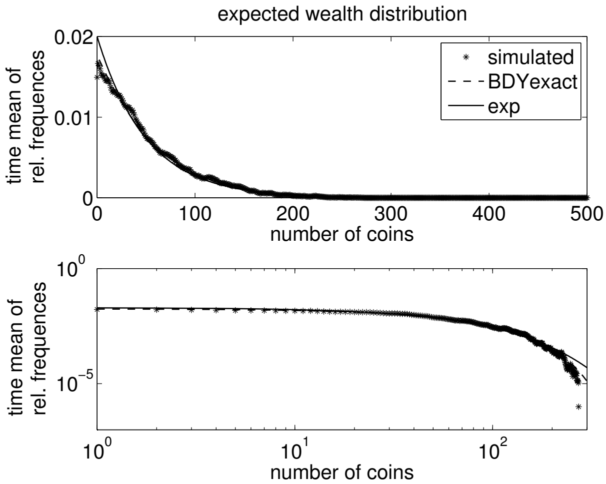


Figure 2: Time mean of relative frequencies of $g = 10$ agents sharing $n = 500$ coins obtained from simulation of the BDY game (stars) compared with theoretical values for expected wealth distribution (dashed line) and with distribution in the limit of large systems, $\text{Exp}(g/n)$ (line), shown in linear (up) and logarithmic scale (down). The values of the random variables $\mathbf{Z}_t$ were sampled and averaged over 10^5 of Monte Carlo steps, after an equilibration of 10^4 steps.

By redistribution, one means a step where a coin is given back to an agent, i.e.

$$\mathbf{n}^j := (n_1, ..., n_j + 1, ..., n_g).$$

If the system is in the state $\mathbf{Y}_t = (n_1, ..., n_g) := \mathbf{n}$, at the next step of this game, possible values of $\mathbf{Y}_{t+1}$ will be $\mathbf{Y}_{t+1} = (n_1, ..., n_i - 1, ..., n_j + 1, .., n_g) := \mathbf{n}_i^j$, corresponding to a loss of the ith agent due to taxation and a gain of the jth agent due to redistribution. The transition probability between these states is:

$$\mathbb{P}(\mathbf{n}_i^j|\mathbf{n}) = \frac{n_i}{n}\frac{\alpha_j + n_j - \delta_{i,j}}{\theta + n - 1}, \tag{3}$$

where $(\alpha_1, ..., \alpha_g)$ are weights for redistribution and $\theta = \sum_j \alpha_j$. Taxation consists of random selection of a coin, in opposition to the BDY move where the selection refers to agents. If a coin is randomly selected out of n coins, the probability of selecting a coin belonging to the agent i is n_i/n, where n_i is the number of coins of the agent i. Hence, the agents are taxed proportionally to their wealth. Then, this coin is redistributed to the agents following the rule that the jth agent will receive a coin with probability proportional to $\alpha_j + n_j$, where n_j is the number of coins of the agent j and α_j is a suitable weight. The redistribution policy is determined by the values of α_j. Positive values make rich agents richer, and the effect is larger the smaller is α_j, $\alpha_j \to \infty$ determines a redistribution where all agents are equivalent, so the redistribution mechanism becomes random, whereas negative values of α_j tend to favor poor agents.

Equation (3) defines the transition probability matrix of an irreducible Markov chain which is also aperiodic. Hence, there exists an invariant probability distribution, which coincides with the equilibrium distribution. In this case it is the Pólya distribution:

$$\pi(\mathbf{n}) = \frac{n!}{\theta^{[n]}} \prod_{i=1}^{g} \frac{\alpha_i^{[n_i]}}{n_i!} \tag{4}$$

and it can be derived by means of detailed balance given that the chain is reversible. $x^{[n]}$ is the Pochhammer symbol representing the rising (or upper) factorial defined by $x^{[n]} = x(x+1)...(x+n-1)$.

Let us suppose that $\alpha_j = \alpha$ for all j. Depending on the choice of α, one can obtain different equilibrium situations. Marginalizing equation (4) for a single agent (all the agents follow the same probability distribution) in the continuous limit of large systems, for α positive, the taxation and redistribution model is approximately described by the gamma distribution (see [3], Chapter 5), whose form factor is just the initial redistribution weight. If α is negative, then the limiting distribution is the hypergeometric distribution and in the case $\alpha \to \infty$ it is the Poisson distribution. Note that in the case of equidistributed agents, one has that $\mathbb{E}(Z_i) = g\mathbb{P}(Y_1 = i)$; in other words, the knowledge of the marginal occupation distribution immediately gives the expected wealth distribution.

