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QCD AT THE HADRON SCALE AND NON-LOCAL THIRRING MODEL

Jiří Souček

1. Introduction

Quantum Chromodynamics has become the favoured candidate for a theory of strong interactions. At short distances the effective coupling strength has been shown to be small, thus for large momentum processes the behaviour of QCD is calculable and essentially known (perturbative QCD). At moderate and large distances the effective coupling is not small, perturbative theory cannot be relied on, and different, in general less controled approximations must be used. Some of them are : variational ansatz of the BCS-type [4], the lattice QCD [9], self-consistent method of Nambu and Jona-Lasinio [6]. The renormalization group method improves our understanding considerably, but also in the perturbative region only.

In a sense, the continuum QCD is defined only at short distances : the usual definition of quantum field theory is based on the perturbative theory with a small coupling constant [7]. It is known from the lattice QCD [9], that the high temperature expansion gives good results at large distances. But at the most interesting region of hadron distances, both perturbative and lattice QCD fall into troubles.

In this paper, we will address the problem of behaviour of QCD at the hadron scale. We propose a new perturbative scheme which combines together the weak-coupling expansion at short distances with the high temperature expansion at large distances. Our scheme is a generalization of the perturbative QCD ; it reduces to it at short distances.

Our main idea is the following one. The reasonable approximation scheme at the hadron scale should be non-local and should break explicitly the scale invariance. We break it through the introduction of the approximation scheme - we use different approximations in the long-wave and short-wave regions.

Generally, any theory describing hadrons must break scale invariance. In the

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"This paper is in final form and no version of it will be submitted for publication elsewhere".

perturbative QCD, the coupling constant is replaced, via dimensional transmutation [1], by a distance scale, which we may for convenience think of as the distance at which the coupling constant attains the value of order unity.

We shall assume that at distances small compared to a certain scale Λ_0, the QCD ground state will have properties of the perturbative vacuum, while at larger distance it will have properties of the ground state of the lattice QCD at a high temperature. (The relation of Λ_0 to Λ_{QCD} is not clear.) This Λ_0 is an input parameter of our scheme; its precise definition will be given bellow.

In our scheme we shall treat the long-wave and the short-wave components of gauge fields differently - this is often used in the superfluidity and the superconductivity theories [5]. Let us consider an Euclidean field configuration $\Phi(x)$. In the momentum representation we can write

$$\Phi(x) = \int d^4p\ \tilde{\Phi}(p)e^{ipx} . \tag{1.1}$$

We shall divide each such Φ into the short-wave (S) and long-wave (L) parts by

$$\Phi^S(x) = \int_{|p|>\Lambda_0} d^4p\ \tilde{\Phi}(p)e^{ipx} , \quad \Phi^L(x) = \int_{|p|<\Lambda_0} d^4p\ \tilde{\Phi}(p)e^{ipx} . \tag{1.2}$$

In our case the parameter Λ_0 defines the theory (contrary to [5], where it plays only a technical role), because it breaks the scale invariance.

We will consider QCD in the covariant gauge

$$L = \overline{\Psi}i\gamma D\Psi - \frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + L_{\text{gauge fixing}} + L_{\text{ghost}} . \tag{1.3}$$

Instead of the decomposition $L = L_0 + L_{int}$ from the perturbative QCD we propose another decomposition

$$L_{QCD} = L_{QCD0} + L_{pert} . \tag{1.4}$$

Our 0-th order approximation QCD^0 is not a free field theory, but the non-local colored Thirring model with the infinitely strong interaction. Its Lagrangian is

$$\begin{aligned} L_{QCD0} &= -\frac{1}{4}(\partial_\mu A^s_\nu - \partial_\nu A^S_\mu)^2 + \overline{\Psi}i\partial\!\!\!/\Psi + m_0^{-2}(J^{aL}_\mu)^2 , \\ m_0 &\to 0 , \quad J^a_\mu = \overline{\Psi}\,\gamma_\mu\,\frac{1}{2}\,\lambda^a\,\Psi \end{aligned} \tag{1.5}$$

where J^{aL}_μ is the long-wave component of the color current. The perturbative reduction of QCD to QCD^0 is the main result of the paper.

