Examples of quantum braided groups

LADISLAV HLAVATÝ *
Department of Physics,
Faculty of Nuclear Sciences and Physical Engineering

12 January 1993

Abstract

A new type of algebras that represent a generalization of both quantum groups and braided groups is defined. These algebras are given by a pair of solutions of the Yang–Baxter equation that satisfy some additional conditions. Several examples are presented.

1 Introduction – quantum groups and braided groups

Matrix groups like $GL(n), SO(n)$ e.t.c. were generalized in two ways recently. Both are based on deformation of the algebra of functions on the groups generated by coordinate functions T_i^j that commute

$$T_i^j T_k^l = T_k^l T_i^j \quad \Leftrightarrow \quad T_1 T_2 = T_2 T_1 \tag{1}$$

*Postal address: Břehová 7, 110 00 Prague 1, Czechoslovakia. E-mail HLAVATY@FJFI.CVUT.CS

In the quantum groups [1, 2] these commutation relations are modified by a matrix $R = \{R^{kl}_{ij}\}$ so that the functions do not commute but satisfy the relations

$$R_{12}T_1T_2 = T_2T_1R_{12} \tag{2}$$

In this relation the elements of matrix R are numbers but the matrix $T = \{T^j_i\}$ is formed by generally noncommuting elements of an algebra.

Another type of deformation of the relations (1) represent the so called braided groups [3] defined by the relation

$$T_1Z_{12}T_2Z^{-1}_{12} = Z^{-1}_{21}T_2Z_{21}T_1 \tag{3}$$

where Z is again a matrix $\{Z^{kl}_{ij}\}$ with number elements.

The quantum groups appeared to be hidden symmetries of many physical models. The relevance of the braided groups for the low–dimensional quantum field theory was explained in [4]. The relation (3) can be also interpreted as constant reflection equation [5] that was recently investigated in [6].

The goal of this paper is to define a concept that unifies both the quantum groups and braided groups. We call these more general objects *quantum braided groups*. Before doing that let us summarize the properties of quantum groups and braided groups.

Both the algebras defined by (2) or (3) can be extended to bialgebras with matrix coproduct and counit

$$\Delta(T^j_i) := T^k_i \otimes T^j_k,\ \epsilon(T^j_i) := \delta^j_i. \tag{4}$$

However, the tensor products of the algebras defined by the relations (2) differ from those defined by the relations (3).

The multiplication in the tensor product $A \otimes A$ of the algebras A defined by the relations (2) (corresponding to quantum groups) is

$$m_{(A\otimes A)} : A \otimes A \otimes A \otimes A \rightarrow A \otimes A$$

$$m_{(A\otimes A)} := (m \otimes m) \circ (id \otimes \tau_{23} \otimes id) \tag{5}$$

where m is the product in A and τ_{23} is the transposition of the second and third factor in $A \otimes A \otimes A \otimes A$. It is then easy to prove that A is a bialgebra.

On the other hand the multiplication in the tensor product $B \otimes B$ of the algebras B defined by the relations (3) (corresponding to braided groups) is more complicated because instead of the simple transposition τ a more general map $\psi : B \otimes B \rightarrow B \otimes B$ called *braiding* appears in the product [3].

$$m_{(B\otimes B)} : B \otimes B \otimes B \otimes B \rightarrow B \otimes B$$

$$m_{(B\otimes B)} := (m \otimes m) \circ (id \otimes \psi_{23} \otimes id) \tag{6}$$

where m is the product in B and

$$\psi(T^m_i \otimes T^n_k) := \psi^{mn}_{ik}{}^{jl}_{rs}(T^r_j \otimes T^s_l) \tag{7}$$

where

$$\psi^{mn\ jl}_{ik\ ,rs} := Z^{aj}_{id}(Z^{-1})^{lb}_{ar} Z^{cn}_{sb} \tilde{Z}^{md}_{ck} \tag{8}$$

and $\tilde{Z} := ((Z^{t_2})^{-1})^{t_2}$.

