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NUMERICAL SIMULATION OF INTERACTION OF FLUIDS AND SOLID BODIES*

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1. Introduction

In this work we focus on the numerical simulation of an aeroelastic problem. We consider two–dimensional viscous incompressible flow around an airfoil with two degrees of freedom. It means that the airfoil can oscillate in the vertical direction and rotate around an elastic axis.

The mathematical model of flow is represented by the Navier–Stokes equations and the continuity equation. The initial condition and mixed boundary conditions are added to this system. The numerical simulation consists of the finite element solution of the Navier–Stokes equations coupled with the system of the ordinary differential equations, which describes the airfoil motion.

Since the computational domain is time dependent and the grid is moving, we use the Arbitrary-Lagrangian-Eulerian (ALE) formulation of the Navier–Stokes equations [7]. High Reynolds numbers (10^5–10^6) require the application of a turbulent model.

2. Formulation of the problem

We assume that $(0, T)$ is a time interval and by Ω_t we denote a computational domain occupied by the fluid at time t. The boundary $\partial\Omega_t$ consists of disjoint parts $\Gamma_D, \Gamma_O, \Gamma_{W_t}$, where Γ_D represents the inlet and inpermeable fixed walls, Γ_O the outlet and Γ_{W_t} is the boundary of the airfoil at time t. The fluid flow is characterised by the velocity $\boldsymbol{u} = \boldsymbol{u}(\boldsymbol{x}, t) = (u_1(\boldsymbol{x}, t), u_2(\boldsymbol{x}, t))$ and the kinematic pressure $p = p(\boldsymbol{x}, t)$. By ρ we denote the fluid density. The ALE method is based on the ALE mapping of the reference domain $\Omega_{ref} = \Omega_0$ onto the current domain Ω_t:

$$\boldsymbol{A}_t : \Omega_{ref} \mapsto \Omega_t, \quad \boldsymbol{X} \mapsto \boldsymbol{x}(\boldsymbol{X}, t) = \boldsymbol{A}_t(\boldsymbol{X})\,. \tag{1}$$

By $\boldsymbol{w}$ we denote the domain velocity: $\boldsymbol{w} = \frac{\partial}{\partial t}\boldsymbol{x}(\boldsymbol{X}, t)$. In the domain Ω_t we consider the Navier–Stokes system written in the following ALE form

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$$\frac{D^A}{Dt}\boldsymbol{u} + [(\boldsymbol{u}-\boldsymbol{w})\cdot\nabla]\,\boldsymbol{u} + \nabla p - \nu\Delta\boldsymbol{u} = 0 \quad \text{in} \quad \Omega_t, \tag{2}$$

$$\operatorname{div}\boldsymbol{u} = 0 \quad \text{in} \quad \Omega_t, \tag{3}$$

equipped with the initial condition

$$\boldsymbol{u}(\boldsymbol{x},0) = \boldsymbol{u}_0, \quad \boldsymbol{x}\in\Omega_0, \tag{4}$$

and the boundary conditions

$$\text{a) } \boldsymbol{u}|_{\Gamma_D} = \boldsymbol{u}_D, \qquad \text{b) } \boldsymbol{u}|_{\Gamma_{W_t}} = \tilde{\boldsymbol{u}}_\Gamma = \boldsymbol{w}|_{\Gamma_{W_t}}, \tag{5}$$

$$\text{c) } -(p-p_{ref})\,\boldsymbol{n} + \nu\frac{\partial\boldsymbol{u}}{\partial\boldsymbol{n}} = 0 \quad \text{on} \quad \Gamma_O.$$

The vertical displacement H and rotation α of the airfoil are described by the system [7]

$$m\ddot{H} + S_\alpha\ddot{\alpha}\cos\alpha + k_{HH}H + d_{HH}\dot{H} - S_\alpha\dot{\alpha}^2\sin\alpha = -L(t),$$
$$S_\alpha\ddot{H}\cos\alpha + I_\alpha\ddot{\alpha} + k_{\alpha\alpha}\alpha + d_{\alpha\alpha}\dot{\alpha} = M(t), \tag{6}$$

where m denotes the mass of the airfoil, S_α, I_α are the static moment and the inertia moment around the elastic axis, k_{HH}, $k_{\alpha\alpha}$ denote the bending stiffness and the torsional stiffness, d_{HH}, $d_{\alpha\alpha}$ are the structural dampings. The aerodynamic lift force $L(t)$ and the aerodynamic torsional moment $M(t)$ are define by the relations

$$L = -\int_{\Gamma_{Wt}}\sum_{j=1}^{2}\tau_{2j}n_j dS, \quad M = -\int_{\Gamma_{Wt}}\sum_{i,j=1}^{2}\tau_{ij}n_j r_i^{\text{ort}} dS, \tag{7}$$

$$\tau_{ij} = \rho\left[-p\delta_{ij} + \nu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\right], \quad r_1^{\text{ort}} = -(x_2 - x_{EO2}),\ r_2^{\text{ort}} = x_1 - x_{EO1}.$$

These relations determine the interaction between the moving fluid and the airfoil.