In our scheme, there are two differences with respect to the perturbative QCD: the free part of the Lagrangian for $F^{aL}_{\mu\nu}$ will become perturbative and the interaction term between the quark field and the long-wave component A^L_μ of the gauge potential will become non-perturbative. It can be summarized that quarks interact strongly with the long-wave part of the gluon field and that the kinetic term for

$F^L_{\mu\nu}$ is considered as a perturbation.

It may seem that we have made some strange tricks with the coupling constant. But, in fact our coupling g plays a double role. At small distances, $g < 1$ is the usual weak-coupling constant from the perturbative theory, while at large distances $g^{-1} > 1$ is the effective coupling constant and we make the expansion in the inverse coupling $(g^{-1})^{-1} = g$. This enables us to suppose that effective coupling g is small at all distances (more precisely: that the perturbative expansion in g could be meaningfull).

It means physically that the "bare" coupling $g_0 << 1$ is renormalized at small distances to $g < 1$, but at large distances to $g^{-1} > 1$. (The relation $g \cdot g^{-1} = 1$ defines Λ_0 , as it will be shown.)

In the part 2. we give the "heuristic" derivation of our scheme, which will clarify its meaning and we will obtain the 0-th order approximation QCD^0 . Nothing better can be done, because the continuum QCD is not defined at large distances - our scheme could be the way how to define it. In the part 3. we discuss the non-locality of the scheme and the main problem how to work with QCD^0 . It is not a free theory, so that some nonperturbative method is needed.

We hope that the proposed perturbative scheme could improve our understanding of the behaviour of QCD at hadron distances. The problem of finding non-perturbative methods is reduced from the full QCD to QCD^0 , which is substantially simpler (e.g. it has only trivial UV-divergences).

2. The derivation of the scheme

We need a slightly unusual point of wiev on the renormalization procedure for our way of reasoning. Let us recall the well-know idea in the case of QED , where we left away all questions connected with the gauge. The Lagrangian expressed in bare quantities is

$$(2.1) \qquad L = \bar{\Psi}^0 (i\partial\!\!\!/ - A\!\!\!/^0 - m_0)\Psi^0 - \frac{1}{4}\frac{1}{g_0^2}(F^0_{\mu\nu})^2 .$$

In the usual procedure we introduce renormalized fields and couplings, express L in them and add counterterms

$$(2.2) \qquad L = \bar{\Psi}(i\partial\!\!\!/ - A\!\!\!/ - m)\Psi - \frac{1}{4}\frac{1}{g^2}F^2_{\mu\nu} + L_{\text{counterterms}} .$$

Everything is then expressed in the renormalized quantities. The "unphysical" bare quantities disappear from the theory.

We need rather the conceptual framework of the Kadanoff-Wilson (block - spin) renormalization procedure [8]. Let us suppose that we are given the bare teory (2.1) with a certain fixed regularization (given by a sufficiently large cut-off). Evidently, it makes no sense to expand it around the bare fields with the bare couplings m_0 and g_0 , because we would obtain large contributions ("divergences"). The resulting theory differs much from the bare one. To have a meaningfull perturbative scheme we must :

(i) to guess the "physical" 0-th order approximation,

(ii) to guess the "physical" interaction terms,

(iii) to add counterterms in such a way that the resulting Lagrangian will be the same as the original one.

All this must be done in such a way that the Lagrangian expressed in the new, renormalized quantities

(2.3) $$L = L_0 + L_{int} + L_{counterterms}$$

would give small contributions in the perturbative theory.

