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ON 4-PLANAR MAPPINGS OF SPECIAL ALMOST ANTIQUATERNIONIC SPACES

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ABSTRACT. In the paper special 4-planar mappings of almost antiquaternionic Hermitian spaces are studied. Fundamental equations of these mappings are expressed in linear Cauchy form.

The 4-quasiplanar mappings of an almost quaternionic space have been studied in [5], [9] and [14]. These mappings generalize the geodesic, quasigeodesic and holomorphically projective mappings of Riemannian and Kählerian spaces, see [4], [12], [13], [15], [17], [18]. Similar problems are studied on complex manifolds in [2]. Antiquaternionic spaces which were studied e.g. in [11], [16] have some properties similar to those of quaternions [1]. This fact can be used in the study of 4-planar mappings of almost antiquaternionic spaces.

I. N. Kurbatova studied a special kind of 4-planar mappings (called 4-quasiplanar, see [9]) from a Riemannian space V_n onto another Riemannian space $\bar{V}_n$ where an almost quaternionic structure on V_n is Hermitian and it satisfies additional conditions so that V_n a $\bar{V}_n$ are Apt spaces.

Analyzing the results of [9] (theorems 2 – 6) we noticed that the space $\bar{V}_n$ is implicitly supposed to be Hermitian and this assumption is essential. The Hermitian structure of $\bar{V}_n$ is more important than the Hermitian structure of V_n and, moreover, it simplifies fundamental equations of 4-planar mappings. In this paper we do not assume V_n to be Hermitian.

1. A well-known definition says that an *almost antiquaternionic* space is a differentiable manifold M_n with almost product structures $\overset{1}{F}$ and $\overset{2}{F}$ satisfying

$$\overset{1}{F}{}^h_\alpha \overset{1}{F}{}^\alpha_i = \delta^h_i\,; \quad \overset{2}{F}{}^h_\alpha \overset{2}{F}{}^\alpha_i = \delta^h_i\,; \quad \overset{1}{F}{}^h_\alpha \overset{2}{F}{}^\alpha_i + \overset{2}{F}{}^h_\alpha \overset{1}{F}{}^\alpha_i = 0\,, \tag{1}$$

where δ^h_i is the Kronecker symbol, see e.g. [3], [14], [16].

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The tensor $\overset{3}{F}{}_i^h \equiv \overset{1}{F}{}_i^\alpha \overset{2}{F}{}_\alpha^h$ defines an almost complex structure. The relations among the tensors $\overset{1}{F}, \overset{2}{F}, \overset{3}{F}$ are the following

$$\overset{1}{F}{}_i^h = -\overset{2}{F}{}_i^\alpha \overset{3}{F}{}_\alpha^h = \overset{3}{F}{}_i^\alpha \overset{2}{F}{}_\alpha^h; \quad \overset{2}{F}{}_i^h = -\overset{3}{F}{}_i^\alpha \overset{1}{F}{}_\alpha^h = \overset{1}{F}{}_i^\alpha \overset{3}{F}{}_\alpha^h; \quad \overset{3}{F}{}_i^h = \overset{1}{F}{}_i^\alpha \overset{2}{F}{}_\alpha^h = -\overset{2}{F}{}_i^\alpha \overset{1}{F}{}_\alpha^h, \tag{2}$$

i.e. that the three structures $\overset{1}{F}, \overset{2}{F}, \overset{3}{F}$ define an *almost antiquaternionic structure.*

Let $A_n \equiv (M_n, \Gamma, \overset{1}{F}, \overset{2}{F}, \overset{3}{F})$ be an almost antiquaternionic space with a torsion-free affine connection Γ.

Definition 1. A curve ℓ: $x^h = x^h(t)$ in A_n is called *4-planar,* if the tangent vector $\lambda^h = dx^h/dt$, being parallely transported along this curve, remains in the linear 4-dimensional space generated by the tangent vector λ^h and the corresponding vectors $\overset{1}{F}{}_\alpha^h \lambda^\alpha$, $\overset{2}{F}{}_\alpha^h \lambda^\alpha$ and $\overset{3}{F}{}_\alpha^h \lambda^\alpha$.

