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Pham Huu Anh Ngoc; Thai Bao Tran; Nguyen Dinh Huy

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NEW CRITERIA FOR EXPONENTIAL STABILITY OF LINEAR NEUTRAL DIFFERENTIAL SYSTEMS WITH DISTRIBUTED DELAYS

Pham Huu Anh Ngoc, Thai Bao Tran and Nguyen Dinh Huy

We present new explicit criteria for exponential stability of general linear neutral time-varying differential systems. Particularly, our results give extensions of the well-known stability criteria reported in [3, 11] to linear neutral time-varying differential systems with distributed delays.

Keywords: linear neutral differential equation, exponential stability, time-varying systems

Classification: 34K20

1. INTRODUCTION

Neutral delay differential equations have many applications in physics, biology and other real world problems. They appear in modelling of the networks containing lossless transmission lines as in high-speed computers where the lossless transmission lines are used to interconnect switching circuits [5], in the study of vibrating mass attached to an elastic bar, in the theory of automatic control and in neuromechanical systems in which inertia plays an important role and etc. (see e. g. [8, 10]). Problems of stability of delay differential equations of neutral type have been studied intensively during the past decades, see e. g. [1, 4, 6, 15] and the references therein. A traditional approach to stability of neutral differential equations is the Lyapunov method, see e. g. [2, 6, 8, 10]. Most existing results in the literature are given in terms of matrix inequalities and not straight forward to use.

In this paper, we investigate the exponential stability of the following linear neutral differential system with distributed delays

$$\frac{\mathrm{d}}{\mathrm{d}t}(x(t) - D(t)x(t-r)) = A(t)x(t) + B(t)x(t-\tau) + \int_{-h}^{0} C(t,\theta)x(t+\theta)\,\mathrm{d}\theta,\ t \geq \sigma \quad (1)$$

where $A(\cdot), B(\cdot), D(\cdot) : \mathbb{R} \to \mathbb{R}^{n\times n}$ and $C(\cdot,\cdot) : \mathbb{R}\times[-h,0] \to \mathbb{R}^{n\times n}$, are given continuous functions and $r, \tau, h > 0$, are delays.

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Generally, it is not easy to tackle the exponential stability of (1). A criterion for the exponential stability of linear neutral differential systems with $C(t,s) = 0$, and D constant but A and B time varying has been found in [14]. The stability condition of [14] is given in terms of the existence of symmetric and positive definite solutions of a continuous Riccati algebraic matrix equation coupled with a discrete Lyapunov equation. As far as we know, there are not many explicit criteria for the exponential stability of (1). Actually, some explicit stability criteria for linear neutral time-invariant differential systems with discrete delays of the form

$$\frac{\mathrm{d}}{\mathrm{d}t}(x(t) - Dx(t-r)) = Ax(t) + Bx(t-\tau), \tag{2}$$

($A, B, D \in \mathbb{R}^{n\times n}$ are constant matrices) have been found in [3, 9, 11]. Particularly, it is well-known that (2) is exponentially stable if

$$\|D\| < 1,\ \mu(A) + \frac{\|AD\| + \|B\|}{1 - \|D\|} < 0, \tag{3}$$

where $\mu(A)$ is a matrix measure of A, see e.g. [3, 9, 11]. But so far, to the best of our knowledge, it is still an open question that what is an extension of (3) to the linear neutral time-varying differential system (1).

Very recently, by a novel approach based on the theory of positive systems, we gave some spectral criteria for the exponential stability of (1), see [13]. In contrast to [13], we present in this paper a simple approach that does not involve positive systems. It is simply based on a comparison principle and a proof by reductio ad absurdum. Consequently, new explicit scalar criteria for the exponential stability of (1) are derived. Our stability criteria are given explicitly in terms of the matrix measure and norms of the system matrices. In particular, they give answers to the open question mentioned above. Finally, the advantage of the present paper over [13] is the fact that the approach is much simpler than that of [13] and importantly, the stability criteria are given directly in terms of scalar inequalities of norms of the system matrices while the stability criteria of [13] are given in terms of vector-valued inequalities involving upper bounds of the system matrices. The present stability criteria are also much easier in use compared to those of [13].

