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A NOTE ON THE DIMENSION OF PRODUCTS

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A subset of a uniform space $(X,\mathcal{U})$ is called $\mathcal{U}$-open if it is the inverse image of an open subset of the space of real numbers under a uniformly continuous function. Complements of $\mathcal{U}$-open sets are called $\mathcal{U}$-closed. We set $\mathcal{U}$- dim X = -1 or $\mathcal{U}$- Ind X = -1 if and only if $X = \emptyset$. For n = 0,1,2,... we write $\mathcal{U}$-dim $X \leq n$ if every finite $\mathcal{U}$-open cover of X can be refined by a finite $\mathcal{U}$-open cover of order $\leq n$; and $\mathcal{U}$- Ind $X \leq n$ if for any two disjoint $\mathcal{U}$-closed sets E_1, E_2 of X there are disjoint $\mathcal{U}$-open sets G_1, G_2 with $E_1 \subset G_1$, $E_2 \subset G_2$ and $\mathcal{U}$- Ind $(X-G_1 \cup G_2) \leq n-1$, where for a subset Y of X we write $\mathcal{U}$- Ind Y rather than $\mathcal{U}_Y$- Ind Y. If $\mathfrak{M}$ is the Čech uniformity on a Tychonoff space X, we set Ind X = $\mathfrak{M}$- Ind X. These dimension functions are rather well-behaved with respect to properties that it is desirable for a dimension function to possess, e.g. subset and sum theorems [1, 2, 3, 4]. Proofs of the following results will appear in a forthcoming paper.

Proposition 1. Every uniform space with $\mathcal{U}$-dim $\leq n$ can be densely embedded in a uniform space with $\mathcal{U}$-dim $\leq n$ and which is the inverse limit of metric spaces with dim $\leq n$.

Proposition 2. For any infinite cardinals α, β, there is a universal space for $\mathcal{U}$-dim $\leq n$ and double weight $\leq (\alpha, \beta)$.

Proposition 3. If one of $(X,\mathcal{U})$, $(Y,\mathcal{V})$ is not empty, then $\mathcal{U} \times \mathcal{V}$ -dim $X \times Y \leq \mathcal{U}$-dim X + $\mathcal{V}$ - dim Y.

Proposition 4. If one of $(X,\mathcal{U})$, $(Y,\mathcal{V})$ is not empty, then $\mathcal{U} \times \mathcal{V}$ - Ind $X \times Y \leq \mathcal{U}$- Ind X + $\mathcal{V}$ - Ind Y.

Proposition 5. If every cozero subset of $(X,\mathcal{U})$ is the union of a σ-locally finite collection of $\mathcal{U}$-open sets of X, then, for any subset Y of X, $\mathfrak{M}$- Ind Y $\leq \mathcal{U}$ - Ind Y, where $\mathfrak{M}$ is the Čech uniformity on X.

Proposition 6. If $X \times Y$ is rectangular [5], i.e. every finite cozero cover of $X \times Y$ can be refined by a σ-locally finite cover consisting of products of cozero sets of X, Y, and one of X, Y is not empty, then $\mathrm{Ind}^* X \times Y \leq \mathrm{Ind}^* X + \mathrm{Ind}^* Y$.

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