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We deduce that in general all natural bilinear concomitants of the type of the Frölicher-Nijenhuis bracket form a 10-dimensional real vector space. - All manifolds and maps are assumed to be infinitely differentiable.

1. Let $\Omega^p(M,TM)$ denote the space of differential forms of order p on a manifold M with values in the tangent bundle TM, which are called the vector valued p-forms on M, [4]. Obviously, $\Omega^o(M,TM)$ coincides with the set $\mathfrak{X}(M)$ of all vector fields on M. We consider, for each n-dimensional manifold M, a bilinear map $B_M: \Omega^p(M,TM)\times \Omega^q(M,TM)\to\Omega^{p+q}(M,TM)$ satisfying $f^*B_M(P,Q) = B_N(f^*P,f^*Q)$ for each local diffeomorphism $f: N\to M$. Such a family $B = (B_M)$ is called a natural concomitant of vector valued differential forms.

2. We will use the following notation, [9]. Let $\Omega^q(M)$ be the space of differential q-forms on M and $\Omega(M) = \bigoplus_q \Omega^q(M)$ be the exterior algebra of M. Every $P\in\Omega^p(M,TM)$ determines a graded derivation $i(P)$ of degree p-1 in $\Omega(M)$ characterized by $i(P)\omega = \omega\circ P$ if $\omega\in\Omega^1(M)$ and $i(P)f = 0$ for all functions $f\in\Omega^o(M)$. The explicit formula for $i(P)$ is

$$(1)\quad i(P)\omega(X_1,\dots,X_{p+q-1}) = \frac{1}{p!(q-1)!}\sum_\sigma (\operatorname{sign}\sigma)\cdot \omega(P(X_{\sigma1},\dots,X_{\sigma p}),X_{\sigma(p+1)},\dots,X_{\sigma(p+q-1)})$$

with $\omega\in\Omega^q(M)$, $X_1,\dots,X_{p+q-1}\in\mathfrak{X}(M)$ and summation with respect to all permutations σ of p+q-1 letters, [9]. This formula makes sense also for a vector valued q-form $Q\in\Omega^q(M,TM)$ and defines a vector valued (p+q-1)-form $i(P)Q\in\Omega^{p+q-1}(M,TM)$.

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Let $\Theta(P) = i(P)\circ d - (-1)^{p-1} d\circ i(P)$ be the graded commutator of $i(P)$ with the exterior differential d. For a vector field $X \in \Omega^0(M,TM)$, $\Theta(X)$ coincides with the Lie derivative with respect to X. According to [4], for every $P \in \Omega^p(M,TM)$ and $Q \in \Omega^q(M,TM)$ there exists a unique element $[P,Q] \in \Omega^{p+q}(M,TM)$ satisfying $\Theta([P,Q]) = \Theta(P)\circ\Theta(Q) - (-1)^{pq}\Theta(Q)\circ\Theta(P)$. This is called the Frölicher-Nijenhuis bracket of P and Q. It can be characterized by

$$[\varphi\otimes X, \psi\otimes Y] = \varphi\wedge\psi\otimes[X,Y] + \varphi\wedge\Theta(X)\psi\otimes Y - \Theta(Y)\varphi\wedge\psi\otimes X$$

$$+(-1)^p(d\varphi\wedge i(X)\psi\otimes Y + i(Y)\varphi\wedge d\psi\otimes X)$$

for all $\varphi\in\Omega^p(M)$, $\psi\in\Omega(M)$, $X,Y\in\mathfrak{X}(M)$, [9]. The coordinate expression of $[P,Q]$ is

$$[P^i_\alpha d^\alpha\otimes\partial_i, Q^j_\beta d^\beta\otimes\partial_j] = (P^j_{\gamma_1\dots\gamma_p}\partial_j Q^i_{\gamma_{p+1}\dots\gamma_{p+q}}$$