2. PRELIMINARIES

For given integers, $l, q \geq 1, \mathbb{R}^l$ denotes the l-dimensional vector space over $\mathbb{R}$ and $\mathbb{R}^{l\times q}$ stands for the set of all $l \times q$-matrices with entries in $\mathbb{R}$. For $1 \leq p \leq \infty$, the p-norm on $\mathbb{R}^n$ is defined by $\|x\|_p = (|x_1|^p + |x_2|^p + ... + |x_n|^p)^{\frac{1}{p}},\ 1 \leq p < \infty;\ \|x\|_\infty = \max_{i=1,2,\dots,n} |x_i|$. Throughout the paper, a norm of $P \in \mathbb{R}^{l\times q}$ is understood as its operator norm associated with a given pair of vector norms on $\mathbb{R}^l$ and $\mathbb{R}^q$, that is $\|P\| = \max\{\|Py\| : \ \|y\| = 1\}$. It is well-known that

$$\|A\|_1 = \max_{1\leq j\leq q} \sum_{i=1}^{l} |a_{ij}|;\ \|A\|_2 = \sqrt{\max_{1\leq j\leq q} \lambda_j(A^T A)};\ \|A\|_\infty = \max_{1\leq i\leq l} \sum_{j=1}^{q} |a_{ij}|.$$

Suppose $\|M\|$ is the norm of $M \in \mathbb{R}^{n\times n}$ induced by a vector norm $\|\cdot\|$ defined on $\mathbb{R}^n$. The matrix measure of A is defined by

$$\mu(A) = \lim_{h\to 0^+} \frac{\|I_n + hA\| - 1}{h},$$

where I_n is the identity matrix, see e. g. [5]. Particularly, one has

$$\mu_\infty(A) = \max_{1\leqslant i\leqslant n} \left\{ a_{ii} + \sum_{j=1, j\neq i}^{n} |a_{ij}| \right\};$$

$$\mu_1(A) = \max_{1\leqslant j\leqslant n} \left\{ a_{jj} + \sum_{i=1, i\neq j}^{n} |a_{ij}| \right\};$$

$$\mu_2(A) = \max\left\{ \lambda : \det\left(\lambda I_n - \frac{A + A^T}{2}\right) = 0 \right\},$$

see e. g. [5]. Let

$$D^+ f(t) := \limsup_{h\to 0^+} \frac{f(t+h) - f(t)}{h},$$

be the upper Dini derivative of f.

3. EXPLICIT CRITERIA FOR EXPONENTIAL STABILITY

Consider the linear neutral time-varying differential system with distributed delay (1), where $A(\cdot), B(\cdot), D(\cdot) : \mathbb{R} \to \mathbb{R}^{n\times n}$ and $C(\cdot,\cdot) : \mathbb{R}\times[-h, 0] \to \mathbb{R}^{n\times n}$, are given continuous functions and $r, \tau, h > 0$, are time delays.
Let $d = \max\{r, \tau, h\}$. Denote by $\mathcal{C} := C([-d, 0], \mathbb{R}^n)$, the Banach space of all continuous functions on $[-d, 0]$ with values in $\mathbb{R}^n$, endowed with the norm $\|\varphi\| := \max_{\theta\in[-d,0]} \|\varphi(\theta)\|$.
Let $\sigma \in \mathbb{R}$ and $\varphi \in \mathcal{C}$ be given. An initial condition to (1) is defined by

$$x(\theta + \sigma) = \varphi(\theta), \quad \theta \in [-d, 0]. \tag{4}$$

Definition 3.1. (Hale and Lunel [8]) A continuous function $x(\cdot) : [-d+\sigma, \infty) \to \mathbb{R}^n$, is said to be a solution of (1) through (σ, φ) if the function $x(t)-D(t)x(t-r)$ is continuously differentiable on (σ, ∞) with a right-hand derivative at σ and $x(\cdot)$ satisfies (1) and (4).