If the resulting coupling g is small, we will obtain the usual QED. From the term $F^2_{\mu\nu}$ in (2.2.) we see that only configurations with a small $F_{\mu\nu} \lesssim g$ contribute significantly to the Euclidean partition function

(2.4) $$\exp\left(- S[\Psi,A]\right) .$$

The term $F^2_{\mu\nu}$ gives the Gaussian damping factor. To normalize the distribution of F's we scale the gauge field A_μ

(2.5) $$A_\mu \to gA_\mu .$$

Then we will obtain the well-known form

(2.6) $$L = \overline{\Psi}(i\not{\partial} - g\not{A} - m)\Psi - \frac{1}{4} F^2_{\mu\nu} + L_{counterterms}$$

in which the weak coupling of electrons and photons is evident.

Let us now consider the opposite case, where the resulting coupling g (given by g_0 and by the assumed regularization) is large, so that we have the strongly coupled QED^{strong} . The scheme (2.2), (2.5), (2.6) is now meaningless and the right choice in (i) - (iii) is completely unclear. One possible choice, the strong coupling expansion, means that we take for a 0-th order approximation the theory with $g^{-1} = 0$ and we shall expand in g^{-1} . From (2.2) we have

(2.7) $$L_0^{strong} = \overline{\Psi}(i\not{\partial} - \not{A} - m)\Psi ,$$

(2.8) $$L_{pert}^{strong} = - \frac{1}{4}(g^{-1})^2 F^2_{\mu\nu} .$$

The partition function in the 0-th order is such that all configurations of A_μ contribute almost equally - there is no damping factor for A_μ . In the 0-th order we have an infinitely strong coupling between electron and positrons. Such a situation is well-known from the high temperature expansion in the lattice QCD [9]. This infinitely strong coupling will become weaker only after taking into account the perturbation (2.8).

But there are two important differences with respect to the lattice theory.

On the lattice the field values belong in the compact group space and thus the high temperature expansion makes sense. In our case, there is also a certain damping factor for A_μ's which could be obtained using functional integration for the fields Ψ. We will not make this integration, because it contradicts to the meaning of the perturbative theory. But we will suppose such a damping factor implicitely. For the practical purposes we introduce the term regularizing the equal distribution of A's

$$L_{reg} = m_0^2 A_\mu^2 , \quad m_0 \to 0 . \tag{2.9}$$

In another interpretation this term mimicks the finite group space from the QCD on the lattice which is shown to describe the behavior of QCD at long distances (our consideration of QED serves only as a motivation for the QCD case). The second difference lies in the fact that the lattice constant regularizes UV-divergences, which we would meet immediately in our QED^{strong}. But this problem is not essential, because the parameter Λ_0 will make the appropriate cut-off in the strong coupling expansion in the QCD case.

The "strong" perturbation theory (2.7), (2.8), (2.9) is an analogue of the high temperature expansion on the lattice and its characteristic feature is that all gauge field configurations contribute equally in the 0-th order. We see that in the strong coupling case the scaling (2.5) makes no sense.

Now we shall come to QCD. Our scheme combines the weak coupling expansion (2.6) at short distances with the strong coupling expansion (2.7), (2.8), (2.9) at long distances. The bare coupling g_0 is given at a certain very small distance (corresponding to an assumed regularization) by the perturbative QCD - like in the Kadanoff-Wilson approach. Thus the affective coupling at large distances is determined and must be large, if QCD has to confine quarks. We will assume this.

The reasonable choice (2.2) and (2.6) of L_0 and L_{int} in QED^{weak} is formally identical with the bare Lagrangian (2.1). But this should not be true in the case of QCD. Our choice of L_0 and L_{int} should respect what we already know about the behavior of QCD at small distances (perturbative QCD) and at large distance (lattice QCD).

We shall assume that the parameter Λ_0 divides these two regions : $p > \Lambda_0$ and $p < \Lambda_0$. The breaking of the scale invariance originates in different approximations used in these two regions. Thus we cannot use the concept of a running coupling. We will use rather the old-fashioned "constant" coupling, fixed by certain (yet unknown) renormalization conditions.