$$- (-1)^{pq} Q^j_{\gamma_1\dots\gamma_q}\partial_j P^i_{\gamma_{q+1}\dots\gamma_{q+p}} - p P^i_{\gamma_1\dots\gamma_{p-1}j}\partial_{\gamma_p} Q^j_{\gamma_{p+1}\dots\gamma_{p+q}}$$

$$+ (-1)^{pq} q Q^i_{\gamma_1\dots\gamma_{q-1}j}\partial_{\gamma_q} P^j_{\gamma_{q+1}\dots\gamma_{p+q}})d^\gamma\otimes\partial_i \tag{2}$$

where $d^\alpha = dx^{\alpha_1}\wedge\dots\wedge dx^{\alpha_p}$ and $\partial_i = \partial/\partial x^i$ in local coordinates $x = (x^1,\dots,x^n)$ on M. This follows from (1.9) of [9] by inserting the basic vector fields ∂_i into the global formula.

The wedge product of a differential q-form and a vector valued p-form is a bilinear map $\Omega^q(M)\times\Omega^p(M,TM)\to\Omega^{p+q}(M,TM)$ characterized by $\omega\wedge(\varphi\otimes X) = (\omega\wedge\varphi)\otimes X$ for all $\omega\in\Omega^q(M)$, $\varphi\in\Omega^p(M)$, $X\in\mathfrak{X}(M)$. Further, let $C:\Omega^p(M,TM)\to\Omega^{p-1}(M)$ be the contraction operator defined by $C(\omega\otimes X) = i(X)\omega$ for all $\omega\in\Omega^p(M)$, $X\in\mathfrak{X}(M)$. In particular, for $P\in\Omega^0(M,TM)$ we have $C(P) = 0$. Clearly $C(i(P)Q)$ is a multiple of $C(i(Q)P)$, $P\in\Omega^p(M,TM)$, $Q\in\Omega^q(M,TM)$. Finally, write $I = Id_{TM}\in\Omega^1(M,TM)$.

3. <u>Theorem.</u> For dim $M > p+q+1$, all natural concomitants of vector valued differential forms $\Omega^p(M,TM)\times\Omega^q(M,TM)\to\Omega^{p+q}(M,TM)$ form a vector space linearly generated by the following 10 operators

$$[P,Q],\ dC(P)\wedge Q,\ dC(Q)\wedge P,$$

$$dC(P)\wedge C(Q)\wedge I,\ dC(Q)\wedge C(P)\wedge I,\ dC(i(P)Q)\wedge I, \tag{3}$$

$$i(P)dC(Q)\wedge I,\ i(Q)dC(P)\wedge I,\ di(P)C(Q)\wedge I,\ di(Q)C(P)\wedge I$$

The proof of this theorem will occupy the rest of the paper.

We remark that for $p<2$ or $q<2$ some expressions in (3) vanish identically. In particular, for $p = q = 0$ we have only one nontrivial generator $[P,Q]$. One sees easily that the Frölicher-Nijenhuis bracket of two vector valued 0-forms, i.e. of two vector fields, coincides with the classical bracket. In this case we rededuce a result by S. van Strien, [14], and D. Krupka with V. Mikolášová, [7], that all natural concomitants of two vector fields are the scalar multiples of the classical bracket. An infinitesimal version of this result was deduced by M. de Wilde and P. Lecomte, [2].

4. From [4] or [9] we see that all expressions in the theorem are indeed natural. For the converse suppose that $B = (B_M)$ is a natural concomitant as in 1. Since B_M is a local operator, it is a (finite order) differential operator by the bilinear Peetre theorem, [1], see also [13]. In [9] it was checked that B is a homogeneous bilinear differential operator of total order 1.