A curve is 4-planar if and only if the equations

$$\frac{d\lambda^h}{dt} + \Gamma_{\alpha\beta}^h \lambda^\alpha \lambda^\beta = \sum_{s=0}^{3} \underset{s}{\rho} \overset{s}{F}{}_\alpha^h \lambda^\alpha$$

hold, where $\overset{0}{F}{}_i^h \equiv \delta_i^h$, $\Gamma_{\alpha\beta}^h$ are components of the affine connection on A_n and $\underset{s}{\rho} = \underset{s}{\rho}(t)$ $(s = 0, \dots, 3)$ denote functions of the parameter t.

Any geodesic curve is a special case of a 4-planar curve where $\rho_1 \equiv \rho_2 \equiv \rho_3 \equiv 0$.

Consider two spaces A_n and $\bar{A}_n$ with the same underlying manifold M_n and the same almost antiquaternionic structure $(\overset{1}{F}, \overset{2}{F}, \overset{3}{F})$ but with two different torsion-free affine connections Γ and $\overline{\Gamma}$, respectively.

Definition 2. A diffeomorphism f: $A_n \to \bar{A}_n$ is called a *4-planar mapping,* if it maps any geodesic of A_n to a 4-planar curve of $\bar{A}_n$.

Remark. In the following we shall attach to each local map φ around a point $p \in A_n$ the local map $\varphi \circ f^{-1}$ around the point $f(p) \in \bar{A}_n$. This means that any point $x \in A_n$ and the corresponding point $f(x) \in \bar{A}_n$ will have the same local coordinates.

The following theorem holds [14]:

Theorem 1. *A diffeomorphism of A_n onto $\bar{A}_n$ is a 4-planar mapping if and only if in every local coordinate system $x = (x^1, x^2, \dots, x^n)$ the conditions*

$$\overline{\Gamma}{}_{ij}^h(x) = \Gamma_{ij}^h(x) + \sum_{s=0}^{3} \underset{s}{\psi}{}_{(i} \overset{s}{F}{}_{j)}^h \tag{3}$$

hold, where Γ_{ij}^h and $\overline{\Gamma}{}_{ij}^h$ are components of the affine connections Γ and $\overline{\Gamma}$, respectively, $\underset{s}{\psi}{}_i(x)$, $s = 0, \dots, 3$, are covectors, and $(i\,j)$ denotes a symmetrization of indices.

Using Theorem 1 one can prove that all 4-planar curves of A_n are mapped onto 4-planar curves of $\bar{A}_n$.

Finally, we will consider a special case of $\bar{A}_n$, namely an almost antiquaternionic Riemannian space $\bar{V}_n \equiv (M_n, \bar{g}, \overset{1}{F}, \overset{2}{F}, \overset{3}{F})$ in which $\bar{\Gamma}$ denote the Levi-Civita connection of $\bar{g}$.

The following theorem holds (see [14]).

Theorem 2. *A diffeomorphism $f\colon A_n \to \bar{V}_n$ is a 4-planar mapping if and only if the metric tensor $\bar{g}_{ij}(x)$ satisfies the following equations:*

$$(4)\qquad \bar{g}_{ij,k} = \sum_{s=0}^{3} \left(\underset{s}{\psi}_k\, \bar{g}_{\alpha(i}\, \overset{s}{F}{}^{\alpha}_{j)} + \underset{s}{\psi}_{(i}\, \bar{g}_{j)\alpha}\, \overset{s}{F}{}^{\alpha}_{k} \right),$$

where comma denotes the covariant derivative in A_n.

Recall that the covariant derivative of $\bar{g}$ in $\bar{V}_n$ is zero.

The proof follows from the fact that formulas (3) and (4) are equivalent in our special case.

2. Now we shall prove the following two lemmas.

Consider the spaces A_n, $\bar{A}_n$ and let "," or "|" before an index denote a covariant derivative w.r. to the corresponding local variable on A_n and $\bar{V}_n$, respectively. Now and further we will suppose that the affinors $\overset{s}{F}{}^h_i$ defining the almost antiquaternionic structure are traceless, i.e. $\overset{s}{F}{}^{\alpha}_{\alpha} = 0$, $s = 1, 2, 3$.