Instead of taxation and redistribution of only one coin at each time step, we will consider *block taxation* in which $m \leq n$ coins are randomly taken from agents and then redistributed according to the same probability, i.e. proportionally to the actual wealth of agents and the chosen weight for redistribution. Block taxation can be written in the following way

$$\mathbf{n}' = \mathbf{n} - \mathbf{m} + \mathbf{m}', \tag{5}$$

where $\mathbf{n} = (n_1, \ldots, n_g)$ is the initial agent description, $\mathbf{m} = (m_1, \ldots, m_g)$ is the taxation vector and $\mathbf{m}' = (m'_1, \ldots, m'_g)$ is the redistribution vector, with the constraints $\sum_{i=1}^{g} m_i = m$ and $\sum_{j=1}^{g} m'_j = m$. This leads to the same equilibrium distribution, given by the equation (4).

Figure 3 presents the expected wealth distribution in the system with dynamics governed by the taxation and redistribution game. It shows time mean of relative frequencies of agents with $0, 1, \ldots, 500$ coins obtained from simulations, compared with theoretical values for the expected wealth distribution and with gamma distribution which is the distribution in the continuous limit of large systems.

2.3. Zipf-Simon-Yule

The third important game is the Zipf-Simon-Yule one, initially described in [5]. In order to consider a system that is conservative, in terms of total wealth and number of agents, as in previous two games, we will modify the given Zipf-Simon-Yule model. This game includes a failure probability, which is independent of agents' wealth. An

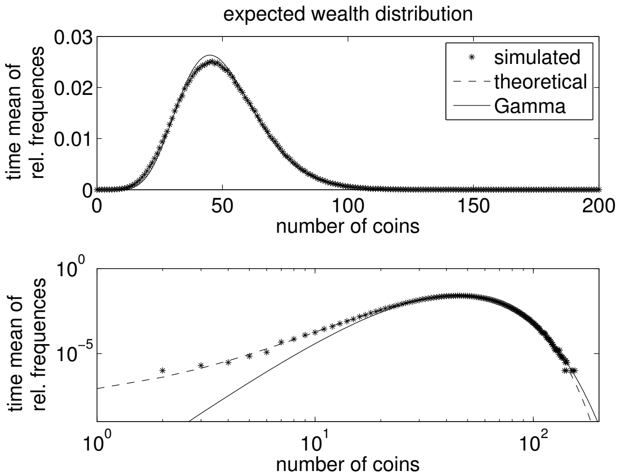


Figure 3: Expected wealth distribution in a system of $g=10$ agents and $n=500$ coins for the 250-block taxation and redistribution game with redistribution weight $\alpha = 10$. Time mean of relative frequencies of agents with i coins obtained from simulation (stars) are compared with theoretical expected wealth distribution (dashed line) and with distribution in the limit of large systems, Gamma$(\alpha, n/\alpha g)$ (line), in linear (up) and logarithmic scale (down). Values from simulation were sampled and averaged over 10^5 Monte Carlo steps, after an equilibration of 10^4 steps.

agent with coins is randomly selected and all his coins are removed. Therefore, the probability of failure for the ith agent is:

$$\mathbb{P}(\mathbf{n}_{(i)}|\mathbf{n}) = \frac{1}{g - z_0(\mathbf{n})}\mathbf{1}_{\{n_i>0\}}, \tag{6}$$

where $\mathbf{n}_{(i)} = (n_1, \ldots, 0, \ldots, n_g)$ is an agent description vector with a zero element on the ith position. The wealth of the failed agent is then redistributed to the agents with probability proportional to their actual wealth, but the last coin is given back to the failed agent. This move can be regarded as a sort of "compassionate capitalism". This is a trick to avoid absorbing states; without this move a failed agent should be cancelled out forever, and after g moves the process would stop. If we assume that the ith agent had m coins that are removed, then the probabilities for redistributing each of those m coins are the following:

$$\begin{cases} \mathbb{P}(X_1 = j|\mathbf{n}_{(i)}) = \frac{n_j}{n-m}\,, \\ \vdots \\ \mathbb{P}(X_{s+1} = j|\mathbf{n}_{(i)}, j_1, \ldots, j_s) = \frac{n_j^{'}}{n-m+s}\,, \\ \vdots \\ \mathbb{P}(X_m = j|\mathbf{n}_{(i)}, j_1, \ldots, j_{m-1}) = \delta_{j,i}\,. \end{cases} \tag{7}$$