It is well-known that for fixed $\sigma \in \mathbb{R}$ and given $\varphi \in \mathcal{C}$, there exists a unique solution of (1) through (σ, φ), denoted by $x(\cdot; \sigma, \varphi)$, see, e. g. [8], Theorem 1.1, p. 256.

Definition 3.2. (Hale and Lunel [8]) The system (1) is said to be exponentially stable if there exist $M, \beta > 0$ such that

$$\|x(t; \sigma, \varphi)\| \leq M e^{-\beta(t-\sigma)} \|\varphi\|, \quad \forall t \geq \sigma, \forall \varphi \in \mathcal{C}.$$

We are now in the position to prove the main result of this note.

Theorem 3.3. Assume that there exists $k \in [0,1)$ such that

$$\|D(t)\| \le k, \qquad t \in \mathbb{R}. \tag{5}$$

Then (1) is exponentially stable if

$$\sup_{t\in\mathbb{R}}\{\|A(t)D(t)\| + \|B(t)\| + \int_{-h}^{0} \|C(t,\theta)\|\mathrm{d}\theta\} < (k-1)\sup_{t\in\mathbb{R}}\mu(A(t)). \tag{6}$$

P r o o f. It follows from $0 \le k < 1$ and (6) that

$$1 - ke^{\beta r} > 0, \tag{7}$$

and

$$\sup_{t\in\mathbb{R}}\mu(A(t)) + \frac{e^{\beta(r+\tau+h)}}{1-ke^{\beta r}}\sup_{t\in\mathbb{R}}\{\|A(t)D(t)\| + \|B(t)\| + \int_{-h}^{0}\|C(t,\theta)\|\mathrm{d}\theta\} < -\beta. \tag{8}$$

for some $\beta > 0$ sufficiently small. Let $q := \dfrac{\min\{e^{\beta r}, e^{\beta\tau}, e^{\beta h}\}}{1-ke^{\beta r}}$. Obviously, one has $1 + kqe^{\beta r} < q$. Then (5) yields

$$1 + q\|D(t)\|e^{\beta r} < q, \quad \forall t \in \mathbb{R}, \tag{9}$$

and (8) implies that

$$\mu(A(t)) + q\big(\|A(t)D(t)\|e^{\beta r} + \|B(t)\|e^{\beta\tau} + \int_{-h}^{0} e^{-\beta\theta}\|C(t,\theta)\|\mathrm{d}\theta\big) < -\beta, \quad \forall t \in \mathbb{R}. \tag{10}$$

In what follows, we will show that (1) is exponentially stable if (9) and (10) hold.

Let $x(t) := x(t;\sigma,\varphi), t \in [\sigma - d, +\infty)$ be the unique solution of (1) through (σ,φ). Set $y(t) := x(t) - D(t)x(t-r), t \in [\sigma,\infty)$. Then $x(\cdot)$ and $y(\cdot)$ satisfy the following equations:

$$\frac{\mathrm{d}y(t)}{\mathrm{d}t} = A(t)y(t) + A(t)D(t)x(t-r) + B(t)x(t-\tau) + \int_{-h}^{0} C(t,\theta)x(t+\theta)\,\mathrm{d}\theta, \quad t \ge \sigma \tag{11}$$

$$x(t) = y(t) + D(t)x(t-r), \quad t \ge \sigma. \tag{12}$$

Let $\mathcal{C}_1 := \{\varphi \in \mathcal{C} \,:\, \|\varphi\| \le 1\}$. Fix $K > 2$ and let us define $u(t) := Ke^{-\beta(t-\sigma)}q, t \in \mathbb{R}$ and $v(t) := Ke^{-\beta(t-\sigma)}, t \in \mathbb{R}$. It follows from (9) that $q > 1$. Note that $\|D(t)\| < 1, \forall t \in \mathbb{R}$. For any $\sigma \in \mathbb{R}$ and for any $\varphi \in \mathcal{C}_1$, we have