We will suppose that in the short-wave region $p > \Lambda_0$ there is an effective coupling constant $g_S < 1$ and in the long-wave region $p < \Lambda_0$ there is an effective coupling $g_L > 1$. We will expect that the renormalization in the Kadanoff--Wilson sense will give

$$g_0 \dashrightarrow g_S < 1 \quad \text{in} \quad p > \Lambda_0 , \qquad (2.10)$$
$$g_0 \dashrightarrow g_L > 1 \quad \text{in} \quad p < \Lambda_0 .$$

At first we must choose a suitable 0-th approximation for gluons. We will suppose that the self-interaction of gluons manifests itself at the 0-th order only in the renormalization (2.10) and otherwise their interaction can be neglected, so that the typical features - damping of $F_{\mu\nu}$ in the weak region, equal distribution of $F_{\mu\nu}$ in the strong region - considered above will remain conserved. A corresponding choice of the gluon part of the Lagrangian (with color indices supressed) will be

$$L = -\frac{1}{4}\left(g_S^{-2}(F_{\mu\nu}^S)^2 + g_L^{-2}(F_{\mu\nu}^L)^2\right) \qquad (2.11)$$

where $F_{\mu\nu} = F_{\mu\nu}^S + F_{\mu\nu}^L$ is the decomposition (1.2) of $F_{\mu\nu}$ into short-wave and long-wave parts. The term (2.11) is exactly the one, which gives the right limits for $p \to \infty$ and $p \to 0$. Neglecting the self-interaction of gluons (apart from (2.10)) we also have

$$F_{\mu\nu}^{S,L} = \partial_\mu A_\nu^{S,L} - \partial_\nu A_\mu^{S,L} . \qquad (2.12)$$

This means that the configurations A_μ with the prevailing short-wave part A_μ^S are supressed as in the perturbation QCD, while those with the prevailing long-wave part contribute equally into the partition function at the 0-th order.

In the relation (2.11) we have neglected the self-interaction of gluons, but when we shall turn it on we can suppose that (2.11) is still true for the free part of $F_{\mu\nu}$ like in (2.11). The formula (2.11) then means that the free part of $F_{\mu\nu}^0$ will come over to

$$\frac{1}{g_0}(\partial_\mu A_\nu^0 - \partial_\nu A_\mu^0) \dashrightarrow \frac{1}{g_S}(\partial_\mu A_\nu^S - \partial_\nu A_\mu^S) + \frac{1}{g_L}(\partial_\mu A_\nu^L - \partial_\nu A_\mu^L) \qquad (2.13)$$

during the Kadanoff-Wilson renormalization.

The relation $\dashrightarrow$ will mean our choice of the corresponding part of the "renormalized" Lagrangian. From the other point of view this transition is expected to be the result of renormalization effects.

Now we must propose the term corresponding to the self-interaction of gluons. Our choice is, as in (2.13)

$$\frac{1}{g_0}[A_\mu^0, A_\nu^0] \dashrightarrow \frac{1}{g_S}[A_\mu^S, A_\nu^S] + \frac{1}{g_L}[A_\mu^L, A_\nu^L] + \frac{1}{g_{SL}}\left([A_\mu^S, A_\nu^L] + [A_\mu^L, A_\nu^S]\right) . \qquad (2.14)$$

The problem we had to solve here was the choice of the $[A^S, A^L]$-term. We have suggested to put in the constant interpolating between g_S and g_L in front of this term

$$g_{SL} = (g_S g_L)^{1/2} . \qquad (2.15)$$

We can suppose g_{SL} to be of order of unity (see (2.10)). The relations (2.14) and (2.15) also represent well our basic idea, that configurations $A \approx A^S$ are supressed and those with $A \approx A^L$ contribute almost equally.