5. Let P^2M be the second order frame bundle of M (i.e. the space of all invertible 2-jets of R^n into M with source 0), which is a principal fibre bundle with structure group G_n^2 of all invertible 2-jets of R^n into R^n with source and target 0. According to a general theory, see e.g. [6], the first order natural operators from $\Omega^p(M,TM)\times\Omega^q(M,TM)$ into $\Omega^{p+q}(M,TM)$ correspond to the G_n^2-equivariant maps from the standard fibre S of the first jet prolongation of the first bundle into the standard fibre $R^n\otimes\Lambda^{p+q}R^{n*}$ of the second bundle. Clearly, S is the product of four spaces $R^n\otimes\Lambda^pR^{n*}$, $R^n\otimes\Lambda^pR^{n*}\otimes R^{n*}$, $R^n\otimes\Lambda^qR^{n*}$, $R^n\otimes\Lambda^qR^{n*}\otimes R^{n*}$ and the action of G_n^2 on S is given by the usual transformation law of tensor fields and their first order partial derivatives. In order to be completely clear we specify this in detail: the transformation law of a tensor field of type $\binom{1}{p}$ is

$$(4)\quad \bar{P}^i_{j_1\dots j_p} = P^{\ell}_{m_1\dots m_p}\frac{\partial\bar{x}^i}{\partial x^{\ell}}\frac{\partial x^{m_1}}{\partial\bar{x}^{j_1}}\cdots\frac{\partial x^{m_p}}{\partial\bar{x}^{j_p}}$$

and the transformation law of its first partial derivatives is

$$(5)\quad \bar{P}^i_{j_1\dots j_p,k} = P^{\ell}_{m_1\dots m_p,n}\frac{\partial\bar{x}^i}{\partial x^{\ell}}\frac{\partial x^{m_1}}{\partial\bar{x}^{j_1}}\cdots\frac{\partial x^{m_p}}{\partial\bar{x}^{j_p}}\frac{\partial x^n}{\partial\bar{x}^k} +$$

$$(5)\quad + P^{\ell}_{m_1\dots m_p}\Big(\frac{\partial^2\bar{x}^i}{\partial x^{\ell}\partial x^n}\frac{\partial x^{m_1}}{\partial\bar{x}^{j_1}}\dots\frac{\partial x^{m_p}}{\partial\bar{x}^{j_p}}\frac{\partial x^n}{\partial\bar{x}^k}$$

$$+\frac{\partial\bar{x}^i}{\partial x^{\ell}}\frac{\partial^2 x^{m_1}}{\partial\bar{x}^{j_1}\partial\bar{x}^k}\frac{\partial x^{m_2}}{\partial\bar{x}^{j_2}}\dots\frac{\partial x^{m_p}}{\partial\bar{x}^{j_p}}$$

$$+\dots+\frac{\partial\bar{x}^i}{\partial x^{\ell}}\frac{\partial x^{m_1}}{\partial\bar{x}^{j_1}}\dots\frac{\partial^2 x^{m_p}}{\partial\bar{x}^{j_p}\partial\bar{x}^k}\Big)$$

Since B is bilinear and of total order 1, its associated map $B_o: S\to R^n\otimes\Lambda^{p+q}R^{n*}$ is a sum $B_o = B_1 + B_2$, where B_1 and B_2 are bilinear maps

$$(6)\quad B_1:\ R^n\otimes\Lambda^pR^{n*}\otimes R^{n*}\times R^n\otimes\Lambda^qR^{n*}\to R^n\otimes\Lambda^{p+q}R^{n*}$$

$$(7)\quad B_2:\ R^n\otimes\Lambda^pR^{n*}\times R^n\otimes\Lambda^qR^{n*}\otimes R^{n*}\to R^n\otimes\Lambda^{p+q}R^{n*}$$

We remark that B_o corresponds to the canonical coordinate expression of the natural operator B on R^n.