To prove that B is bialgebra namely that Δ and ϵ are morphisms of the algebra B and $B \otimes B$ is a bit more complicated than for the quantum groups but there are no principal problems. The identities

$$\tilde{Z}^{kn}_{il} Z^{ml}_{kj} = Z^{kn}_{il} \tilde{Z}^{ml}_{kj} = \delta^m_i \delta^n_j \tag{9}$$

which follow immediately from the definition of $\tilde{Z}$, is used for that. If antipodes on the bialgebras are defined we get Hopf algebras.

Finally let us remark that the relations (3) are invariant under the transformation $T' = A^{-1}TA$ where A^j_i are generators of the quantum group's defined by

$$Z_{12}A_1A_2 = A_2A_1Z_{12} \tag{10}$$

and $A^{-1} = S(A)$, i.e. the matrix with entries $S(A^j_i)$ where S is the antipode. In other words, B is A(Z)–comodule algebra [7].

2 Quantum braided groups

As mentioned in the beginning, our goal is to define an object that will unify the properties of both quantum and braided groups or more precisely, that will contain both of them as special cases. Prototypes for that are quantum supergroups.

The supergroups are special cases of the braided groups where $Z = \eta := diag(+,+,\ldots,-,-,\ldots)$ and $\psi(x \otimes y) = (-)^{|x||y|} y \otimes x$. The defining relations of a quantum supergroup can be written in a form that reminds (2) but the super-commuting nature of its elements is expressed by inclusion of the matrix η into the defining relation [8]

$$\mathcal{R}_{12}T_1\eta_{12}T_2\eta_{12} = \eta_{12}T_2\eta_{12}T_1\mathcal{R}_{12} \tag{11}$$

Comparing (2), (3) and (11) leads us quite naturally to the investigation of algebras given by a pair of $n^2 \times n^2$ matrices $(\mathcal{R}, Z)$ that define relations

$$\mathcal{R}_{12}T_1Z_{12}T_2Z^{-1}_{12} = Z^{-1}_{21}T_2Z_{21}T_1\mathcal{R}_{12} \tag{12}$$

which include the cases of both quantum and braided groups.

The experience with the quantum groups, braided groups, and quantum super-groups teaches us that the matrices $\mathcal{R}$ and Z cannot be arbitrary but will be restricted by conditions of Yang–Baxter type. These conditions follow from two possible ways to transpose expressions containing triples of generators T^j_i. In order that the relations (12) can be applied we shall consider triples of the form

$$T_1Z_{12}T_2Z^{-1}_{12}Z_{23}Z_{13}T_3Z^{-1}_{13}Z^{-1}_{23}. \tag{13}$$

They can be transposed to expressions with the transposed order of T_1, T_2, T_3 if matrices $\mathcal{R}$ and Z are invertible and satisfy

$$Z_{12}Z_{13}Z_{23} = Z_{23}Z_{13}Z_{12}, \tag{14}$$

$$\mathcal{R}_{12}Z_{23}Z_{13} = Z_{23}Z_{13}\mathcal{R}_{12}, \tag{15}$$

$$Z_{12}Z_{13}\mathcal{R}_{23}^{-1}Z_{32}^{-1} = \mathcal{R}_{23}^{-1}Z_{32}^{-1}Z_{13}Z_{12}. \tag{16}$$

Under these conditions the expression (13) can be transposed by two ways and we require that the results be equal

$$\mathcal{R}_{12}^{-1}Z_{21}^{-1}\mathcal{R}_{13}^{-1}Z_{31}^{-1}\mathcal{R}_{23}^{-1}Z_{32}^{-1}T_3Z_{32}T_2Z_{31}Z_{21}T_1\mathcal{R}_{23}Z_{23}\mathcal{R}_{13}Z_{13}\mathcal{R}_{12}Z_{13}^{-1}Z_{23}^{-1} =$$

$$\mathcal{R}_{23}^{-1}Z_{32}^{-1}\mathcal{R}_{13}^{-1}Z_{31}^{-1}\mathcal{R}_{12}^{-1}Z_{21}^{-1}T_3Z_{32}T_2Z_{31}Z_{21}T_1\mathcal{R}_{12}Z_{12}\mathcal{R}_{13}Z_{13}\mathcal{R}_{23}Z_{13}^{-1}Z_{12}^{-1}. \tag{17}$$

