

# Mathematics throughout the ages

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# ON A SMALL COMENIUS' WORK "GEOMETRY AND GEODESY"

KAREL MAČÁK

## 1 Introduction

As for the title of the considered Comenius' work, this small work was published in Latin in the collected works of Comenius [1] with the title *Geometria* and its Czech translation appeared in the selected works of Comenius [2] with the title *Geometrie a geodézie*. The Czech title corresponds well to the two parts of the work and therefore it will be used (translated into English) in this article, if the whole work will be discussed; the titles *Geometry* and *Geodesy* will be used for the two parts of the whole work.

This work is not very known and therefore at first the main circumstances of the emergence of the work will be mentioned.<sup>1</sup>

At the beginning of the year 1628 Comenius with his family and other Czech Brethren emigrated to the Polish town Leszno. Leszno had about 12000 inhabitants at that time (i.e. more than Berlin, the capital of the electorate of Brandenburg) and the owners of the demesne was the protestant house of Leszynsky,<sup>2</sup> who derived his descent from Bohemia and had the same coat of arms as the Czech house of Pernštejn.<sup>3</sup> Leszno obtained the middle-age privileges in 1547 and its development is connected with the Czech Brethren who emigrated to Leszno at first after the unsuccessful uprising of the Czech Estates in 1547; a new wave of emigrants came after the battle of White Mountain.

In the time when Comenius came to Leszno, the demesne belonged to Rafael V Leszynsky. He was a cultured man, who spent his young days

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<sup>1</sup>The facts concernig the life of Comenius are adopted from [3].

<sup>2</sup>It was an important house in Poland, what can be seen from the fact that Stanislaw I. Leszynsky was elected in 1704 the King of Poland.

<sup>3</sup>It is a head of an European bison with a withy in the nostrils.

in Paris, London, Florence and Prague, studied at Calvinist universities in Basel and Strassbourg and spoke fluently five languages. From the confessional point of view he was very tolerant; not only did Protestants of five different denominations but also Catholics and Eastern Orthodox believers live on his demesne. From our point of view it is important that under his rule also the grammar school in Leszno developed well; this school was founded in 1555 by his grandfather Rafael IV and in 1624 it was transformed to a higher Latin school which was intended to prepare aristocratic sons and wealthy citizens for possible studies abroad.

When the emigrants came to Leszno, the town was overpopulated and the cost of living rose steeply. Moreover, there are about forty clergymen among the emigrants and they could not earn their living by practising their profession. Therefore Comenius accepted the offer to teach at Latin school (for 140 florins per year). The small work *Geometry and geodesy* arose probably from 1628 to 1631 as Comenius' draft of his lessons; by the postscript to [2] it could also be a fair copy of student's notes from the lessons.<sup>4</sup> Comenius never edited the work and this fact can be seen on the quality of the text; in the postscript to [2] we can find that not only formal, but also factual mistakes can be found in the text and also the figures are sometimes quite careless.

The manuscript of the *Geometry and geodesy* was found by the Czech research worker Stanislav Souček with the help of the Russian Slavist V. G. Černobajev in 1931 as a part of the so-called Leningrad's manuscript in the Library of Saltykov-Ščedrin in today's Petrohrad; the details about the publishing of the text of the manuscript can be found in [1, 2].

It is clear from the given facts that the manuscript of *Geometry and geodesy* meant in the life's work of Comenius only a side issue, which influenced nobody and nothing. Nevertheless in our opinion the work is worth reading because it gives us quite concrete information about the teaching of mathematics in the first half of the 17<sup>th</sup> century; after all, it is a draft of lessons or notes from lessons. Moreover, from the point of view of teaching mathematics Comenius on one hand was not a professional mathematician, but on the other hand he represented a teacher of a common mathematical level at that time (see the following part of this article) and from the point of view of the knowledge of the general pedagogic theory he was over the average of the teachers of that time. In our opinion the work could be held for a "snap" of the teaching

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<sup>4</sup>The question of the authorship of Comenius is discussed in [4].

mathematics at that time, which was made by a top photographer; from this point of view it is not a fault that Comenius never edited the work, rather the reverse is the case: with a bit of imagination we can say that thanks to all the circumstances we have the possibility to see how the great pedagogical theorist Comenius taught the mathematics in practice.