6. Consider the canonical inclusion $GL(n,R)\hookrightarrow G^2_n$ transforming every matrix into the 2-jet at 0 of the corresponding linear transformation of R^n. By (4) and (5), the linear maps associated with the bilinear maps B_1 and B_2, which will be denoted by the same symbol, are GL(n,R)-equivariant, so that we can apply the classical theory of invariant tensors, [3]. Consider first the following diagram

$$(8)\quad\begin{array}{ccc} R^n\otimes\Lambda^pR^{n*}\otimes R^{n*}\otimes R^n\otimes\Lambda^qR^{n*} & \xrightarrow{\ B_1\ } & R^n\otimes\Lambda^{p+q}R^{n*} \\ {\scriptstyle id\otimes Alt_p\otimes id\otimes Alt_q}\ \uparrow\downarrow & & {\scriptstyle id\otimes Alt_{p+q}}\ \uparrow\downarrow \\ R^n\otimes\overset{p}{\otimes}R^{n*}\otimes R^{n*}\otimes R^n\otimes\overset{q}{\otimes}R^{n*} & \longrightarrow & R^n\otimes\Lambda^{p+q}R^{n*} \end{array}$$

where Alt denotes the alternator of the indicated degree. Since the vertical maps are also GL(n,R)-equivariant, it suffices to determine all GL(n,R)-equivariant maps in the bottom row and to restrict them and to take the alternator of the result. By the theory of invariant tensors, [3], all GL(n,R)-equivariant maps $\overset{2}{\otimes}R^n\otimes\overset{p+q+1}{\otimes}R^n\to R^n\otimes\overset{p+q}{\otimes}R^{n*}$ are given by all kinds of permutations of the indices, all contractions and tensorizing with the identity. Since we apply this to alternating forms and use the alternator on the result, permutations do not play a role.

7. In what follows we discuss the case $p \geq 2$, $q \geq 2$ only and we leave the other cases to the reader. (A direct discussion shows that in the remaining cases the list (9) below should be reduced by those terms that do not make sense, but our next procedure leads to (3) as well.) Constructing B_1, we may contract the vector field part of P into a non-derivation entry of P or into the derivation entry of P or into Q, and we may contract the vector field part of Q into Q or into a non-derivation entry of P or into the derivation entry of P, and then tensorize with $I = id_{R^n}$. This gives 9-1=8 possibilities. If we perform only one contraction, we get 3+3=6 further possibilities, so that we have a 14-parameter family, which corresponds to the lower case letters in the list (9) below. Constructing B_2, we obtain analogously another 14-parameter family denoted by upper case letters in the list (9) below. Hence by $GL(n,R)$-equivariancy we deduce the following expression for B_o: $S \to R^n \otimes \Lambda^{p+q} R^{n*}$ (we do not indicate alternation in the subscripts and we write α, β for any kind of free form-index on the right hand side)

$$
\begin{aligned}
B^i_\gamma = {} & aP^m_{m\alpha,k}Q^n_{n\beta}\delta^i_\ell + bP^m_{\alpha,m}Q^n_{n\beta}\delta^i_\ell + cP^m_{\alpha,k}Q^n_{nm\beta}\delta^i_\ell + dP^m_{mn\alpha,k}Q^n_\beta\delta^i_\ell \\
& + eP^m_{n\alpha,m}Q^n_\beta\delta^i_\ell + fP^m_{n\alpha,k}Q^n_{m\beta}\delta^i_\ell + gP^m_{m\alpha,n}Q^n_\beta\delta^i_\ell + hP^m_{\alpha,n}Q^n_{m\beta}\delta^i_\ell \\
(9) \quad & + iP^m_{m\alpha,k}Q^i_\beta + jP^m_{\alpha,m}Q^i_\beta + kP^m_{\alpha,k}Q^i_{m\beta} + \ell P^i_{\alpha,k}Q^n_{n\beta} + mP^i_{n\alpha,k}Q^n_\beta + nP^i_{\alpha,n}Q^n_\beta \\
& + AP^m_{m\alpha}Q^n_{n\beta,k}\delta^i_\ell + BP^m_{m\alpha}Q^n_{\beta,n}\delta^i_\ell + CP^m_{mn\alpha}Q^n_{\beta,k}\delta^i_\ell + DP^m_\alpha Q^n_{nm\beta,k}\delta^i_\ell \\
& + EP^m_\alpha Q^n_{m\beta,n}\delta^i_\ell + FP^m_{n\alpha}Q^n_{m\beta,k}\delta^i_\ell + GP^m_\alpha Q^n_{n\beta,m}\delta^i_\ell + HP^m_{n\alpha}Q^n_{\beta,m}\delta^i_\ell \\
& + IP^i_\alpha Q^n_{n\beta,k} + JP^i_\alpha Q^n_{\beta,n} + KP^i_{n\alpha}Q^n_{\beta,k} + LP^m_{m\alpha}Q^i_{\beta,k} + MP^m_\alpha Q^i_{m\beta,k} + NP^m_\alpha Q^i_{\beta,m}
\end{aligned}
$$