In order that the equation (17) does not impose additional relations for T we require that the matrix $\mathcal{R}$ satisfy the "braided Yang–Baxter equations"

$$\mathcal{R}_{12}Z_{12}\mathcal{R}_{13}Z_{13}\mathcal{R}_{23}Z_{23} = \mathcal{R}_{23}Z_{23}\mathcal{R}_{13}Z_{13}\mathcal{R}_{12}Z_{12} \tag{18}$$

$$\mathcal{R}_{12}^{-1}Z_{21}^{-1}\mathcal{R}_{13}^{-1}Z_{31}^{-1}\mathcal{R}_{23}^{-1}Z_{32}^{-1} = \mathcal{R}_{23}^{-1}Z_{32}^{-1}\mathcal{R}_{13}^{-1}Z_{31}^{-1}\mathcal{R}_{12}^{-1}Z_{21}^{-1} \tag{19}$$

Introducing $R := \mathcal{R}Z$ we immediately see that (18) is the ordinary Yang–Baxter equation (YBE) for R

$$R_{12}R_{13}R_{23} = R_{23}R_{13}R_{12} \tag{20}$$

and the equations (15) and (16) can be rewritten to simpler forms

$$R_{12}Z_{13}Z_{23} = Z_{23}Z_{13}R_{12} \tag{21}$$

$$Z_{12}Z_{13}R_{23} = R_{23}Z_{13}Z_{12} \tag{22}$$

The equation (19) is then satisfied due to

Lemma: If R and Z are solutions of the YBE that satisfy (21) and (22) then $PZPRZ^{-1}$ and $Z^{-1}RPZP$, where P is the permutation matrix $P_{ij}^{kl} = \delta_i^l\delta_j^k$, are also solutions of the YBE.

Proof can be done by direct check. Let us note that the condition $PZPZ^{-1} = 1$ required in [9] is not necessary here.

Conclusion then is that when we have a pair (R, Z) of solutions of the YBE that satisfy the relations (21), (22), we can define the algebra

$$B(R,Z) := C < T_i^j >_{i,j=1}^n /\{R_{ab}^{cd}Z_{cd}^{-1}{}^{gh}T_g^iZ_{ih}^{ml}T_l^k - Z_{ba}^{-1}{}^{ji}T_j^gZ_{gi}^{hc}T_c^dR_{dh}^{mk}\}_{a,b,m,k=1}^n \tag{23}$$

that do not impose additional relations of cubic or higher degree. The compact form of the relations in (23) is

$$R_{12}Z_{12}^{-1}T_1Z_{12}T_2 = Z_{21}^{-1}T_2Z_{21}T_1R_{12} \tag{24}$$

that is equivalent to (12).

One can show that matrix coproduct and counit (4) are morphisms of $B(R,Z)$ into $B(R,Z)\otimes B(R,Z)$ where the product in $B(R,Z)\otimes B(R,Z)$ is defined by (6), (8). To do that one must prove that

$$R_{12}Z_{12}^{-1}\Delta(T_1)Z_{12}\Delta(T_2)=Z_{21}^{-1}\Delta(T_2)Z_{21}\Delta(T_1)R_{12} \tag{25}$$

which is simple but tedious exercise with indices where the identity (9) is used. It means that the algebra (23) can be extended to the bialgebra with the coproduct (4). That enable us to define the dual algebra of functionals $L^{\pm}=\{L^{\pm}{}_i{}^j\}$ on $B(R,Z)$ by

$$< L_1^{\pm}, T_2T_3\dots T_n >:= \mathcal{R}_{12}^{\pm}\mathcal{R}_{13}^{\pm}\dots\mathcal{R}_{1n}^{\pm} \tag{26}$$

where

$$\mathcal{R}_{12}^{+}:=Z_{12}\mathcal{R}_{21}Z_{21}=Z_{12}R_{21},\quad \mathcal{R}_{12}^{-}:=\mathcal{R}_{12}^{-1}=Z_{12}R_{12}^{-1} \tag{27}$$