Lemma 1. *Let a 4-planar mapping $A_n \to \bar{A}_n$ be given and let $\underset{s}{\psi}_i$ denote the corresponding covectors from (3). Then*

$$(5)\qquad \overset{s}{F}{}^{\alpha}_{i,\alpha} = \overset{s}{F}{}^{\alpha}_{i|\alpha}, \qquad s = 1, 2, 3.$$

holds if and only if the covectors $\underset{s}{\psi}_i$ are expressed by formulas

$$(6)\quad \underset{s}{\psi}_i = \frac{n^2+2n}{n^2-2n+8}\psi_\alpha\, \overset{s}{F}{}^{\alpha}_{i},\ s = 1, 2, \qquad \underset{3}{\psi}_i = -\frac{n^2-6n}{n^2-2n+8}\psi_\alpha\, \overset{3}{F}{}^{\alpha}_{i}, \quad \psi_i \equiv \underset{0}{\psi}_i.$$

The proof of the Lemma 1 is a consequence of (5) and the fundamental equations of 4-planar mappings (3). We use also the algebraic properties (1) and (2) of antiquaternionic structures.

A manifold with an affine connection Γ and an almost complex structure F is said to be an *Apt space* (see [4], [9], or *nearly Kählerian space* or *Tachibana space* [4], [6], [7], [8], [10], [19]), if its structure F satisfies $F^{\alpha}_{i,\alpha} = 0$; a space $A_n = (M_n, \Gamma, \overset{1}{F}, \overset{2}{F}, \overset{3}{F})$ to be an *almost antiquaternionic Apt space*, if

$$\overset{s}{F}{}^{\alpha}_{i,\alpha} = 0, \qquad s = 1, 2, 3.$$

Lemma 1 implies that an Apt space A_n is 4-planarly mapped on an Apt space $\bar{A}_n$ iff (6) holds. Obviously, antiquaternionic Kählerian spaces are Apt spaces.

Contracting (3) with respect to h and j we got the lemma

Lemma 2. *If for a 4-planar mapping $A_n \to \bar{A}_n$ the formulae (6) hold and the spaces A_n and $\bar{A}_n$ are equiaffine, then the vector ψ_i is a gradient, i.e. there exists a function ψ such that $\psi_i = \psi_{,i}$.*

3. Now we shall show that if a 4-planar mapping from A_n onto a Riemannian space $\bar{V}_n$ is given, then the formulae (3) and (4) are both equivalent to the following formula:

$$\bar{g}^{ij}_{,k} = -\sum_{s=0}^{3} \left(\underset{s}{\psi}_k \bar{g}^{\alpha(i} \overset{s}{F}{}^{j)}_{\alpha} + \underset{s}{\psi}_{\alpha} \bar{g}^{\alpha(i} \overset{s}{F}{}^{j)}_{k} \right), \tag{7}$$

where $\bar{g}^{ij}$ is the inverse matrix of metric tensor $\bar{g}_{ij}$. In fact, (7) is a consequence of the identity $\bar{g}^{ij}_{,k} = -\bar{g}_{\alpha\beta,k}\bar{g}^{\alpha i}\bar{g}^{\beta j}$.

In what follows we shall asumme an antiquaternionic structure on $\bar{V}_n$ which is *Hermitian*, i.e. we have

$$\bar{g}_{i\alpha} \overset{s}{F}{}^{\alpha}_{j} + \bar{g}_{j\alpha} \overset{s}{F}{}^{\alpha}_{i} = 0, \qquad s = 1,2,3. \tag{8}$$