## 2 The mathematical education of Comenius

As was said in the previous part Comenius was not a professional mathematician, but he represented a teacher of a common mathematical level at that time. To explain the level of his mathematical knowledge an overview of his university studies will be given now.

Jan Amos Komenský (Comenius) was born in 1592 and after having finished his studies at the grammar school in Přerov he was sent with a group of other young people to study at universities in Herborn and Heidelberg.<sup>5</sup>

The academy in Herborn was founded in 1548 and it existed till 1847, when it was changed to a Protestant seminary. It never obtained the official university status, but its level was allegedly better than the level of many established universities.<sup>6</sup> It had all four common university faculties<sup>7</sup> and at the turn of the 16<sup>th</sup>–17<sup>th</sup> century it was the centre of the Calvinist education in Germany. Comenius studied here at the Faculty of Arts from spring 1611 till spring 1613, then he peregrinated with his friends about three months in Germany and the Netherlands and from summer 1613 till spring 1614 he studied at the theological faculty of the university in Heidelberg. He finished his studies with a defence of theological thesis; in the sources there are not any facts about conferring an academic degree on Comenius.

From our point of view his studies at the Faculty of Arts are important, because on the one hand the faculties of arts were at that time the “preparatory” faculties for the other three faculties, i.e. all university students had to study at first at least two years at this faculty; on the other hand, the faculties of arts prepared the prospective teachers for

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<sup>5</sup>Comenius' study was financially supported by the Czech Brethren and the nobleman Charles the Elder of Žerotín with the assumption of the prospective priesthood of Comenius.

<sup>6</sup>For example, it is known that the level of the Charles university in Prague was not good at that time (see [5], p. 25, 74).

<sup>7</sup>They were the faculties of arts and theology and the faculties of medicine and of Law.

grammar schools.<sup>8</sup> At these faculties the seven liberal arts were studied, which were divided into two parts: the so-called *trivium* consisting of grammar, rhetoric and dialectics, and the so-called *quadrivium* consisting of arithmetic, geometry, astronomy and the co-called *musica* (the mathematical theory of musical intervals). Comenius himself wrote (see [3], p. 27) that during his studies he was influenced most by two professors of Herborn Academy: by the theologian Johann Fischer-Piscator and by the philosopher Johann Heinrich Alsted (1588–1638; from 1609 an extraordinary professor at Herborn’s academy ([3], p. 28)). Alsted aimed at an encyclopedic overview and summation of all old and new knowledge in a coherent and clearly arranged system and he is the author of the first encyclopaedia edited in Germany *Encyclopaedia septem tomis distincta*, which appeared in Herborn in 1630 and which is mentioned also in the history of mathematics ([6], p. 719). Alsted prepared the material for his encyclopaedia earlier of course and Comenius could probably use the Alsted’s mathematical works [7, 8] as a base for his work.

Therefore it can be said that although Comenius probably did not formally obtain the academic bachelor’s degree, his academic mathematical education was on the level which was usual for qualified teachers at that time, i.e. two-year studies at the faculty of arts (see [5], p. 26). When he came to Leszno in 1628, he had, in addition to his university education, also some years of the teaching practice at schools in Přerov and Fulnek, hence he could write his work *Geometry and geodesy* not only on the basis of his theoretical opinions, but also on the basis of his own teaching experience.

Comenius’ interest in the natural sciences can be documented by one more fact: at the beginning of the year 1614 Comenius bought in Heidelberg the original manuscript of the Copernicus’ epochal work *De revolutionibus orbium coelestium libri sex*, but he did not have money enough for his journey home then and therefore he had to go home on foot ([3], p. 34); it is interesting in this connection that Comenius himself was an opponent of Copernicus’ heliocentric theory.<sup>9</sup>

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<sup>8</sup>Of course, the structure of the schools at that time was not the same as it is today and the terms for the various types of schools do not correspond exactly to the today’s terms (see e.g. [5]), p. 25, 73).