and

$$< ab,c >:=< a\otimes b,\Delta(c) > . \tag{28}$$

The functionals then satisfy

$$\mathcal{R}_{21}L_1^{\epsilon}Z_{21}L_2^{\sigma}Z_{21}^{-1}=Z_{12}^{-1}L_2^{\sigma}Z_{12}L_1^{\epsilon}\mathcal{R}_{21} \tag{29}$$

or equivalently

$$R_{21}Z_{21}^{-1}L_1^{\epsilon}Z_{21}L_2^{\sigma}=Z_{12}^{-1}L_2^{\sigma}Z_{12}L_1^{\epsilon}R_{21} \tag{30}$$

where $(\epsilon,\sigma)=(+,+),(+,-),(-,-)$.

The relations of the quantum braided group are invariant under the transformation $T'=\tilde{A}TA$ where A_i^j and $\tilde{A}_i^j$ are generators of the quantum group given by

$$R_{12}A_1A_2=A_2A_1R_{12} \tag{31}$$

and its twisted version

$$\tilde{R}_{12}\tilde{A}_1\tilde{A}_2=\tilde{A}_2\tilde{A}_1\tilde{R}_{12} \tag{32}$$

where $\tilde{R}=PZPRZ^{-1}$ and beside that the generators must satisfy braiding relations

$$\tilde{A}_1Z_{12}A_2=A_2Z_{12}\tilde{A}_1, \tag{33}$$

$$A_1T_2=T_2A_1,\quad \tilde{A}_1T_2=T_2\tilde{A}_1. \tag{34}$$

On the other hand we can construct quantum spaces invariant under action of the quantum braided group. They are defined as

$$V(B(\hat{R}))=C< x^i >_{i=1}^{N} /\{x^ix^j-x^kx^lB(\hat{R})_{kl}^{ij}\}_{i,j=1}^{N} \tag{35}$$

where B is a polynomial of $\hat{R}=PR$. These algebras are invariant under the action of the quantum braided group $x'^i:=x^jT_j^i$ if x and T satisfy the braiding relation

$$T_1x_2=x_2Z_{12}^{-1}T_1Z_{12}. \tag{36}$$

The proof of the invariance of

$$x_1 x_2 = x_1 x_2 B(\hat{R})_{12} \tag{37}$$

is straightforward when the commutation relation

$$[\hat{R}_{12}, Z_{12}^{-1} T_1 Z_{12} T_2] = 0 \tag{38}$$

that follows from (24) is used.

Similarly, if the antipode $T^{-1} := S(T)$ exists, the covector quantum spaces

$$V(F(\check{R})) = C < v_i >_{i=1}^{N} / \{v_i v_j - F(\check{R})_{ij}^{kl} v_k v_l\}_{i,j=1}^{N} \tag{39}$$

where F is a polynomial of $\check{R} = RP$, are invariant uder the action $v_i' := T_i^{-1\ j} v_j$ if v and T^{-1} satisfy

$$v_1 T_2^{-1} = Z_{21}^{-1} T_2^{-1} Z_{21} v_1. \tag{40}$$

In general, the polynomials B, F can be arbitrary singular polynomials [10] but if we require moreover the invariance of the quantum spaces under the addition $x'' = x + x'$, $v'' = v + v'$, where x, x' and v, v' are two copies of generators that satisfy braiding relations [7]

$$x_1' x_2 = x_2 x_1' R_{12}, \; v_1' v_2 = R_{12} v_2 v_1', \tag{41}$$

then the polynomials B, F must satisfy conditions

$$(\hat{R} + 1)(B(\hat{R}) - 1) = 0, \tag{42}$$

$$(F(\check{R}) - 1)(\check{R} + 1) = 0, \tag{43}$$

that can be solved by virtue of the minimal polynomial of the matrix $\hat{R}$. Remarkable fact is that they are the same conditions as those that determine the quantum spaces where covariant differential calculi can be defined [11].

3 Examples

The problem that we have to solve for determination of a quantum braided group is to find solutions of the system (14,20,21,22).

There are several simple solutions of the system. One of them is $Z = 1, R$ - any solution of the YBE. This gives the algebras that correspond to the ordinary (unbraided) quantum groups [2]. Other solutions are $Z = R$ or $Z = PR^{-1}P$, R being any solution of the YBE. They correspond to the (unquantised) braided groups.

