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In: Josef Novák (ed.): General Topology and its Relations to Modern Analysis and Algebra, Proceedings of the Third Prague Topological Symposium, 1971. Academia Publishing House of the Czechoslovak Academy of Sciences, Praha, 1972. pp. 215--216.

Persistent URL: http://dml.cz/dmlcz/700724

DISCONNECTED BOUNDED *PL* MANIFOLDS IN EUCLIDEAN SPACES

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In this paper we consider the following problem: Under what conditions one can extend a given embedding $g\colon \partial M \to E^q$ to an embedidng of the whole M, where M is disconnected bounded PL (piecewise linear) manifold of dimension n and all embeddings are assumed to be piecewise linear. Under a disconnected bounded PL manifold we understand a compact PL manifold such that each component of M has a nonempty boundary. We say that M unknots rel ∂M in E^q, if every two extensions are isotopic keeping the boundary fixed.

Proposition 1. *Every PL embedding $g\colon \partial M \to E^{2n+1}$ extends to a PL embedding of M. Furthermore, M unknots* rel ∂M *in E^{2n+1}.*

Proposition 2. *Every PL embedding $g\colon \partial M \to E^{2n}$ extends to a PL embedding of M. M in general knots* rel ∂M.

Denote by $M_1 \cup M_2 = M$ a disjoint union of two manifolds.

Theorem 1. *Let $M = M_1 \cup M_2$ be a closed orientable PL n-manifold in E^{2n+1}, $n \geqq 2$. Then the linking number $L(M_1, M_2)$ classifies, up to an ambient isotopy, the embeddings of M into E^{2n+1}.*

Theorem 2. *Let $M = M_1 \cup M_2$ be a compact bounded orientable PL n-manifold, $n \geqq 3$, such that each ∂M_i $(i = 1, 2)$ is nonempty and connected, and $g\colon \partial M = \partial M_1 \cup \partial M_2 \to E^{2n-1}$ is a PL embedding. Then g extends to a PL embedding of M if and only if the linking number $L(g(\partial M_1), g(\partial M_2)) = 0$.*

Theorem 3. *Let $M = M_1 \cup M_2$ be a compact bounded PL n-manifold. Assume that M_1 is $(k + 1)$-connected and M_2 is k-connected, $2k + 2 < n$, $k \leqq n - 4$. Let $g\colon \partial M \to E^{2n-k-1}$ be a PL embedding. If the embeddings $g_1 = g|\partial M_1 \colon \partial M_1 \to E^{2n-k-1} - g(\partial M_2)$ and $g_2 = g|\partial M_2 \colon \partial M_2 \to E^{2n-k-1} - g(\partial M_1)$ are inessential (homotopic to a constant map), then g extends to a PL embedding of M.*

Using the same technique as in the proofs of the above statements one can prove results about unlinking in Euclidean spaces. We say that two polyhedra X

and Y in E^q are geometrically unlinked if there is a q-ball which contains one of them and does not intersect the other.

Proposition 3. *Let M_1 and M_2 be compact bounded PL n-manifolds in E^{2n}. Then M_1 and M_2 are geometrically unlinked.*

Theorem 4. *Let M_1 and M_2 be two compact closed PL n-manifolds in E^{2n+1}, $n \geqq 2$. M_1 and M_2 are geometrically unlinked in E^{2n+1} if and only if the given embedding $M_1 \cup M_2 \to E^{2n+1}$ extends to an embedding of a cone $C(M_1 \cup M_2) \to \to E^{2n+1}$.*