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INTEGRAL REPRESENTATION OF FUNCTIONS AND IMBEDDING THEOREMS FOR DOMAINS WITH THE FLEXIBLE HORN PROPERTY

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Let $\lambda = (\lambda_1,\dots,\lambda_n) \in (0,\infty)^n$. A domain $G \subset \mathbb{R}^n$ will be said to have *the flexible λ-horn property* (*the flexible cone property* if $\lambda_1 = \dots = \lambda_n$) if, for some $\delta > 0$, $T > 0$ and for any $x \in G$, there exists a curve $\rho(t^\lambda) = \rho(t^\lambda, x) \overset{\text{def}}{=} (\rho_1(t^{\lambda_1}),\dots,\rho_n(t^{\lambda_n}))$, $0 \leq t \leq T$, possessing the following properties:

a) $\rho_i(u)$ are absolutely continuous on $[0,T^{\lambda_i}]$; $|\rho_i'(u)| \leq 1$ for a.a. $u \in [0,T^{\lambda_i}]$;

b) $\rho(0,x) = 0$, $x + \bigcup_{0<t\leq T} [\rho(t^\lambda,x) + t^\lambda \delta^\lambda (-1,1)^n] \subset G$.

The concept of a domain with the flexible cone property is more general than that of a domain with the cone property, with the F. John property, and of an (ε,δ)-domain.

We get an integral representation of functions in terms of their derivatives and differences. On this basis imbeddings of anisotropic Sobolev spaces $W^{\ell}_{p^0;p^1,\dots,p^n}(G) \subsetneq L_q(G)$ are established, as well as estimates for L_q-moduli of continuity of functions, leading to imbeddings of spaces defined via differences.

A necessary condition is obtained for the Fourier multipliers from $L_p(\mathbb{R}^n)$ into $L_p(\mathbb{R}^n)$, $1 < p < \infty$. This generalizes the Hörmander criterion, relaxing the requirement on the smoothness in L_q from $1 + [\frac{n}{2}]$ to $\frac{n}{2}$.

References

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ON STABILIZATION OF FUNCTIONS AND FREE BOUNDARY VARIATIONAL PROBLEMS ON UNBOUNDED INTERVALS

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We consider the class of functions $u : (1,\infty) \to \mathbb{R}$ which stabilize to polynomials $P(t;u) = \sum_{m=0}^{r-1} a_m t^m$ ($r \in \mathbb{N}$ is fixed) as $t \to +\infty$. For functions from this class the inequality

$$|u^{(s)}(t)| \leq c\Big(\sum_{\mu=1}^{k} |u^{(i_\mu)}(1)| + \sum_{\nu=1}^{\ell} |a_{j_\nu}| + ||\phi u||_{L_p(1,+\infty)}\Big) ,$$
$$1 \leq p \leq +\infty , \quad j = 0,1,\dots,r-1 , \quad t \in (1,+\infty) ,$$

is established where ϕ is a given function (a weight), $t^{\alpha}\phi^{-1} \in L_q(1,+\infty)$, $\alpha > r - 1$, $1/p + 1/q = 1$, $k + \ell \geq r$; $\{i_\mu\}_{\mu=1}^{\mu=k}$ and $\{j_\nu\}_{\nu=1}^{\nu=\ell}$ are admissible sets of indices $i, j \in \overline{0, r-1}$, connected with the Pólya problem [1], a_{j_ν} are the coefficients of the polynomial $P(t;u)$, the constant $c > 0$ is independent of the function u [2,3].

In the case $p = 2$ we prove existence and uniqueness of a function minimizing the corresponding quadratic functional in the class considered, $u^{(i_\mu)}(1)$, $\mu = 1,\dots,k$, and a_{j_ν}, $\nu = 1,\dots,\ell$, being fixed.

The conditions are explained which are satisfied by the solution to this problem with arbitrary values of i and j at the ends of the interval $(1,+\infty)$.