Now we can make the scaling (2.5), but with respect to our discussion of QED^{strong} we shall scale only A^S_μ :

(2.16) $$A^S_\mu \to g_S A^S_\mu \ , \quad A^L_\mu \to A^L_\mu \ .$$

After this redefinition the relations (2.13) and (2.14) will read

(2.13') $$\frac{1}{g_0}(\partial_\mu A^0_\nu - \partial_\nu A^0_\mu) \text{ - - > } (\partial_\mu A^S_\nu - \partial_\nu A^S_\mu) + g_L^{-1}(\partial_\mu A^L_\nu - \partial_\nu A^L_\mu) \ ,$$

(2.14') $$\frac{1}{g_0}[A^0_\mu, A^0_\nu] \text{ - - > } g_S[A^S_\mu, A^S_\nu] + g_L^{-1}[A^L_\mu, A^L_\nu] + \frac{g_S}{g_{SL}}([A^S_\mu, A^L_\nu] + [A^L_\mu, A^S_\nu]) \ .$$

Let us denote parts of $F_{\mu\nu}$ by (f = free)

(2.17) $$\begin{aligned} F^{S,L}_{\mu\nu f} &= \partial_\mu A^{S,L}_\nu - \partial_\nu A^{S,L}_\mu \ , \\ F^{S,L}_{\mu\nu I} &= [A^{S,L}_\mu, A^{S,L}_\nu] \ , \\ F^{SL}_{\mu\nu I} &= [A^S_\mu, A^L_\nu] + [A^L_\mu, A^S_\nu] \ , \end{aligned}$$

where in the first two relations the superscript is either S or L . Together we have obtained

(2.18) $$\begin{aligned} &\frac{1}{g_0} F^0_{\mu\nu} \text{ - - > } \tilde{F}_{\mu\nu} \ , \\ &\tilde{F}_{\mu\nu} = F^S_{\mu\nu f} + g_S(F^S_{\mu\nu I} + g_{SL}^{-1} F^{SL}_{\mu\nu I}) + g_L^{-1}(F^L_{\mu\nu f} + F^L_{\mu\nu I}) \ . \end{aligned}$$

We have to add the regularization term (2.9) for the long-wave part of A_μ

(2.19) $$L_{gluon} = -\frac{1}{4}\tilde{F}^2_{\mu\nu} + L_{reg} \ ,$$

(2.20) $$L_{reg} = \frac{1}{2} m_0^2 (A^L_\mu)^2 \ , \quad m_0 \to 0 \ .$$

The scaling (2.16) gives for the quark terms

(2.21) $$\overline{\Psi}_0 i\partial\!\!\!/ \Psi_0 \text{ - - > } \overline{\Psi} i\partial\!\!\!/ \Psi = L_{quark} \ ,$$

(2.22) $$\overline{\Psi}_0 \gamma_\mu A^0_\mu \Psi_0 \text{ - - > } \overline{\Psi}\gamma_\mu(g_S A^S_\mu + A^L_\mu)\Psi = L_{qg} \ .$$

Thus our resulting Lagrangian

(2.23) $$L = L_{gluon} + L_{quark} + L_{qg}$$

can be expanded in the coupling constants g_S and g_L^{-1} (see (2.18) and (2.22)). From (2.10) we can expect that

(2.24) $$g_S \ , \ g_L^{-1} \lesssim 1 \ ,$$

so that the expansion in the both constants g_S and g_L^{-1} could make sense at all distances.

We shall not write down all interaction terms. But the 0-th order approximation ($g_S = g_L^{-1} = 0$) is interesting

$$L_{QCD^0} = \bar{\Psi}\gamma\cdot(i\partial + A^L)\Psi - \frac{1}{4}(\partial_\mu A_\nu^S - \partial_\nu A_\mu^S)^2 + \frac{1}{2}m_0^2(A_\mu^L)^2 \ . \tag{2.25}$$

Let us now suppose that the couplings g_S and g_L^{-1} are fixed by some renormalization conditions. Let us consider the change of our basic parameter Λ_0

$$\Lambda_0 \to \Lambda_0' = \Lambda_0 + \delta\Lambda_0 \ , \quad \delta\Lambda_0 > 0 \ .$$

We can expect that the corresponding change of g_S and g_L will be

$$\delta g_S < 0 \ , \quad \delta g_L < 0 \ ,$$

because g_S and g_L are the effective couplings in the regions $p > \Lambda_0$ and $p < \Lambda_0$. We have $\delta(g_L^{-1}) > 0$ and making an appropriate variation of Λ_0 we obtain

$$g_S = g_L^{-1} \equiv g \ , \tag{2.26}$$

where the common value of coupling constants is denoted g . This condition fixes Λ_0 and we shall take (2.26) as a definition of Λ_0 .