<sup>9</sup>As for Copernicus’ manuscript, Comenius sold it in 1637 to the Czech nobleman Otto Nostic and so it got to the Nostic’ library in Prague. In 1957 it was exchanged with Poland for the manuscript of the Comenius’ work *Labyrinth světa a ráj srdce*.

### 3 The main parts of the work

It has already been said that the whole work *Geometry and geodesy* is divided into two parts: the first part *Geometry*.<sup>10</sup> with the volume of 14 pages of the format B 5 and the second part *Geodesy*<sup>11</sup> with the volume of 4,5 pages of the same format. The first part contains an introduction (*Proemium*) and the following seven chapters:<sup>12</sup>

*I. Definition and parts of geometry.*

*II. About a point and a line.*

*III. About an angle.*

*IV. About a figure.*

*V. About a circle.*

*VI. About a triangle.*

*VII. About a body.*

The text of the chapters II.-V. can be divided into two parts: the “explanation” of the matter and the “problems”. Although it could be interesting to comment all of Comenius’ work here, attention will be paid only to the problems, because almost all are “classic” and – in our opinion – the level of the work can be well illustrated by them.

As for the second part of the work, it is divided into the following seven chapters:<sup>13</sup>

*I. About the construction of a geometrical quadrant.*

*II. About the general use of the quadrant.*

*III. About geometrical measures.*

*IV. About planimetry.*

*V. About the measurement of height.*

*VI. About the measurement of depth.*

*VII: About the measurement of height without an apparatus.*

From the mathematical point of view, all problems of these parts are solved with the use of the concept of similar triangles; the technical basis for solving these problems is the use of the well-known apparatus called *quadrans*. The matter of these chapters is quite common in the textbooks of geometry of that time and we will not deal with it here.

<sup>10</sup>In Latin *Geometria theoretica*

<sup>11</sup>In Latin *Secunda pars geometriae geodesia dicta*

<sup>12</sup>*I. Geometriae definitio et divisio, II. De puncto et linea, III. De angulo, IV. De superficie, V. De circulo, VI. De triangulo, VII. De corpore.*

<sup>13</sup>*I. De quadrantis geometrici structura, II. De quadrantis usu in genere, III. De mensuris geometricis, IV. De planimetria, V. De altimetria, VI. De profundimetria, VII. De mensuranda altitudine absque instrumento.*

## 4 The problems in the *Geometry*

### 4.1 Problems in Chapter II

In the chapter *About a point and a line* there are three problems:<sup>14</sup>

1. To divide a given straight line<sup>15</sup> into two equal parts.
2. To draw a parallel line to a given straight line.
3. To draw a perpendicular to a given line.<sup>16</sup>

All these problems are classic,<sup>17</sup> but the solution of the second problem given by Comenius is a little surprising:<sup>18</sup>

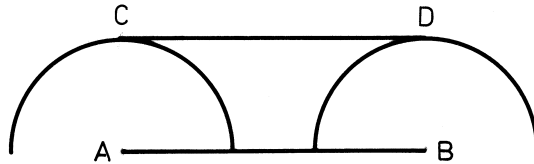


Figure 1

A line AB is given, to which a parallel line CD is to be drawn. Therefore circumscribe two semicircles<sup>19</sup> from the points A, B and so the line touching the both semicircles is the parallel line CD.

Drawing a common tangent to two given circles is probably regarded by Comenius as an elementary problem which could be solved “experimentally”.

It is not clear why Comenius used this method, because he (as a former university student) had to know the solution of this problem given in the first book of Euclid’s *Elementa*, where at first (I/23) the problem of moving an angle is solved and then, with the help of this solution, the problem of drawing in a given point the parallel line to a given line is solved (I/31). Nevertheless, the method described by

<sup>14</sup>1. *Datam lineam rectam in partes duas aequales dividere.* 2. *Datae lineae rectae parallelam ducere.* 3. *Lineam perpendicularem excitare.*

<sup>15</sup>(i.e. a given finite straight line)

<sup>16</sup>The perpendicular ought to be drawn in a given point on the given line; it follows from the solution given by Comenius.