(8) is equivalent with

$$\bar{g}^{i\alpha} \overset{s}{F}{}^{j}_{\alpha} + \bar{g}^{j\alpha} \overset{s}{F}{}^{i}_{\alpha} = 0, \qquad s = 1,2,3, \tag{9}$$

or with

$$\bar{g}^{\alpha\beta} \overset{s}{F}{}^{i}_{\alpha} \overset{s}{F}{}^{j}_{\beta} = e_s \bar{g}^{ij}, \quad s = 1,2,3, \quad e_1 = e_2 = -1, \ e_3 = 1. \tag{10}$$

Using (9), the equations of 4-planar mappings are simplified to

$$\bar{g}^{ij}_{,k} = -2\psi_k \bar{g}^{ij} - \sum_{s=0}^{3} \underset{s}{\psi}_{\alpha} \bar{g}^{\alpha(i} \overset{s}{F}{}^{j)}_{k}. \tag{11}$$

Suppose now that the covector ψ_i is a gradient, i.e. $\psi_i \equiv \underset{0}{\psi}_i \equiv \psi_{,i}$ where ψ is a function. We define the tensor

$$a^{ij} \equiv e^{2\psi}\bar{g}^{ij}.$$

Then (11) can we rewritten in the form

$$a^{ij}_{,k} = \sum_{s=0}^{3} \underset{s}{\lambda}{}^{(i} \overset{s}{F}{}^{j)}_{k}, \tag{12}$$

where

$$\underset{s}{\lambda}{}^{i} \equiv - \underset{s}{\psi}_{\alpha}\bar{g}^{\alpha i}. \tag{13}$$

By the definition of the tensor a^{ij} (10) is equivalent with

$$a^{\alpha\beta} \overset{s}{F}{}^{i}_{\alpha} \overset{s}{F}{}^{j}_{\alpha} = e_s a^{ij}, \qquad s = 1,2,3. \tag{14}$$

Due to the fact that $\bar{V}_n$ is Hermitian and using (13) we see that the formula (6) is equivalent with

$$\underset{s}{\lambda}{}^{i} = \frac{n^2+2n}{n^2-2n+8}\lambda^{\alpha} \overset{s}{F}{}^{i}_{\alpha}, \ s = 1,2, \quad \underset{3}{\lambda}{}^{i} = -\frac{n^2-6n}{n^2-2n+8}\lambda^{\alpha} \overset{3}{F}{}^{i}_{\alpha}, \quad \lambda^{i} \equiv \underset{0}{\lambda}{}^{i}. \tag{15}$$

Now we come back to the affine case. Let a space A_n be given as before and let the system of equations (12), (14) and (15) have a solution for a regular matrix function a^{ij} and a vector function λ^i. Then one can prove that the inverse matrix $||\tilde{g}_{ij}|| = ||a^{ij}||^{-1}$ defines a Riemannian metric $\tilde{g}$ on M_n and the covector $\lambda^\alpha \tilde{g}_{\alpha i}$ is a gradient grad ψ. By the conformal change $\bar{g}_{ij} = e^{2\psi}\tilde{g}_{ij}$ we obtain a new metric $\bar{g}$ for which A_n becomes a almost antiquaternionic Hermitian space $\bar{V}_n$. Moreover, there exists a 4-planar mapping $A_n \to \bar{V}_n$.

Now we can conclude the above results with

Theorem 3. *Under the condition (5) an equiaffine space A_n admits a 4-planar mapping on an antiquaternionic Hermitian space $\bar{V}_n$ if and only if there exists a regular tensor a^{ij} on A_n satisfying (12), (14), and (15).*

This result coincides with the result by N.S. Sinyukov for geodesic mappings and the results by V.V. Domashev and J. Mikeš for holomorphically projective mappings of Kählerian spaces etc., see [12], [13], [18].