Now the interaction terms in our "physical" Lagrangian simplify considerably; $g_{SL} = 1$ and our decomposition (1.4) has the form

$$L = -\frac{1}{4}\left[F_{\mu\nu f}^S + g(F_{\mu\nu f}^L + F_{\mu\nu I}^S + F_{\mu\nu I}^{SL} + F_{\mu\nu I}^L)\right]^2 + \bar{\Psi}\gamma_\mu(i\partial_\mu + gA_\mu^S + A_\mu^L)\Psi + \frac{1}{2}m_0^2(A_\mu^L)^2 \tag{2.27}$$

so that the only changes with respect to the perturbation QCD are

$$F_{\mu\nu f}^L \to gF_{\mu\nu f}^L \ , \qquad g\bar{\Psi}\not{A}^L\Psi \to \bar{\Psi}\not{A}^L\Psi$$

and the introduction of L_{reg} .

This is the motivation of our choice of the "physical" Lagrangian of QCD . We are not able to introduce counterterms (besides the UV-counterterms, which are the same as in the perturbative QCD), because the 0-th order approximation QCD^0 is not a free theory and we do not known any non-perturbative method to solve it.

We are aware that we have used many assumptions which are only plausible (at the best), but not proved. We shall show now that the resulting scheme is quite reasonable, especially that QCD^0 could be a meaningfull 0-th order approximation at the hadron scale.

The short-wave component A_μ^S of the gauge field is not coupled in QCD^0 and the long-wave component A_μ^L can be integrated out (by functional integration). In this way we shall obtain the non-local colored Thirring model with an infinitely strong interaction

(2.28) $$L_{QCD^0} = \overline{\Psi}_i i \not\partial \Psi_i + \frac{1}{2} m_0^{-2} (J_\mu^{aL})^2 , \quad m_0 \to 0 , \quad J_\mu^a = \overline{\Psi}_i \gamma_\mu \frac{1}{2} \lambda^a \Psi_i ,$$

where we have reintroduced the flavor indices. Note that there are only trivial UV-divergences in (2.28), because Λ_0 is fixed.

In the regularization limit $m_0 \to 0$ we will obtain (roughly) a model of free quarks with a constraint

(2.29) $$L = \overline{\Psi} i \not\partial \Psi$$

(2.30) $$J_\mu^{aL} = 0 ,$$

because contributions of configurations with $J_\mu^{aL} \neq 0$ are supressed in the partition function. More exactly, the second term in (2.28) goes to the δ-functional (we have neglected the corresponding Fadeev - Popov determinant). The constraint (2.30) is in a sense momentum space analogue of the usual boundary condition in the bag model. This means that essential configurations are those with the vanishing long-wave part of the color current.

The physical meaning of the model (2.29 , (2.30) is clear. The interaction between quarks mediated by a gluon exchange is vanishing if the gluon carries a momentum greater than Λ_0 and is infinitely strong if the gluon carries a momentum smaller than Λ_0 . Clearly, in QCD^0 quarks are confined.

3. Discussion

The condition (2.30) looks non-natural, because there is a sharp cut-off at the momentum space. But this is as non-natural as the sharp cut-off at the x-space in the usual bag model. The sharp division of S- and L- regions seems to be necessary in our method; but this holds only for QCD^0 .

The behavior of QCD at intermediate distances is the most complicated (and most important) problem in QCD. Our method tries to describe this region by approximating it simultaneously from both sides. In QCD^0 this region is infinitely thin and it will be enlarged by perturbative contributions. The fact that the intermediate region is infinitely thin seems to prevent QCD^0 to be a realistic theory. But the 0-th order theory need not be realistic - it must only satisfy the condition that the "distance" between QCD^0 and the full QCD can be treated perturbatively. See QED as an example : the 0-th order non-interacting theory is completely non-realistic. We think that QCD^0 can be considered as a basic non--perturbative part of QCD parametrized by Λ_0 which should be related directly to the properties of hadrons. QCD^0 describes well the basic non-perturbative feature of QCD - the confinament of quarks.