To present some nontrivial examples we are going to solve the system (14,20,21,22) for $n = 2$ i.e. for matrices R and Z of the dimension 4×4. In this dimension we have at our disposal the complete list of the YBE solutions [12] so that in principle it is easy to check whether pairs of the solutions satisfy (21, 22), however there are two obstacles. First, the solutions of the YBE are rather too many. Even if we restrict ourselves to the invertible ones that form eleven classes [13, 14] they give

121 pairs and it takes a lot of time to check them. Second and more important, even if we do that we can anyway miss some solutions. The reason is that the solutions of the YBE are given up to symmetries of the YBE but the cartesian product of the group of symmetries is not the group of symmetry of the system (14,20,21,22). Nevertheless, checking many solutions of the YBE we have been able to find several solutions of the system (14,20,21,22) that give nontrivial examples of the quantum braided groups in two dimensions.

The first type of nontrivial examples is given by diagonal Z and six–or–less vertex solutions of the YBE like e.g.

$$R_5 = \begin{pmatrix} q & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & q-t & qt & 0 \\ 0 & 0 & 0 & q \end{pmatrix}, \quad R_6 = \begin{pmatrix} q & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & q-t & qt & 0 \\ 0 & 0 & 0 & -t \end{pmatrix}, \tag{44}$$

(The numbering corresponds to that in classifications given in [13, 14].)

The defining relations of the quantum braided group given by $R = R_5$, $Z = diag(x, u, v, y)$ are

$$\begin{aligned} \tau AB &= BA, \\ \kappa CA &= AC, \\ \kappa DB &= \xi BD, \\ \tau CD &= \xi DC, \\ \xi BC &= \kappa\tau CB, \\ (AD - DA) &= (\kappa - \tau)CB, \end{aligned} \tag{45}$$

where A, B, C, D are generators of the algebra

$$T = \begin{pmatrix} A & B \\ C & D \end{pmatrix}. \tag{46}$$

and τ, κ, ξ are parameters related to those in R and Z by $\tau = tx/v$, $\kappa = qx/v$, $\xi = xy/uv$. This quantum braided group have a structure similar to the well known quantum group $GL_{q,s}(2)$ [16] which is obtained when $y = uv/x$.

On the other hand, the quantum braided group given by $Z = diag(x, u, v, y)$, $R = R_6$, where $q \neq -t$ reminds the quantum supergroup $GL_{q,t}(1|1)$ [17]. The defining relations are

$$\begin{aligned} B^2 &= 0 = C^2 \\ \tau AB &= BA, \\ \kappa CA &= AC, \\ \tau DB &= -\xi BD, \\ \kappa CD &= -\xi DC, \\ \xi BC &= \kappa\tau CB, \\ (AD - DA) &= (\kappa - \tau)CB, \end{aligned} \tag{47}$$

The braiding relations for the above given quantum braided groups are given by matrix $Z = R'_8 = diag(x,u,v,y)$ and read

$$\begin{array}{rcl} \psi(A\otimes X)=X\otimes A,\ \psi(X\otimes A) & = & A\otimes X,\ \ X\in\{A,B,C,D\},\\ \psi(B\otimes B) & = & \xi B\otimes B,\\ \psi(B\otimes C) & = & \xi^{-1}C\otimes B,\\ \psi(C\otimes B) & = & \xi^{-1}B\otimes C,\\ \psi(C\otimes C) & = & \xi C\otimes C,\\ \psi(D\otimes X)=X\otimes D,\ \psi(X\otimes D) & = & D\otimes X,\ \ X\in\{A,B,C,D\} \end{array} \tag{48}$$

where $\xi = xy/uv$. Note that A, D are always bosonic. Only B and C can have anomalous statistics for this Z.

