ON MILD AND WICKED EMBEDDINGS

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A subset A of a triangulated space X is said to be *tame* iff there is an autohomeomorphism of X mapping A onto a polyhedron in X. Otherwise A is said to be *wild* in X [1]. It is well-known that every tame subset of X is a deformation retract of a neighborhood in X (which may be taken arbitrarily small) [4]. However, as simple polyhedra as the arc, the simple closed curve, the disk, the sphere and the ball may be so embedded in some 3-manifolds that they are deformation retracts of no neighborhood [2]. This gives rise to the following definition. A subset A of a topological space X is *mildly embedded* in X (for short – is *mild* in X) iff A is a deformation retract of a neighborhood in X. Otherwise, A is said to be *wickedly embedded* in X (is *wicked* in X). It is clear that a wicked topological polyhedron in a triangulated space must be wild.

Obviously enough, if A and X are AR-spaces, then A is mild in X. But, if A or X is only an ANR-space, the situation is quite different as we have already mentioned. Perhaps the most interesting case is the one where X is the Euclidean n-dimensional space E^n or the n-sphere S^n, and A is an m-dimensional manifold with $1 \leqq m \leqq n$. It has been proved [2] that a topological $(n-1)$-sphere is always mild in S^n and there are some indications to support the conjecture that such is the case of any ANR-set which is homotopy equivalent to S^m with $0 \leqq m < n$. On the other hand, any orientable closed surface of positive genus may be wicked in S^3 [3].

It can be proved that an orientable closed surface is mild in S^3 if it can be homeomorphically approximated by unknotted surfaces in the sense that each of them bounds a cube with handles in every complementary domain.

Thus, one is led to ask the following questions.

1. Let A be an ANR-subset of S^n which is homotopy equivalent to S^m (or simply, $A \underset{\text{top}}{=} S^m$). Is A mild in S^n?

If $m = 1$ and $n = 3$, the answer is yes.

2. Do there exist wicked embeddings of an m-manifold in S^n for $1 < m \leqq n$ and $n > 3$?.

3. Does there exist a sufficient condition for mild embedding of an $(n-1)$-manifold in S^n $(n > 3)$ similar to the one for closed orientable surfaces in S^3?

References

[1] *R. H. Fox and E. Artin:* Some wild cells and spheres in 3-space. Ann. of Math. *49* (1948), 979—990.

[2] *A. Kirkor:* Every topological (n — 1)-sphere is a deformation retract of an open neighborhood in the n-sphere. Bull. Acad. Polon. Sci. Sér. Sci. Math. Astronom. Phys. *17* (1969), 801—807.

[3] *A. Kirkor:* A positional characterization of the 2-sphere. Bull. Acad. Polon. Sci. Sér. Sci. Math. Astronom. Phys. *18* (1970), 437—442.

[4] *E. C. Zeeman:* Seminar on combinatorial topology. Inst. Hautes Études Sci. Publ. Math., 1963.

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