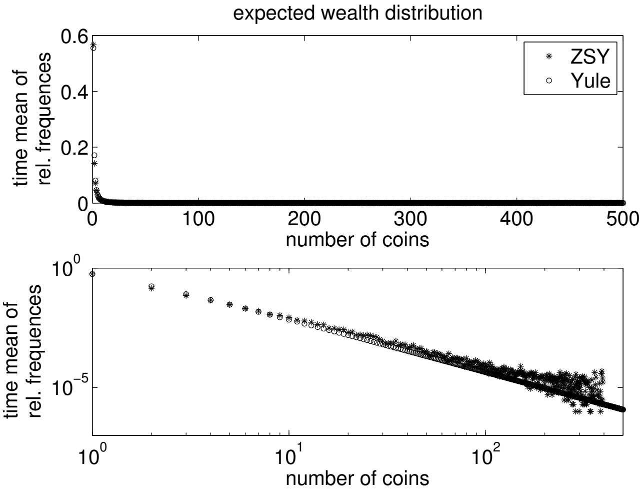


Figure 4: Expected wealth distribution in a system of $g = 100$ agents sharing $n = 500$ coins, shown in linear (up) and logarithmic scale (down). Stars represent the time mean of relative frequencies of agents with i coins obtained from simulation of the ZSY game, sampled and averaged over 10^4 Monte Carlo steps, after an equilibration of 10^4 steps. Circles represent the Yule distribution with parameter $\rho \approx 1$.

The sequence $\mathbf{Y}_0, \mathbf{Y}_1, \ldots, \mathbf{Y}_n, \ldots$ is a finite Markov chain with an irreducible set of states and no periodicity. Therefore, the invariant and equilibrium distribution exists, but it is not easy to find, because the chain is irreversible, and one cannot apply detailed balance.

The described mechanism alone produces power-law tails. Figure 4 presents the expected wealth distribution obtained from simulation of this game and fitted with a Yule distribution, which is the discrete counterpart of the Pareto distribution and has the following form

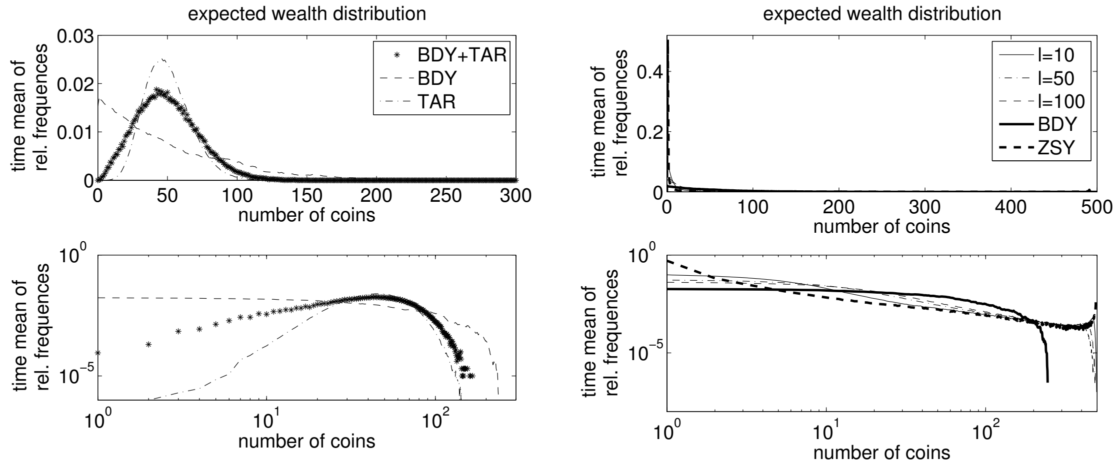
$$\mathbb{P}(U = i) = \rho B(i, \rho + 1) \quad \text{for } i \in \mathbb{N}, \rho \in \mathbb{R}_+ ,$$

where U denotes a random variable and B is the Beta function.

3. "Super-moves"

In the case of the first two models (BDY and TAR), computer simulations are not really necessary because the equilibrium distributions can be analytically derived. On the contrary, they are necessary in the case of the Zipf-Simon-Yule model. However they are necessary if one thinks of a system where two or three games are played sequentially.

In order to know what is the shape of the expected wealth distribution in the equilibrium state in the long-term limit of an economy, we perform Markov chain Monte Carlo simulations for various combinations of three games described in previous section.



(a) (b)

Figure 5: Expected wealth distribution in a system of 10 agents sharing 500 coins for the mixture of the two games. (a) One step consists of $l = 50$ steps of BDY game and one step of 250-block TAR game with redistribution weight $\alpha = 10$. (b) One step consists of one step of ZSY game and $l = 10$, $l = 50$, $l = 100$ steps of BDY game. Time mean of relative frequencies of agents with i coins obtained from simulations of two games are compared with distribution from pure BDY and TAR games (a) and BDY and ZSY games (b). Values from simulation were sampled and averaged over 10^4 Monte Carlo steps, after an equilibration of 10^4 steps.