$$\|x(\theta+\sigma)\| = \|\varphi(\theta)\| \le 1 < Ke^{-\beta\theta}q = u(\theta+\sigma), \theta \in [-d,0],$$

and

$$\begin{aligned}\|y(\sigma)\| &= \|x(\sigma) - D(\sigma)x(\sigma-r)\| \le \|x(\sigma)\| + \|D(\sigma)\|\|x(\sigma-r)\| \\ &\le \|\varphi(0)\| + \|\varphi(-r)\| \le 2 < K = v(\sigma).\end{aligned}$$

It is to show that for any $\varphi \in \mathcal{C}_1 : \|x(t)\| = \|x(t;\sigma,\varphi)\| \le u(t),\ \forall t \ge \sigma$, and $\|y(t)\| = \|x(t) - D(t)x(t-r)\| \le v(t),\ \forall t \ge \sigma$. Assume on the contrary that there exists $t_0 > \sigma$ such that either $\|x(t_0)\| > u(t_0)$ or $\|y(t_0)\| > v(t_0)$. Set $t_* := \inf\{t \in (\sigma,+\infty) : \|x(t)\| > u(t)$ or $\|y(t)\| > v(t)\}$. By continuity, $t_* > \sigma$ and one of the following statement holds:

(C_1) $\|x(t)\| \le u(t), t \in [\sigma, t_*]$ and

$$\begin{cases} \|y(t)\| \le v(t), \forall t \in [\sigma, t_*), \\ \|y(t_*)\| = v(t_*), \\ \exists\ t_n \in (t_*, t_* + \frac{1}{n}), n \in \mathbb{N} :\ \|y(t_n)\| > v(t_n). \end{cases} \tag{13}$$

(C_2) $\|y(t)\| \le v(t), t \in [\sigma, t_*]$ and

$$\begin{cases} \|x(t)\| \le u(t), \forall t \in [\sigma, t_*), \\ \|x(t_*)\| = u(t_*), \\ \exists t'_n \in (t_*, t_* + \frac{1}{n}), n \in \mathbb{N} :\ \|x(t'_n)\| > u(t'_n). \end{cases} \tag{14}$$

Suppose (C_1) holds. Taking (11) into account, we get the following estimate

$$D^+\|y(t_*)\| := \limsup_{h\to 0^+} \frac{1}{h}\Big(\|y(t_*+h)\| - \|y(t_*)\|\Big) =$$

$$\limsup_{h\to 0^+} \frac{1}{h}\Big[\|y(t_*) + h\frac{\mathrm{d}y}{\mathrm{d}t}(t^*)\| - \|y(t_*)\|\Big] \leqslant \limsup_{h\to 0^+} \frac{1}{h}\Big[\|I_n + hA(t_*)\| - 1\|\Big]\|y(t_*)\| +$$

$$\|A(t_*)D(t_*)\|\|x(t_*-r)\| + \|B(t_*)\|\|x(t_*-\tau)\| + \int_{-h}^{0} \|C(t_*,\theta)\|\|x(t_*+\theta)\|\,\mathrm{d}\theta$$

$$= \mu(A(t_*))\|y(t_*)\| + \|A(t_*)D(t_*)\|\|x(t_*-r)\| + \|B(t_*)\|\|x(t_*-\tau)\| + \int_{-h}^{0} \|C(t_*,\theta)\|\|x(t_*+\theta)\|\,\mathrm{d}\theta.$$