3. Discrete problem

Time discretization. We consider a partition $0 = t_0 < t_1 < \cdots < T$, $t_k = k\tau$. On each time level we approximate the solution $\boldsymbol{u}(t_n) \approx \boldsymbol{u}^n$ and $p(t_n) \approx p^n$ and use the second order two step scheme to approximate the ALE derivative. The unknown functions $\boldsymbol{u}^{n+1} : \Omega_{t_{n+1}} \mapsto I\!R^2$ and p^{n+1}: $\Omega_{t_{n+1}} \mapsto I\!R$ satisfy the system

$$\frac{3\boldsymbol{u}^{n+1} - 4\hat{\boldsymbol{u}}^n + \hat{\boldsymbol{u}}^{n-1}}{2\tau} + \Big((\boldsymbol{u}^{n+1} - \boldsymbol{w}^{n+1})\cdot\nabla\Big)\,\boldsymbol{u}^{n+1} + \nabla p^{n+1} - \nu\Delta\boldsymbol{u}^{n+1} = 0,$$

$$\operatorname{div}\boldsymbol{u}^{n+1} = 0, \tag{8}$$

and the boundary conditions (5). The function $\hat{\boldsymbol{u}}^j$ denotes the velocity at time t_j transformed to the domain $\Omega_{t_{n+1}}$.

Space discretization. System (8) is discretized by the finite element metod, based on the weak formulation of our problem: on each time level we want to find the weak solution $U = (\boldsymbol{u}, p) = (\boldsymbol{u}^{n+1}, p^{n+1}) \in W \times Q$, which satisfies

$$a(U, U, V) = f(V), \quad \text{for all}\, V = (\boldsymbol{v}, q) \in X \times Q, \tag{9}$$

and $\boldsymbol{u}$ fulfills the boundary conditions (5), a)–b). Here

$$\begin{aligned}
&W = (H^1(\Omega))^2, \quad X = \{\boldsymbol{v} \in W; \boldsymbol{v}|_{\Gamma_D \cup \Gamma_{W_t}} = 0\}, \quad Q = L^2(\Omega), \qquad (10)\\
&a(U^*, U, V) = \frac{3}{2\tau}\,(\boldsymbol{u}, \boldsymbol{v})_\Omega + \nu\,(\nabla \boldsymbol{u}, \nabla \boldsymbol{v})_\Omega + (((\boldsymbol{u}^* - \boldsymbol{w}^{n+1}) \cdot \nabla)\boldsymbol{u}, \boldsymbol{v})_\Omega \\
&\quad - (p, \nabla \cdot \boldsymbol{v})_\Omega + (\nabla \cdot \boldsymbol{u}, q)_\Omega, \\
&f(V) = \frac{1}{2\tau}(4\hat{\boldsymbol{u}}^n - \hat{\boldsymbol{u}}^{n-1}, \boldsymbol{v})_\Omega - \int_{\Gamma_O} p_{\mathrm{ref}} \boldsymbol{v} \cdot \boldsymbol{n}\, dS, \\
&U = (\boldsymbol{u}, p),\ V = (\boldsymbol{v}, q),\ U^* = (\boldsymbol{u}^*, p).
\end{aligned}$$

(The symbol $(\cdot, \cdot)$ denotes the $L^2(\Omega)$-scalar product.) In order to apply the finite element method, we approximate the spaces W, X, Q by finite dimensional subspaces W_h, X_h, Q_h, which are defined on a triangulation $\mathcal{T}_h$, and we want to find the approximate solution $U_h = (\boldsymbol{u}_h, p_h) \in W_h \times Q_h$ such that

$$a(U_h, U_h, V_h) = f(V_h) \quad \forall V_h \in X_h \times Q_h, \tag{11}$$

and $\boldsymbol{u}_h$ satisfies an approximation of conditions (5), a)–b). In our computations we use

$$Q_h = \{q \in Q \cap C(\bar{\Omega});\, q|_K \in P^1(K), \forall K \in \mathcal{T}_h\},$$

$$W_h = \{\boldsymbol{v} \in W \cap (C(\bar{\Omega}))^2;\, \boldsymbol{v}|_K \in (P^2(K))^2, \forall K \in \mathcal{T}_h\}, \qquad X_h = W_h \cap X.$$

The couple (X_h, Q_h) satisfies the Babuška-Brezzi condition. Because the Reynolds numbers are high, we use a suitable stabilization of the FEM. Here we apply the approach proposed by Lube in [4]. (For more details, see [7].) The solution of the nonlinear discrete problem is realized by the Oseen iterations.

