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A NOTE ON RUDIN'S EXAMPLE OF DOWKER SPACE

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One assumption on a topological space occurs very frequently in mathematics – the property of being normal and countably paracompact. E.g. in such a space every Baire measure can be extended into a Borel measure [3]. In normal and countably paracompact space realcompactness can be described without (explicit or implicit) use of the notion of zero-set [1].

Last year Mrs. M. E. Rudin gave an example of a normal Hausdorff space Y, which is not countably paracompact ([4], [5], [6]). It seemed quite natural to study some other properties of the space Y in order to show the importance of the assumption of countable paracompactness.

Let us recall the following definitions from [1] and [2].

A topological space will be called *almost realcompact*, iff, whenever $\mathscr{A}$ is a maximal centered collection of open sets such that $\{\bar{A} \mid A \in \mathscr{A}\}$ has the countable intersection property (abbr. CIP), then $\bigcap\{\bar{A} \mid A \in \mathscr{A}\}$ is non-void. A topological space will be called *closed complete*, iff, whenever $\mathscr{A}$ is a maximal centered collection of closed sets with CIP, then $\bigcap\mathscr{A}$ is non-void. A topological space will be called *Baire-Borel complete*, if every maximal centered collection of zero sets $\mathscr{Z}$ with CIP has non-void intersection whenever there exists some maximal centered collection of Borel sets $\mathscr{B}$ with CIP such that $\mathscr{B} \supset \mathscr{Z}$.

Theorem 1. *The space Y is neither almost realcompact nor realcompact.*

Theorem 2. *The space Y is not Baire-Borel complete.*

Theorem 3. *There exists maximal centered collection $\mathscr{Z}$ with* CIP, *consisting of zero-sets in Y, which cannot be extended to maximal centered collection of closed sets with* CIP.

Theorem 4. *The space Y is closed complete.*

References

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