8. The map B_o is also equivariant with respect to the kernel of the canonical projection $G^2_n \to GL(n,R)$ of 2-jets into 1-jets, which coincides with the abelian group $R^n \otimes S^2 R^{n*}$, provided S^2 denotes the second symmetric tensor power. By (4) and (5), the action of an element $(s^i_{jk}) \in R^n \otimes S^2 R^{n*}$ on $R^n \otimes \Lambda^p R^{n*}$ is just the identity and its action on $R^n \otimes \Lambda^p R^{n*} \otimes R^{n*}$ is

$$(10)\quad \bar{P}^i_{j_1\dots j_p,k} = P^i_{j_1\dots j_p,k} + P^t_{j_1\dots j_p}S^i_{tk} - P^i_{tj_2\dots j_p}S^t_{j_1k} - \dots - P^i_{j_1\dots j_{p-1}t}S^t_{j_pk}$$

Hence the condition that (9) is equivariant under the action of $R^n\otimes S^2R^{n*}$ has the following form (the alternation in the subscripts is not indicated explicitely)

$$\begin{aligned}
0 = {} & a(P^t_{mj_2\dots j_p}S^m_{tk} - P^m_{tj_2\dots j_p}S^t_{mk} - P^m_{mtj_3\dots j_p}S^t_{j_2k} - \dots \\
& - P^m_{mj_2\dots j_{p-1}t}S^t_{j_pk})Q^n_{n\beta}\delta^i_{\ell} + \dots + n(P^t_{j_1\dots j_p}S^i_{tn} \\
(11)\quad & - P^i_{tj_2\dots j_p}S^t_{j_1n} - \dots - P^i_{j_1\dots j_{p-1}t}S^t_{j_pn})Q^n_{\beta} \\
& + AP^m_{m\alpha}(Q^t_{nj_2\dots j_q}S^n_{tk} - Q^n_{tj_2\dots j_q}S^t_{nk} - Q^n_{ntj_3\dots j_q}S^t_{j_2k} - \dots \\
& - Q^n_{nj_2\dots j_{q-1}t}S^t_{j_qk})\delta^i_{\ell} + \dots + NP^m(Q^t_{j_1\dots j_q}S^i_{tm} \\
& - Q^i_{tj_2\dots j_q}S^t_{j_1m} - \dots - Q^i_{j_1\dots j_{q-1}t}S^t_{j_qm})
\end{aligned}$$

We remark that every term containing an S with both free subscripts vanishes after applying the alternator.

9. The right hand side of (11) represents a trilinear map $R^n\otimes\Lambda^pR^{n*}\times R^n\otimes\Lambda^qR^{n*}\times R^n\otimes S^2R^{n*}\to R^n\otimes\Lambda^{p+q}R^{n*}$ depending on the parameters a,...,N, which can be rewritten in the following form