Invoking (C_1) and (10), we have

$$D^+\|y(t_*)\| \le \mu(A(t_*))v(t_*) + \|A(t_*)D(t_*)\|u(t_*-r) + \|B(t_*)\|u(t_*-\tau) + \int_{-h}^{0} \|C(t_*,\theta)\|u(t_*+\theta)\,\mathrm{d}\theta$$

$$= Ke^{-\beta(t_*-\sigma)}\Big[\mu(A(t_*)) + q\big(\|A(t_*)D(t_*)\|e^{\beta r} + \|B(t_*)\|e^{\beta\tau} + \int_{-h}^{0} e^{-\beta\theta}\|C(t_*,\theta)\|\,\mathrm{d}\theta\big)\Big]$$

$$\overset{(10)}{<} -\beta K e^{-\beta(t_*-\sigma)} = \frac{\mathrm{d}v}{\mathrm{d}t}(t_*).$$

On the other hand, (13) yields,

$$D^+\|y(t_*)\| \ge \limsup_{n\to\infty} \frac{\|y(t_n)\| - \|y(t_*)\|}{t_n - t_*} \ge \limsup_{n\to\infty} \frac{v(t_n) - v(t_*)}{t_n - t_*} = \frac{\mathrm{d}v}{\mathrm{d}t}(t_*). \tag{15}$$

This is a contradiction. Assume that (C_2) holds. Then (9) and (12) yield,

$$\|x(t_*)\| \le \|y(t_*)\| + \|D(t_*)\|\|x(t_* - r)\| \le Ke^{-\beta(t_*-\sigma)} + Ke^{-\beta(t_*-\sigma)}e^{\beta r}\|D(t_*)\|q$$
$$= Ke^{-\beta(t_*-\sigma)}\Big(1 + q\|D(t_*)\|e^{\beta r}\Big) < Ke^{-\beta(t_*-\sigma)}q = u(t_*),$$

which conflicts with (14). Therefore,

$$\|x(t)\| \le u(t), \quad t \ge \sigma; \ \|y(t)\| \le v(t), \quad t \ge \sigma.$$

In particular, we get

$$\|x(t;\sigma,\varphi)\| \le Ke^{-\beta(t-\sigma)}q, \quad t \ge \sigma, \ \varphi \in \mathcal{C}_1.$$

Since (1) is linear, it follows that for any $\varphi \in \mathcal{C}, \varphi \ne 0$,

$$\|x(t;\sigma,\frac{\varphi}{\|\varphi\|})\| = \frac{1}{\|\varphi\|}\|x(t;\sigma,\varphi)\| \le Ke^{-\beta(t-\sigma)}q, \qquad t \ge \sigma.$$

Hence,

$$\|x(t;\sigma,\varphi)\| \le Me^{-\beta(t-\sigma)}\|\varphi\|, \qquad t \ge \sigma, \ \varphi \in \mathcal{C},$$

where $M := Kq$. So (1) is exponentially stable. This completes the proof. □

As mentioned in the proof of Theorem 3.3, (1) is exponentially stable if (9) – (10) hold for some $\beta > 0, q > 0$. This leads to the following result.

Theorem 3.4. Assume that there exist $\beta, q > 0$ such that

$$\mu(A(t)) + q\big(\|A(t)D(t)\|e^{\beta r} + \|B(t)\|e^{\beta\tau} + \int_{-h}^{0} e^{-\beta\theta}\|C(t,\theta)\|\,\mathrm{d}\theta\big) < -\beta, \quad t \in \mathbb{R}, \tag{16}$$

and

$$1 + q\|D(t)\|e^{\beta r} < q, \quad t \in \mathbb{R}. \tag{17}$$

Then (1) is exponentially stable.

Remark 3.5. Let (1) be considered on the half-line $\mathbb{R}_+$. Then (1) is exponentially stable on $\mathbb{R}_+$ provided that (16) – (17) holds for any $t \in \mathbb{R}_+$. That is,

$$\|x(t;\sigma,\varphi)\| \le Me^{-\beta(t-\sigma)}\|\varphi\|, \qquad t \ge \sigma \ge 0, \ \varphi \in \mathcal{C}.$$