<sup>17</sup>They are the problems I/10, I/31 and I/11 of Euclid.

<sup>18</sup>*Sit data linea AB, cui aequidistans debet poni CD. Describere itaque ex puncto A et ex puncto B semicirculos duos, et tum linea semicirculos tangens dabit parallelam lineam CD* The solution is illustrated in [1] by a figure on page 16 and this figure is the same as the following Figure 1.

<sup>19</sup>They must have the same radii, which is not said in the text, but it is clear from the figure.

Comenius is not original; it can be found e.g. in jesuit mathematical manuscripts from that time<sup>20</sup> and L. Nový ([11], p. 169) found it in a textbook of the so-called practical geometry used in Bohemia in the 18<sup>th</sup> century. Therefore it is obvious, that this method was common and widespread at that time, but we do not know where it originated.

## 4.2 Problems in Chapter III

In the chapter *About an angle*, there are four problems:<sup>21</sup>

1. *To draw an angle equal to a given angle.*
2. *To divide an angle into two equal parts.*
3. *To divide an angle into any number of equal or unequal parts; let the angle BAC is divided into three parts.*
4. *To decide whether a given angle is perpendicular.*

The first two problems are classic<sup>22</sup> and the third problem is well-known, too, but in another sense; Comenius deals here with the problem of the trisection of an angle. This problem originates from the ancient world and it was known that its general solution is not elementary ([12], vol. I, p. 235-244); therefore it is surprising that Comenius gives this problem in an elementary course of geometry. On the other hand, in practice such problems occur and a simple approximate method for solving them can be important and interesting even in an introductory course of practical geometry. In the quoted Alsted's books [7, 8] this problem is not mentioned, hence Comenius on one hand did not only take up the problems from Alsted, but on the other hand it is seen that his own knowledge of mathematics was rather practical than theoretical. His method of the trisection is simple:<sup>23</sup>

*From the centre A draw a circle BC and the drawn arc divide with a pair of compasses into three equal parts and from these points D, E draw lines to A.*

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<sup>20</sup>(e.g. in the manuscript XII G 7 in the National library of the Czech Republic in Prague on the f. 4v; for details see [9, 10])

<sup>21</sup>1. *Dato angule aequalem describere.* 2. *Datum angulum in duas aequales partes dividere.* 3. *Angulum datum in quotvis partes aequales vel inequales dividere; sit angulus BAC dividendus in tres angulos aequales.* 4. *Datum angulum, num rectus sit, examinare.*

<sup>22</sup>They are the problems I/23 and I/9 in Euclid.

<sup>23</sup>*Ex centro A describe peripheriam BC et arcum interceptum divide circino in partes aequales tres atque ex punctis illis (D et E) lineas duc ad A.* The solution is illustrated in [1] by a figure on page 20 and this figure is the same as the following Figure 2.



Dividing an arc into three equal parts is probably regarded by Comenius again as an elementary problem which could be solved “experimentally”.

As for the fourth problem, Comenius supposed that the sides of the angle are line segments of equal length and solved the problem simply with the help of the Thalet’s theorem, but the theorem itself does not occur in the Comenius’ text.

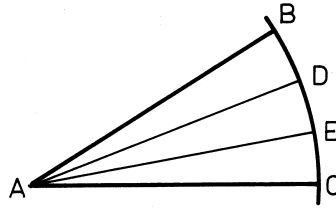


Figure 2

### 4.3 Problems in Chapter IV

In the chapter *About a figure* there is only one problem: <sup>24</sup>

*To find the centre of any given figure.*

The definition of the term “centre of a figure” is not quite clear at Comenius;<sup>25</sup> from the figures it seems that the centre of gravity is meant and in the Czech translation [2] the Czech term “*těžiště*” is sometimes used.<sup>26</sup> If Comenius really had in mind the term “centre of gravity”, then he touched here on a classical problem which e.g. Archimedes dealt with and which was very contemporary in the Comenius’ time in the connection with the emergence of the calculus of the infinitesimal quantities, but the problem of finding the centre of gravity of any given figure is not elementary and cannot be solved generally with geometrical tools. Comenius in fact does not a solution of the mentioned problem; he gives only some elementary figures.

