Tomasz Byczkowski
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ON THE OPEN MAPPING THEOREM

T. BYCZKOWSKI

Wrocław

Let X, Y be two complete metric linear spaces and let $T:X \to Y$ be a continuous linear mapping from X onto Y. It is well known that T is then open. This theorem, proved many years ago by Banach, have been generalized by several authors. This theorem is closely related to another one, the so-called 'closed graph theorem ': if X and Y are complete metric linear spaces then a linear mapping $T:X \to Y$ is continuous if and only if it has the closed graph in $X \times Y$. These theorems have been generalized by several authors, among them by Weston [4], Brown [1] and Pettis [3].

In order to formulate a generalization of the open mapping theorem for mappings between topological spaces we recall some notions.

A subset of a topological space is called nearly open if it is contained in the interior of its closure. A mapping from one topological space to another is called nearly open (nearly continuous) if the image (the inverse image) of every open set is nearly open (see [3,4]). A topological space is called complete in the sense of Čech if it is homeomorphic to a G_δ subset of a compact Hausdorff space.

The following theorem holds:

Theorem. Every nearly open continuous bijection on a complete in the sense of Čech space to a Hausdorff space is open.

Proof of slightly strengthened version of this theorem is contained in [2]. The theorem generalizes Theorem 8 in [4] as well as results of Pettis [3].

From the theorem we can deduce the following corollaries:

1. If X and Y are complete in the sense of Čech and $F:X \to Y$ is one-to-one, has the closed graph and is nearly open then F is open as the mapping to F(X); moreover, if Y is a topological group and F(X) is a subgroup then F is open.

2. If X and Y are complete in the sense of Čech then F on X to Y is continuous if and only if it is nearly continuous and has the closed graph.

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INSTITUTE OF MATHEMATICS, TECHNICAL UNIVERSITY OF WROCŁAW