$$\begin{aligned}
& \Big[BP^m_{m\alpha}Q^t_{\beta}S^n_{tn} + (-1)^q qBP^m_{m\alpha}Q^n_{t\beta}S^t_{nk} + bP^t_{\alpha}Q^n_{n\beta}S^m_{tm} - (-1)^{p+q}pbP^m_{t\alpha}Q^n_{n\beta}S^t_{mk} \\
& + ((-1)^q c - D - (-1)^q(q-1)G)P^m_{\alpha}Q^n_{nt\beta}S^t_{mk} + (C-(-1)^q d \\
& - (-1)^{p+q}(p-1)g)P^m_{mt\alpha}Q^n_{\beta}S^t_{nk} + eP^m_{n\alpha}Q^n_{\beta}S^t_{mt} + (H-e)P^m_{t\alpha}Q^n_{\beta}S^t_{mn} \\
(12)\quad & + EP^m_{\alpha}Q^t_{m\beta}S^n_{tn} + (h-E)P^m_{\alpha}Q^n_{t\beta}S^t_{mn} + ((-1)^q qH - (-1)^q f - F)P^m_{n\alpha}Q^n_{t\beta}S^t_{mk} \\
& + (F + (-1)^q f - (-1)^{p+q}ph)P^m_{n\alpha}Q^t_{m\beta}S^n_{tk} - (-1)^{p+q}(p-1)eP^m_{nt\alpha}Q^n_{\beta}S^t_{mk} \\
& - (-1)^q(q-1)EP^m_{\alpha}Q^n_{mt\beta}S^t_{nk}\Big]\delta^i_{\ell} + jP^t_{\alpha}Q^i_{\beta}S^m_{tm} + (-1)^{p+q}pjP^m_{t\alpha}Q^i_{\beta}S^t_{mk} +
\end{aligned}$$

$$+ JP^i_\alpha Q^t_\beta S^m_{tm} + (-1)^q qJP^i_\alpha Q^m_{t\beta} S^t_{mk} - ((-1)^q k - (-1)^q qN+M)P^m_\alpha Q^i_{t\beta} S^t_{mk}$$

$$(12) \quad + (K+(-1)^{p+q}pn - (-1)^q m)P^i_{m\alpha} Q^t_\beta S^m_{tk} - \ell(-1)^q P^t_\alpha Q^m_{m\beta} S^i_{tk}$$

$$+ LP^m_{m\alpha} Q^t_\beta S^i_{tk} - (-1)^q mP^t_{m\alpha} Q^m_\beta S^i_{tk} + MP^m_\alpha Q^t_{m\beta} S^i_{tk} + (n+N)P^m_\alpha Q^t_\beta S^i_{mt}$$

The equivariancy of B_o with respect to $R^n \otimes S^2 R^{n*}$ is equivalent to the fact that (12) is the zero map.

10. For dim $M > p+q+1$ the indices has at least $p+q+2$ different values, which implies easily that all individual terms in (12) are linearly independent in $\mathrm{Hom}(R^n \otimes \Lambda^p R^{n*} \otimes R^n \otimes \Lambda^q R^{n*} \otimes R^n \otimes S^2 R^{n*}, R^n \otimes \Lambda^{p+q} R^{n*})$. Hence (12) is the zero map if and only if all the coefficients vanish. This leads to the following equations

$$b = B = e = E = h = H = j = J = \ell = L = m = M = 0,$$

$$(13) \quad c = (q-1)G + (-1)^q D, \; C = (-1)^q d + (-1)^{p+q}(p-1)g$$

$$F = (-1)^{q-1} f, \; k = -qn, \; K = (-1)^{p+q-1} pn, \; N = -n$$

while a, A, d, D, f, g, G, n, i, I are independent parameters.

11. Thus, for dim $M > p+q+1$ we have deduced 10-parameter family (13) and one verifies easily that (9) with (13) represents the coordinate expression of the linear combinations of the elements of (3). This completes the proof of our Theorem.

Obviously, for dim $M < p+q$ it holds $\Omega^{p+q}(M,TM) = 0$, so that we can construct the zero operator only. The study of the two remaining cases dim $M = p+q$ and dim $M = p+q+1$ consists in detailed discussion of the conditions for (12) to be the zero map, which is not included into the present paper.

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