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In: (ed.): General Topology and its Relations to Modern Analysis and Algebra, Proceedings of the second Prague topological symposium, 1966. Academia Publishing House of the Czechoslovak Academy of Sciences, Praha, 1967. pp. 235.

Persistent URL: http://dml.cz/dmlcz/700857

ON DISCONTINUOUS SELECTORS

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Let X be an arbitrary set and $\mathscr{L}$ a field of subsets of X. Let $\mathscr{S}$ denote the family of countable unions of members of L.

Let, further, 2^Y denote the space of all closed non void subsets of a topological space Y (with the Vietoris topology).

Theorem. *Let Y be complete separable and let $F : X \to 2^Y$ be such that*

$$\{x : F(x) \cap G \neq \emptyset\} \in \mathscr{S} \quad \text{whenever} \quad G \subset Y \quad \text{is open}.$$

Then there is a selector $f : X \to Y$ (i.e. $f(x) \in F(x)$) such that

$$f^{-1}(G) \in \mathscr{S} \quad \text{whenever} \quad G \subset Y \quad \text{is open}.$$

Corollary. *For each complete separable space Y there is a choice function $f : 2^Y \to Y$ of the first class of Baire.*

f may be assumed continuous if $\dim Y = 0$ (i.e. if Y contains a countable base composed of closed-open sets).

There are also given further applications of these statements.

The main results presented in this communication appeared in the paper of K. Kuratowski and C. Ryll-Nardzewski in the Bull. of the Polish Academy of Sciences *13* (1965), pp. 397–403, under the title "A general theorem on selectors".