4. Modelling of turbulence

The flow with a sufficiently small Reynolds number Re is laminar, but if Re increases, the flow loses its stability and becomes turbulent. We apply the algebraic turbulent model [5], which is based on the Reynolds averaging leading to the Reynolds averaged Navier-Stokes equations

$$\operatorname{div}\, \overline{\boldsymbol{u}} = 0, \tag{12}$$

$$\frac{\partial \overline{u_i}}{\partial t} + (\overline{\boldsymbol{u}} \cdot \nabla)\, \overline{u_i} + \frac{\partial \overline{p}}{\partial x_i} - \nu \Delta \overline{u_i} - \sum_{j=1}^{2} \frac{\partial R_{ji}}{\partial x_j} = 0, \quad i = 1, 2, \tag{13}$$

for averaged quantities $\overline{\boldsymbol{u}}, \overline{p} + p'$. The components $R_{ji} = -\overline{u_i' u_j'}, \quad i, j = 1, 2$, of the Reynolds stress tensor are expressed by Boussinesq's hypothesis in the form

$$R_{ij} = \nu_T \left(\frac{\partial \overline{u_i}}{\partial x_j} + \frac{\partial \overline{u_j}}{\partial x_i} \right), \tag{14}$$

(see, e.g. [3]). Here ν_T is called the turbulent viscosity. It depends on the coordinates, velocity and other variables. To compute ν_T we use two algebraic models designed by Baldwin-Lomax and Rostand [5].

System (12), (13) and (14) is again rewritten in the ALE form and discretized similarly as in Section 3 with the only difference in the definition of the form $a(U^*, U, V)$. The details are contained in [2].

5. Numerical results

5.1. Flow along a flat plate

In order to validate the proposed technique, we compare our numerical results of the simulation of flow along a flat plate with the theory of turbulent flow [6], using the Baldwin-Lomax model and the Rostand model. Let us define the function Y^+ and u^+

$$Y^+(Y) = \frac{u_\tau Y}{\nu}, \qquad u^+ = \frac{\mathcal{U}_\infty}{u_\tau},$$

where Y is the distance from the plate, $\mathcal{U}_\infty$ is the far field velocity and u_τ is the wall-shear velocity.

Figure 1 (left) shows the comparison of the numerical results with theory. Figure 1 (right) shows the comparison of theoretical dependence of the friction coefficient C_f on the local Reynolds number $Re_x = U_\infty x_1/\nu$ with our computations. The agreement of the computation with theory is very good.

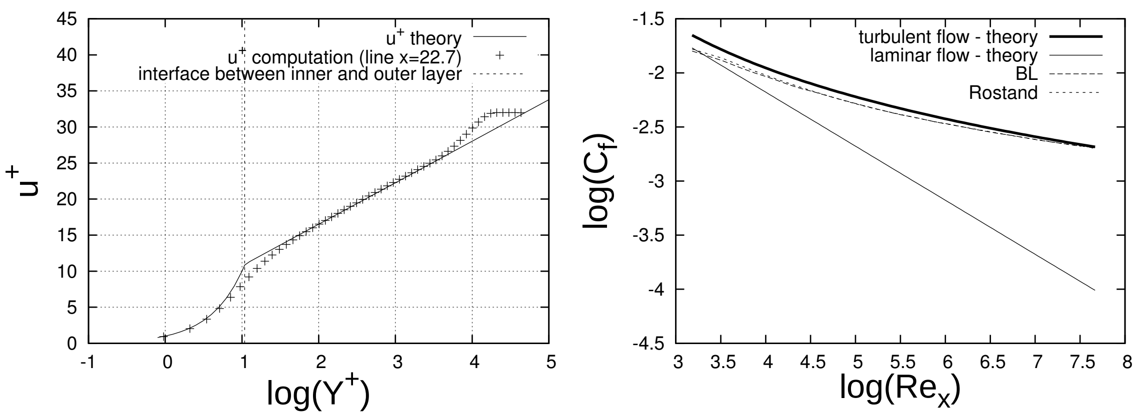

Fig. 1: *The function u^+ in dependence on Y^+ (left); The friction coefficient (right).*

5.2. Flow along the airfoil NACA 0012

Now let us consider flow past the airfoil NACA 0012, which oscillates around the elastic axis (25% of the length of the airfoil) with prescribed frequence $f = 30\,\mathrm{Hz}$ and total amplitude $\alpha^* = 5°$. We compute the pressure coefficient

$$C_p = \frac{P}{\frac{1}{2}\,\rho\,\mathcal{U}_\infty^2},$$

and evaluate $C_{p_{\mathrm{mean}}}$, the time mean value of $C_p(t)$ and the so-called real and imaginary components of the amplitudes C_p' and C_p'' from the relation

$$C_p(t) = C_{p_{\mathrm{mean}}} + C_p' \sin(\omega t) + C_p'' \cos(\omega t).$$

In Figures 2 and 3, there is the comparison of the numerical results with experiments [1]. Although the algebraic model of turbulence is very simple, it gives good results.