<sup>24</sup>*Data quacunq̄ue superficie centrum invenire.*

<sup>25</sup>[1], p. 21: *Centrum est medium superficiei punctum.* for comparison Alsted see ([7], p. 44): *Centrum est punctum in figura medium.* of course, from the mathematical point of view it is not a definition.

<sup>26</sup>The quoted definition of Comenius is translated in [2], p. 29: *Těžiště je střední bod útvaru.*

#### 4.4 Problems in Chapter V

In the chapter *About a circle*, there are two problems:<sup>27</sup>

1. *The quadrature of a circle.*
2. *To draw a circle with the area equal to two given equal circles (i.e. the duplication of a circle).*

As for the second problem, Comenius solved the problem simply with the help of the Pythagoras' theorem, but the theorem itself does not occur in Comenius' text.

As for the first problem, the quadrature of a circle is a well-known problem in the history of mathematics and all that was said about the trisection of an angle can be repeated here. It originates from the ancient world and it was known that its solution is not elementary ([12], vol. I. p. 220–235); therefore it is surprising, that Comenius gives this problem in an elementary course of geometry. On the other hand, in practice such problems occur (as well as the connected problem of the rectification of a circle) and a simple approximate method for solving them could be important and interesting even in an introductory course of practical geometry.

The solution given by Comenius is extraordinarily bad and in both editions [1, 2] it is widely commented; we begin here with the quotations of the Comenius' formulations of the problem and its solution. In [1] on page 23 Comenius wrote:<sup>28</sup>

*If anybody would like to construct the square equal to a given circle, in the same proportion to the diameter, and he will have what he asked (because if the diameter of the circle has 14 feet and the square 11, the areas will be the same). Divide then the diameter of any given circle into 14 parts<sup>29</sup> and take 11 of them, draw four sides and you have the square equal to the circle.*

This solution corresponds to the approximation  $\pi = 121/49 \doteq 2,47$  and it is so bad, that – in our opinion – it cannot be taken seriously under any circumstances; it is a mistake and we ought to ask how Comenius (or another author of the manuscript) could make such a great mistake. Our commentary will be divided into six parts.

<sup>27</sup>1. *Circulum quadrare.* 2. *Datis duobus aequalibus circulis tertium capacitae ambobus parem excitare.*

<sup>28</sup>*Siquis quadratum circulo dato aequalem exstruere velit, aequae proportionem diametro, et habebit quaesitum (nam si diameter circuli habet pedes 14 et quadrati 11, areae aequales erunt). Datum ergo cujuscunque circuli diametrum divide in partes 14, et sumptis ex illis 11, constitue 4 latera; habebis quadratum circulo aequalem.*

<sup>29</sup>(i.e. into 14 equal parts)

A) First, we should say, that Comenius gives only a few lines over his solution of the problem of the quadrature the following approximation:  $\pi = 87/27 \doteq 3,107$ ; it is not very good,<sup>30</sup> but it is anyway much better than the approximation in the quadrature. Nevertheless, it is surprising that Comenius' teacher Alsted used ([7], p. 66, [8], p. 119) the well-known Archimedes' approximation  $\pi = 22/7 \doteq 3,1428$  and Comenius did not use it. It is possible that Comenius did not have the books of his teacher [7, 8] in Leszno and quoted them only by heart (and therefore not quite correctly).<sup>31</sup>

B) Secondly we should try to explain the quite unclear second part of the first sentence, i.e. the part *in the same proportion to the diameter*. We have already said, that Alsted knew Archimedes' approximation for  $\pi$ ; he knew also the Archimedes' result that the area of a circle is equal to the product of the radius and one half of the circumference of the circle ([7], p. 95). Alsted could then probably know also the third result of the Archimedes' work *Measurement of a circle* (see e.g. [12], vol. II, p. 50–56), i.e. the fact that the ratio of the area of a circle is to the square on its diameter approximately as 11 to 14. With the help of this result the problem of the quadrature of a circle could be formulated as follows: “To construct the square with the area equal to 11/14 of the area of the square on the diameter of the given circle”. In our opinion, Comenius' assertion in the brackets could indicate that he had here in mind the mentioned result of Archimedes, but he used it erroneously.