4. In the following we will analyze the equations (12), (14) and (15). We consider the covariant derivatives of (14) in A_n, i.e.

$$a^{\alpha\beta}_{\ \ ,k}\overset{r}{F}{}^i_\alpha \overset{r}{F}{}^j_\beta + a^{\alpha\beta}\overset{r}{F}{}^i_{\alpha,k}\overset{r}{F}{}^j_\beta + a^{\alpha\beta}\overset{r}{F}{}^i_\alpha \overset{r}{F}{}^j_{\beta,k} = e_r a^{ij}_{\ ,k}\,, \qquad r = 1,2,3.$$

Putting (12) into the above equation we get

$$\sum_{s=0}^{3}\left(e_r \underset{s}{\lambda}{}^{(i}\overset{s}{F}{}^{j)}_k - \underset{s}{\lambda}{}^{\alpha}\overset{r}{F}{}^{(i}_\alpha \overset{r}{F}{}^{j)}_\beta \overset{s}{F}{}^\beta_k\right) = a^{\alpha\beta}\overset{r}{F}{}^{(i}_{\alpha,k}\overset{r}{F}{}^{j)}_\beta\,. \tag{16}$$

For $r = 3$, using (1), (2) and (15) we have

$$\lambda^{(i}\delta^{j)}_k - \lambda^\alpha \overset{3}{F}{}^{(i}_\alpha \overset{3}{F}{}^{j)}_k = \frac{n^2-2n+8}{4(n+2)}\, a^{\alpha\beta}\overset{3}{F}{}^{(i}_{\alpha,k}\overset{3}{F}{}^{j)}_\beta \tag{17}$$

and contracting (17) with respect to j and k we have the following expression of the vector λ^i:

$$\lambda^i = \frac{n^2-2n+8}{4(n+2)^2}\, a^{\alpha\beta}\left(\overset{3}{F}{}^i_{\alpha,\gamma}\overset{3}{F}{}^\gamma_\beta + \overset{3}{F}{}^i_\alpha \overset{3}{F}{}^\gamma_{\beta,\gamma}\right)\,. \tag{18}$$

It implies that λ^i can be expressed as linear functions in a^{ij}. It implies

Theorem 4. *Under the condition (5) an equiaffine space A_n admits a 4-planar mapping onto a Hermitian almost quaternionic space $\bar{V}_n$ if and only if the following system of differential equations of Cauchy type is solvable with respect to the unknown functions a^{ij}:*

$$a^{ij}_{\ ,k} = \sum_{s=0}^{3}\underset{s}{\lambda}{}^{(i}\overset{s}{F}{}^{j)}_k\,, \tag{19}$$

where

$$\lambda^i \equiv \underset{0}{\lambda}{}^i\,, \qquad \underset{s}{\lambda}{}^i = \frac{n^2+2n}{n^2-2n+8}\lambda^\alpha \overset{s}{F}{}^i_\alpha\,,\ s=1,2, \qquad \underset{3}{\lambda}{}^i = -\frac{n^2-6n}{n^2-2n+8}\lambda^\alpha \overset{3}{F}{}^i_\alpha\,,$$

$$\lambda^i = \frac{n^2-2n+8}{4(n+2)^2}\, a^{\alpha\beta}\left(\overset{3}{F}{}^i_{\alpha,\gamma}\overset{3}{F}{}^\gamma_\beta + \overset{3}{F}{}^i_\alpha \overset{3}{F}{}^\gamma_{\beta,\gamma}\right)\,, \tag{20}$$

and the matrix (a^{ij}) *satisfies the algebraic condition*

$$(21) \qquad |a^{ij}| \neq 0 \qquad \text{and} \qquad a^{\alpha\beta} \overset{s}{F}{}^{i}_{\alpha} \overset{s}{F}{}^{j}_{\alpha} = e_s a^{ij}, \quad s = 1,2,3.$$

The system (19) does not have more than one solution for the initial Cauchy conditions $a^{ij}(x_o) = \underset{o}{a}{}^{ij}$ under the conditions (21). Therefore the general solution of (19) does not depend on more than $N_o = (n/2)^2$ parameters. The question of existence of a solution of (19) leads to the studium of integrability conditions, which are linear equations w.r. to the unknowns $a^{ij}(x)$ with coefficients from the space A_n.