Finally, the coupled problem of flow induced airfoil vibrations is solved using the finite element method for the flow problem, combined with the Runge-Kutta method

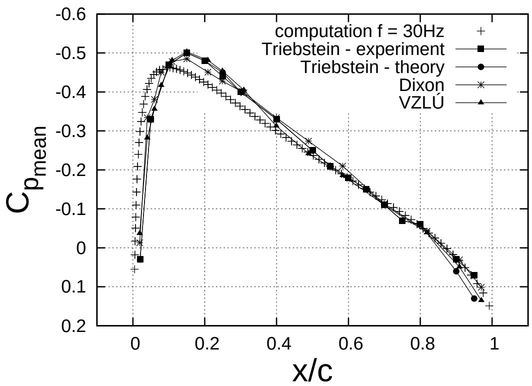

Fig. 2: *The mean value of $C_p(t)$.*

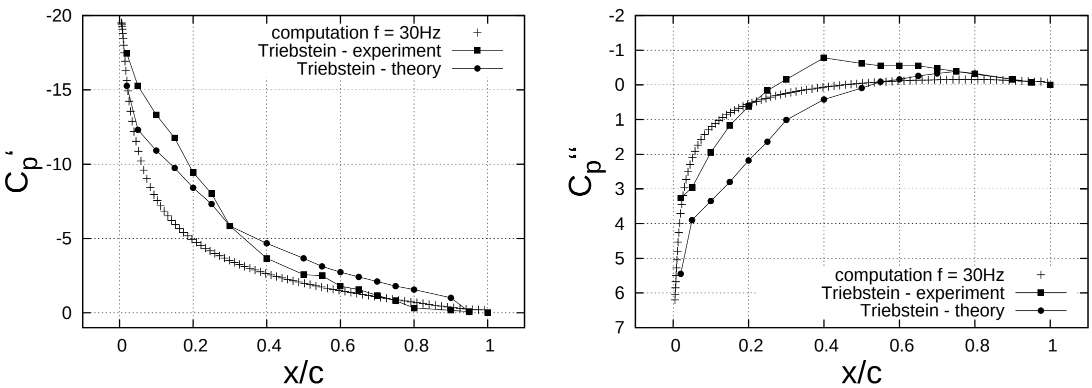

Fig. 3: *The real and imaginary components of the amplitudes.*

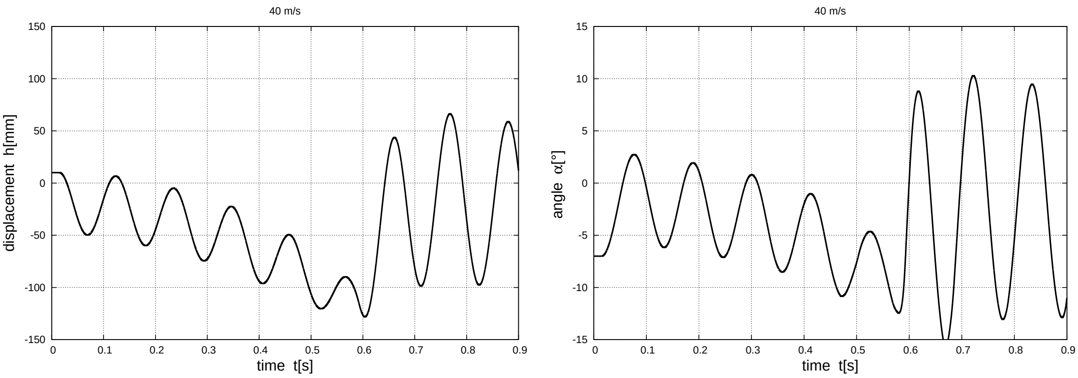

Fig. 4: *Flow induced airfoil vibrations for* $\mathcal{U}_\infty = 40\,m/s$

for system (6) transformed to a first order system. In Figure 4, the displacement H and rotation angle α are plotted in dependence on time for the far field velocity $\mathcal{U}_\infty = 40\,\mathrm{m/s}$. In this case the vibrations are not damped and we get the regime called flutter.

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