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In: Zdeněk Frolík (ed.): Abstracta. 9th Winter School on Abstract Analysis. Czechoslovak Academy of Sciences, Praha, 1981. pp. 15--16.

Persistent URL: http://dml.cz/dmlcz/701216

NINTH WINTER SCHOOL ON ABSTRACT ANALYSIS (1981)

Multipliers on complex Banach spaces

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Let X be a complex Banach space. By E_X we denote the set of extreme functionals on X, i.e. the extreme points of the unit ball of X'.

Definition: An operator $T : X \longrightarrow X$ is called a multiplier, if every $p \in E_X$ is an eigenvector for T', i.e. if there is a function $a_T : E_X \longrightarrow \mathbb{C}$ such that $p \circ T = a_T(p)p$ for $p \in E_X$. Mult(X) means the collection of all multipliers on X. For $T,S \in \text{Mult}(X)$ we say that S is an adjoint for T (and we write $S = T^*$ in this case) if $a_S = \overline{a_T}$ (complex conjugate).

In our talk we discuss conditions on T and/or X such that T^* exists (in general, T will not have an adjoint; consider for example $X :=$ the disk algebra and $T : f \mapsto gf$ with nonconstant g). Among other facts we show that $T \in \text{Mult}(X)$ has an adjoint T^* if any one of the following conditions is satisfied:

(1) X is finite-dimensional

(2) X is smooth

(3) X can be embedded as a self - adjoint subspace of a CK-space

(4) $\sigma(T)$ is contained in the closure of the unbounded component of $\mathbb{C} \setminus \sigma(T)$

(5) X is an L^1-predual space, and $\overline{E_X}$ (weak*-closure) is contained in the convex hull of E_X; this is satisfied, for example, if X is an abstract G-space

(6) X is an L^1-predual space, and the unit ball of X has an extreme point

(7) X can be represented as a space
$X = \{f \mid f \in CK\ ,\ f(k_i) = \int f_i \, d\mu_i \quad \text{for} \quad i=1,\dots,n\}$,
where K is a compact Hausdorff space, $k_1,\dots,k_n$ are distinct elements of K , $\mu_1,\dots,\mu_n$ are (signed) measures on K such that $\|\mu_i\| \leq 1$, $|\mu_i|(\{k_1,\dots,k_n\}) = o$ for $i=1,\dots,n$.

Problems:

1. Is it true that T^* exists whenever X is reflexive (or strictly convex) and $T \in Mult(X)$?
2. Has every $T \in Mult(X)$ an adjoint if X is an L^1-predual space ?

Basic facts concerning multipliers as well as a development of the theory of M-structure where multipliers and their adjoints are of interest can be found in the Lecture Notes volume of the author ("M-Structure and the Banach-Stone theorem"; Lecture Notes in Mathematics 736, Springer-Verlag 1979)