Other examples are provided by the solutions of the system (14,20,21,22) where

$$Z = R'_{10} = \begin{pmatrix} 1 & 0 & 0 & 0\\ x & 1 & 0 & 0\\ y & 0 & 1 & 0\\ z & y & x & 1 \end{pmatrix} \tag{49}$$

and

$$R = R_{10} = \begin{pmatrix} 1 & 0 & 0 & 0\\ g & 1 & 0 & 0\\ h & 0 & 1 & 0\\ f & h & g & 1 \end{pmatrix} \quad \text{or} \quad R = R_{11} = \begin{pmatrix} 1 & 0 & 0 & 0\\ -g & 1 & 0 & 0\\ g & 0 & 1 & 0\\ -gh & h & -h & 1 \end{pmatrix}, \tag{50}$$

The braiding ψ in these cases is

$$\begin{array}{rcl} \psi(C\otimes X) & = & X\otimes C,\ \psi(X\otimes C)=C\otimes X,\ \ X\in\{A,B,C,D\},\\ \psi(A\otimes A) & = & A\otimes A+\tau C\otimes C,\\ \psi(A\otimes B) & = & B\otimes A-\tau(A-D)\otimes C,\\ \psi(A\otimes D) & = & D\otimes A-\tau C\otimes C,\\ \psi(B\otimes A) & = & A\otimes B-\tau C\otimes(A-D),\\ \psi(B\otimes B) & = & B\otimes B+\tau(A-D)\otimes(A-D)+2\tau^2 C\otimes C,\\ \psi(B\otimes D) & = & D\otimes B+\tau C\otimes(A-D),\\ \psi(D\otimes A) & = & A\otimes D-\tau C\otimes C,\\ \psi(D\otimes B) & = & B\otimes D+\tau(A-D)\otimes C,\\ \psi(D\otimes D) & = & D\otimes D+\tau C\otimes C, \end{array} \tag{51}$$

where $\tau = z - xy$. Note that there are again two bosonic elements, namely C and $A + D$. For $z = xy$ the braiding is bosonic even though $Z \neq 1$.

The quantum braided group given by $Z = R'_{10}$, $R = R_{11}$ is defined by the relations

$$BA = AB + \chi B^2,$$

$$\begin{aligned} DB &= BD + \gamma B^2, \\ CB &= BC + \gamma AB - \chi BD, \\ CA &= AC + \tau AB + \gamma A(A-D) + \gamma BC + \tau(\chi-\gamma)B^2 - \gamma\chi BD, \\ DC &= CD - \chi(A-D)D + \chi BC - \tau(\chi-\gamma)B^2 + (\tau-\chi^2)BD, \\ DA &= AD + \gamma AB + \chi BD + (\tau+\gamma\chi)B^2, \end{aligned} \tag{52}$$

where $\tau = z - xy$, $\chi = y - h$, $\gamma = y - g$. Note that for $\tau = 0$ we get the nonstandard unbraided deformation of $GL(2)$ [18, 19] even if $Z \neq 1$.

When investigating the quantum braided group given by $R = R_{10}$, $Z = R'_{10}$ we can assume that the parameters of R_{10} satisfy $g+h \neq 0$ or $f \neq gh$ because otherwise we get a special case of the previous example. Under this assumption the defining relations read

$$\begin{gathered} AB = BA = DB = BD = B^2 = 0, \\ AD = DA,\ BC = CB,\ A^2 = D^2, \\ CA = AC + (y-h)A(A-D) + (y+g)BC \\ CD = DC + (y-h)D(A-D) - (y+g)BC \end{gathered} \tag{53}$$

If $g + h \neq 0$ then moreover

$$BC = A(A-D), \quad AC = CD. \tag{54}$$

4 Conclusions

We have written down the defining relations of a new type of bialgebras that generalize both the quantum groups and braided groups as well as the quantum supergroups. The relations of the algebras are determined by a pair of matrices (R, Z) that solve a system of Yang–Baxter–type equations. The matrix coproduct and counit are of standard matrix form, however, the multiplication in the tensor product of the algebras is defined by virtue of the braiding map given by the matrix Z.

Besides simple solutions of the system of the Yang–Baxter–type equations that generate either quantum groups or braided groups, we have found several solutions that generate genuine quantum braided groups that by a choice of parameters give quantum or braided groups as a special cases.

This work was supported in part by the grant CSAV No. 11 086.

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