QCD^0 has an important property of non-locality. It is the theory in the Euclidean space and it has the Euclidean symmetry. The non-locality is clear from

(2.28) and (2.30) - only long-wave components of J^a_μ enter these formulae - it means also the non-locality in the Euclidean time. Thus QCD^0 is not a Hamiltonian theory. It is only approximately local in time having an approximate evolution operator in time intervals greater than Λ_0^{-1} . The same must be expected at any finite order approximation in our scheme; only by summing the whole expansion we recover the original local QCD .

We think that this is exactly what should be expected for the approximation scheme to QCD at intermediate distances. The local QCD can describe non-local hadrons only by using substantially infinitely many degrees of freedom. Reasonably simple description of hadrons should be non-local in the space and then the Euclidean invariance implies a non-locality in the Euclidean time. Thus it cannot be a Hamiltonian theory. There is a problem, how to define the particle spectrum for the non-hamiltonian theory. But QCD^0 is the asymptotically Hamiltonian theory : we can define the Euclidean evolution operator $U_E(0,T)$ for $T >> \Lambda_0^{-1}$ by

$$(3.1) \qquad < \Phi' \mid U_E(0,T) \mid \Phi > = \int\limits_{\substack{\phi(\vec{x},0)=\Phi(\vec{x}) \\ \phi(\vec{x},T)=\Phi'(\vec{x})}} \mathcal{D}\phi e^{-S[\phi]} ,$$

and an asymtotic Hamiltonian by

$$(3.2) \qquad H_{as} = \lim_{T\to\infty} \left[- \frac{1}{T} \lg U_E(0,T)\right] .$$

Our scheme generalizes the perturbative QCD . To the perturbative QCD there corresponds the region where distances are $<< \Lambda_0^{-1}$ what means the limit $\Lambda_0 \to 0$. In this limit our scheme reduces to the perturbative QCD . We see that the problem of UV-renormalization is in our scheme exactly the same as in the perturbative QCD and can be solved equally well. For QCD^0 this problem is completely absent. The problem of IR-renormalization is not clear in our scheme, because we are not able to solve QCD^0 . Our scheme is at large distances similar to the strong coupling expansion in the lattice QCD and we hope that the IR-divergences in our scheme will be milder that in the perturbative QCD , perhaps absent.

Up to now we have not discussed the gauge symmetry. In our approach we have considered the Euclidean QCD in a fixed covariant gauge. We have supposed that the gauge symmetry was changed for the BRS symmetry, which gives the Slavnov - Taylor's identities - all goes like in the perturbative QCD . Fixing the gauge is necessary in our approach, because the division (1.2) of field configurations into short-wave and long-wave parts is not gauge invariant. Parameters Λ_0 and g in our scheme may generally depend on the choice of the gauge. We must use the covariant gauge, because we want to obtain a relativistically invariant scheme.

Our scheme is defined by the two parameters Λ_0 and g , while in the perturbative QCD there is only one parameter Λ_{QCD} . In principle both Λ_0 and g are determined by Λ_{QCD} through the Kadanoff-Wilson renormalization procedure -

but this procedure cannot be done explicitly. So we suggest to use both Λ_0 and g and to postpone the question of their relation to a future investigation.

The best method how to treat QCD^0 would be the bosonization [2], especially in the functional integral form [3]. Unfortunately, it is known only in the two--dimensional case. Nevertheless, QCD_2^0 can be solved by the methods of Refs. [3] and it may give an usefull insight into the properties of QCD_4^0 .

We left many important questions away, among them : the gauge-fixing and the ghost terms in the Lagrangian, the structure of counterterms, the renormalization conditions etc.

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