C) If Comenius' text of the solution is compared with the corresponding text in Alsted's books [7, 8], then almost the same text as by Comenius can be found by Alsted. In [7] on page 90 under the title *Problemata agrimensorum hoc ordine commemorantur* Alsted writes:

1. *Dato circulo aequale quadratum reperire. Divisa diametro in 14 partes aequales, undecim ex illis erunt latus quadrati circulo aequalis.*

In [8] on page 144 he writes exactly the same and he adds moreover: *intellige aequalitatem non-exactam.*

However, in our opinion Alsted solves here the so-called “false” quadrature, i.e. the problem: to draw the square with the perimeter equal to the circumference of a given circle.<sup>32</sup> In [7] on page 66 he begins the explication about the quadrature of a circle with the statements about the ratio of the diameter to the circumference of the circle;<sup>33</sup> in [8] on

<sup>30</sup>We do not know where Comenius found this approximation.

<sup>31</sup>Comenius the value  $87/27$  gives for  $\pi$ , the well-known Archimedes' approximation  $22/7$  can be written in the form  $88/28$ .

<sup>32</sup>It is in fact the problem of the rectification of a circle.

<sup>33</sup>... *ita diameter ter continetur in peripheria cum 1/7.*

page 119 he distinguishes between the quadrature of a circle (*quadratura sive quadratio circuli*) and the finding of the area of a circle (*inventio area*) and says: *The quadrature of a circle depends on the ratio of the diameter to the circumference.*<sup>34</sup> Therefore it seems that the mistake in Comenius' text could arise from a terminological confusion: Comenius could quote Alsted's solution of the quadrature and did not notice that the term "quadrature" by Alsted means, surprisingly, not the "usual" quadrature but only the "false" quadrature.

As for the rightness of the Alsted's "false" quadrature, it is "correct" in the sense that it corresponds exactly to the value  $\pi = 22/7$ .

D) It is possible that Comenius used not only the Alsted's books, but also some other sources as a source for his text; it was shown already that Comenius wrote about the trisection of an angle and this problem is not discussed by Alsted. Under this assumption another explanation of the "bad" quadrature by Comenius can be given: it could be a "correct" quadrature which was known and used in that time (but not by Alsted), but Comenius quoted it by heart and therefore erroneously (or his pupil wrote it incorrectly).

The "correct" quadrature which we have in mind is the following one (see figure 3):

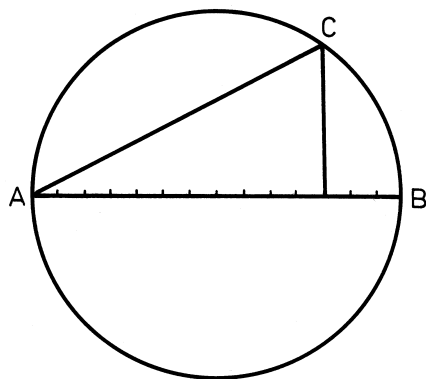


Figure 3

"Divide the diameter AB of the given circle in 14 equal parts. Then draw a perpendicular to the diameter on the end of the 11<sup>th</sup> part from the point A and denote the point of intersection of the perpendicular with the circle as C. The line segment AC is the side of the square with the area equal to the area of the given circle under the assumption, that  $\pi = 22/7$ ."

<sup>34</sup> *Quadratura circuli pendet a ratione diametri & peripheria.*

The proof of the “correctness” of the construction is simple and we omit it. We do not know who the author of this construction is, but in Comenius’ time it was well-known; it can be found e.g. in some jesuit mathematical manuscripts<sup>35</sup> or in the work *Articuli adversus mathematicos* written and edited by Giordano Bruno in Prague in 1588.<sup>36</sup> Therefore in our opinion it is possible, that Comenius could know this “correct” quadrature and that the quadrature in the manuscript could be this one, but written incorrectly.