Remarks. If the antiquaternionic structure is covariantly constant in A_n, i.e. $\overset{s}{F}{}^{h}_{i,j} = 0$, $s = 1,2,3$, the under the conditions of the Theorem 4, the 4-planar mapping is only affine. This follows from the fact, that by (20) $\lambda^i = 0$, $\underset{s}{\lambda}{}^i = 0$ and by (19) $a^{ij}{}_{,k} = 0$. This is equivalent to $\bar{g}_{ij,k} = 0$, which is the condition for the mapping $A_n \to \bar{V}_n$ to be affine.

References

[1] Alekseevsky D. V., Marchiafava S., *Transformation of a quaternionic Kähler manifold,* C.R. Acad. Sci., Paris, Ser. I, 320, No. 6 (1995), 703–708.

[2] Bailey, T. N., Eastwood, M. G., *Complex Paraconformal Manifolds - their Differential Geometry and Twistor Theory,* Forum Math., 3 (1991), 61–103.

[3] Bejancu A., *Geometry of CR-submanifolds,* Math. and Its Appl., 23, Kluwer Acad. Publ. Group. XII (1986), 169p.

[4] Beklemishev D. V., *Differential geometry of spaces with almost complex structure.* Geometria. Itogi Nauki i Tekhn., All-Union Inst. for Sci. and Techn. Information (VINITI), Akad. Nauk SSSR, Moscow (1965), 165–212.

[5] Bělohlávková J., Mikeš J., Pokorná O., *4-planar mappings of almost quaternionic and almost antiquaternionic spaces,* Proc. of the Third Int. Workshop on Diff. Geometry and its Appl. and the First German - Romannian Seminar on Geometry, Sibiu - Romania, Sept. 18 - 23, 1997. General Mathematics, vol. 5 (1997), 101–108.

[6] Gray A., *The structure of nearly Kaehler manifolds,* Math. Ann. 223 (1976), 233–248.

[7] Gray A., Hervella L. M., *The sixteen classes of almost Hermitian manifolds and their linear invariants,* Ann. Mat. Pura Appl., IV. Ser. 123 (1980), 35–58.

[8] Koto S., *Some theorems on almost Kaehlerian spaces,* J. Math. Soc. Japan 12 (1960), 422-433.

[9] Kurbatova I. N., *4-quasi-planar mappings of almost quaternion manifolds,* Sov. Math. 30, 100–104 (1986); translation from Izv. Vyssh. Uchebn. Zaved., Mat., No. 1 (1986), 75–78.

[10] Libermann P., *Sur la classification des structures presque hermitiennes,* Proc. IV. int. Colloq. Differential geometry, Santiago de Compostela 1978 (1979), 168–191.

[11] Marchiafava S., Nagy P. T., *3-webs and pseudo hypercomplex structures,* (manuscript).

[12] Mikeš J., *Geodesic mappings on affine-connected and Riemannian spaces,* J. Math. Sci., New York, 78, 3 (1996), 311–333.

[13] Mikeš J., *Holomorphically-projective mappings and its generalizations,* J. Math. Sci., New York, 89, 3 (1998), 1334–1353.

[14] Mikeš J., Němčíková J., Pokorná O., *On the theory of the 4-quasiplanar mappings of almost quaternionic spaces,* Proc. of Winter School "Geometry and Physic", Srní, January 1997, Suppl. Rend. Circ. Mat. Palermo, II. Ser. 54 (1998), 75–81.

[15] Mikesh J., Sinyukov N. S., *On quasiplanar mappings of space of affine connection,* Sov. Math. 27, 63–70 (1983); translation from Izv. Vyssh. Uchebn. Zaved., Mat., No. 1 (1983), 55–61.

[16] Nagy P. T., *Invariant tensor field and the canonical connection of a 3-webs,* Aequationes Math., 35 (1988), 31–44.

[17] Petrov A. Z., *Simulation of physical fields,* Grav. i teor. Otnos., Issues 4-5, Kazan State Univ. Press, Kazan (1968), 7-21. (1968).

[18] Sinyukov N. S., *Geodesic mappings on Riemannian spaces,* Nauka, Moscow, 1979.
[19] Tachibana, S.-I., *On automorphisms of certain compact almost-Hermitian spaces,* Tohoku Math. J., II. Ser. 13 (1961), 179–185.

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