Let us first consider an alternation of the BDY and TAR game. Suppose that the BDY game is played l times and then one step of the TAR game is performed. The first game shifts the initial expected wealth distribution towards an exponential, and such a distribution becomes the initial one for the second game. Then, the second game shifts the distribution towards a gamma and these steps can be iterated many times. One can guess that the equilibrium distribution of the joint process will be a mixture of the two pure ones, with weights proportional to the frequency of the two processes. This conjecture is qualitatively corroborated in figure 5 for alternating the BDY and TAR games (a) and also for alternating ZSY and BDY games (b), where the resulting expected wealth distribution is compared with distributions from pure games.

In order to mimic what happens in a real economy, we will use the following combination of steps. On each "day" we run a move of BDY, at the end of each "month=l days" we have a ZSY failure, at the end of each "year=k months" we run a TAR, i.e. taxation and redistribution of the coins. After the failure of an agent, modeled by the ZSY game, we have l iterations of coin exchanges between agents, a process giving to the failed agent the opportunity to recover some wealth; in this way the "compassionate capitalism" mechanism can be avoided. These random coin exchanges give the background noise of the economy. Once per year taxation and

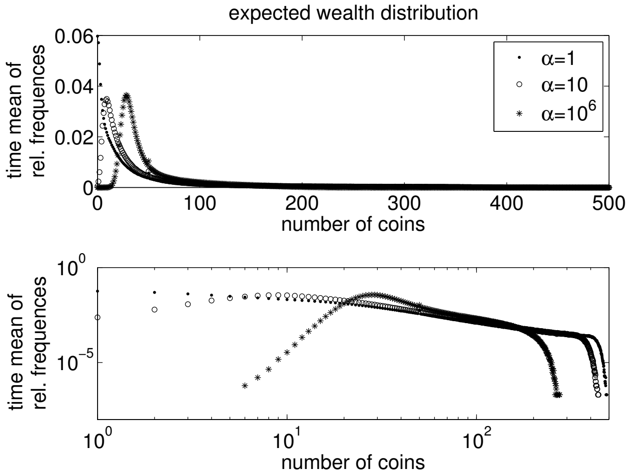


Figure 6: Expected wealth distribution in the system for the mixture of the three games described by the equation (8) with $l = 100$ and $k = 10$. Time mean of relative frequencies of 10 agents sharing 500 coins sampled and averaged over 10^3 realizations of 500 Monte Carlo steps and different redistribution parameter $\alpha = 1$ (dots), $\alpha = 10$ (circles) and $\alpha = 10^6$ (stars).

redistribution of the coins appear as centralized mechanisms depending on weights which characterize the redistribution policy. In summary, the "super-move" for the mixture of the three games at time step t (the last "day" of each "year") can be presented as

$$\mathbf{P}_t = (\mathbb{P}(Y_{1,t} = \cdot), \dots, \mathbb{P}(Y_{g,t} = \cdot)) = \mathbf{P}_{t-1} \cdot ((BDY^l \cdot ZSY)^k \cdot TAR),\ t = 1, 2, ... \quad (8)$$

and $(BDY^l \cdot ZSY)^k \cdot TAR$ is the stochastic matrix for an irreducible, aperiodic Markov chain representing the described alternation of the three games.

Figure 6 shows the resulting expected wealth distribution in the system with dynamics given by the equation (8). Details of the simulations can be found in the caption.

4. Summary and outlook

In this paper we proposed a representation of a conservative economic system, where total wealth and number of agents do not change, using the interplay among three games, previously described separately. The exchange dynamics in the system is composed of pair-wise interactions between economic agents, a mechanism for occasional failures of agents including redistribution of their wealth and a centralized mechanism, which presents redistribution policy. Depending on the relative strength of these mechanisms, the nature of the interplay between them, the specification of redistribution, various outcomes are possible.

The presented model is general enough to be applied to the description of both aggregate wealth distribution, and to the distribution of firm sizes. It can be extended in several directions. One can take into account a heterogeneous population of agents and investigate the presence of asymmetry in the initial endowments on the long run dynamics of the model. This case becomes relevant when one wants to describe the aggregate effect of a policy switch between different redistributive regimes. Another extension could include saving propensity and analyze the resulting distributional properties. Even random failures can be easily taken into account. Future work will investigate the correspondence of the model with real data. There is only one probabilistic parameter, namely α, the weight of the redistribution policy, to be estimated. Other parameters, such as l and k are to be considered fully phenomenological.

Acknowledgements

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