**E.** In [2], p. 36 and [4], p. 408, Karel Čupr’s opinion is cited<sup>37</sup> that Comenius later appreciated the incorrectness of the quadrature in the manuscript and tried to correct it by solving the problem in a “roundabout way”, i.e. he wanted at first to construct a rectangle with one side equal to the diameter of the circle and the other side equal to  $11/14$  of the diameter; the area of this rectangle would be equal to the area of the given circle, if  $\pi$  would be equal to  $22/7$ , and this rectangle could be transformed in the square with the same area. This opinion is based only on one quite unclear figure in the manuscript and moreover it is in contradiction with the text of the manuscript, in which the term “*quadratum*” is used. The opinion, that Comenius could have in mind the word “rectangle”, if he wrote “*quadratum*”, is theoretically possible, because Comenius gives no definition of the term “rectangle”, but it is in contradiction with the definition of “*quadratum*” given by Alsted, who writes: *Quadratum est parallelogrammum rectangulum & aequilaterum* ([7], p. 51, [8], p. 114). We do not know whether Comenius appreciated the incorrectness of his quadrature or not, but in any case the quadrature in the manuscript is wrong.

**F.** The main idea of the quadrature given by Comenius can be expressed as follows: “Divide the diameter of the given circle into  $n$  equal parts and then draw a square whose side is equal to  $k$  of this parts; the area of this square is (approximately) equal to the area of the given circle.” Such methods of the quadrature of a circle are known in the history of mathematics; e.g. in [6], pp. 602–603 the following quadratures of this type are described:

a) an old Egyptian quadrature, by which the square with one side equal to  $8/9$  of the diameter of the given circle is used; it corresponds to the value  $\pi \doteq 3.1605$ .

<sup>35</sup>(e.g. in the manuscripts XIV G 8, XI D 13, I F 27 in the National library in Prague; for details see [10])

<sup>36</sup>The quadrature is described in the *Opus 118* of the work; in the edition [13] it is on the page 57.

<sup>37</sup>It was published in *Zeměměřičský obzor*, 30, 1942, p. 171.

b) two old Indian quadratures, by which the squares with one side equal to  $7/8$  or  $13/15$  of the diameter of the given circle are used; they correspond to the values  $\pi \doteq 3.0625$  or  $\pi \doteq 3.004$ .

In the history of mathematics many similar approximate methods for the quadrature of a circle can be found. There is not any evidence that Comenius knew some of them, but he could have been influenced by some of them.

## 5 Conclusion

In this article the geometrical problems of Comenius' work *Geometry and geodesy* were discussed. The editors of this work of Comenius' write that the problem of the quadrature of a circle was one of the preferred subjects of the mature Comenius ([1], p. 35, [2], p. 36, [4], pp. 407–8),<sup>38</sup> therefore we paid more attention to it.

Following the editions [1, 2] of Comenius' work *Geometry and geodesy* we paid attention mainly to the possible connections between this work of Comenius' and the works [7, 8] of his teacher J. H. Alsted. Nevertheless, in our opinion Alsted's books may not have been the only source of the mathematical knowledge of Comenius. He wrote this work in Poland and from this point of view it could be interesting that in 1566 the first Polish textbook of geometry was edited;<sup>39</sup> the possible connections between Comenius and the Polish authors have not been studied yet.

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<sup>38</sup>Nevertheless, it cannot be taken as evidence of Comenius' interest in mathematics, because in [2], p. 36, his interest in the quadrature is connected with his interest in the construction of the "perpetuum mobile" and in the mystery of numbers. As for Comenius' interest for the "perpetuum mobile", details can be found in [14, 15].

<sup>39</sup>It was the book *Geometria, to jest miernicka nauka* written by Stanislav Grzepski (1524–1570); for details see [16], p. 276-285.

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<sup>40</sup>In this article the edition from the year 1640 was used for quotations.