Remark 3.6. Consider a linear neutral differential system with time-varying delays of the form

$$\frac{\mathrm{d}}{\mathrm{d}t}(x(t) - D(t)x(t - r(t))) = A(t)x(t) + B(t)x(t - \tau(t)) + \int_{-h(t)}^{0} C(t,\theta)x(t+\theta)\,\mathrm{d}\theta, \ t \ge \sigma \tag{18}$$

where $A(\cdot), B(\cdot), D(\cdot) : \mathbb{R} \to \mathbb{R}^{n\times n}$ and $C(\cdot,\cdot) : \mathbb{R} \times [-h, 0] \to \mathbb{R}^{n\times n}$ and $r(t), \tau(t), h(t) : \mathbb{R} \to \mathbb{R}_+$ are continuous functions satisfying $0 \le r(t) \le r, 0 \le \tau(t) \le \tau, 0 \le h(t) \le h, t \in \mathbb{R}$, for some $r, \tau, h > 0$.

Assume that for fixed $\sigma \in \mathbb{R}$ and given $\varphi \in \mathcal{C}$, (18) always has a unique solution satisfying the initial condition (4). For example, if $D(t)$ is bounded on $\mathbb{R}$ and $0 < r_1 \leq r(t) \leq r, t \in \mathbb{R}$, see Theorem 1.1, page 256 of [8] for a general context.

It is worth noticing that the proof of Theorem 3.3 also works for (18). Thus, Theorems 3.3−3.4 are still valid for (18).

The following corollary, which is immediate from Theorem 3.3, generalizes (3) to linear neutral time-invariant differential systems with distributed delays.

Corollary 3.7. Suppose $A, B, D \in \mathbb{R}^{n\times n}$ and $C(\cdot) : [-h, 0] \to \mathbb{R}^{n\times n}$ is a continuous function. Then the linear neutral time-invariant differential system

$$\frac{\mathrm{d}}{\mathrm{d}t}(x(t) - Dx(t-r)) = Ax(t) + Bx(t-\tau) + \int_{-h}^{0} C(\theta)x(t+\theta)\,\mathrm{d}\theta, \tag{19}$$

is exponentially stable for any $r > 0, \tau > 0, h > 0$ if $\|D\| < 1$ and

$$\mu(A) + \Big(\|AD\| + \|B\| + \int_{-h}^{0} \|C(\theta)\|\,\mathrm{d}\theta\Big)\frac{1}{1-\|D\|} < 0. \tag{20}$$

To end this note, we give an example to illustrate Theorem 3.3.

Example 3.8. Consider a linear neutral time-varying differential system with delays of the form (1) on $\mathbb{R}_+$, where

$$A(t) = \begin{pmatrix} -2-e^t & \sin t \\ \cos t & -2-e^t \end{pmatrix}; \quad D(t) = \frac{1}{20e}\begin{pmatrix} \sin t & \cos t \\ \sin 2t & \cos 2t \end{pmatrix};$$

$$C(t,\theta) = \frac{e^\theta}{60}\begin{pmatrix} \sin 5t & \cos 5t \\ \sin 6t & \cos 6t \end{pmatrix}; \quad B(t) = \frac{1}{40e^2}\begin{pmatrix} \sin 3t & \cos 3t \\ \sin 4t & \cos 4t \end{pmatrix},$$

for $t \in \mathbb{R}_+, \theta \in [-3, 0]$ and $r = 1, \tau = 2, h = 3$.

Let $\mathbb{R}^2$ be endowed with $\|\cdot\|_\infty$ and the norm on $\mathbb{R}^{2\times 2}$ is the induced norm. It is easy to check that

$$\mu_\infty(A(t)) \leq -1 - e^t, \ \|D(t)\|_\infty \leq \frac{1}{10e}, \qquad t \in \mathbb{R}_+$$

$$\|A(t)D(t)\|_\infty \leq \frac{3+e^t}{10e}, \ \|B(t)\|_\infty \leq \frac{1}{20e^2}, \quad t \in \mathbb{R}_+$$

$$\|C(t,\theta)\|_\infty \leq \frac{e^\theta}{30}, \quad t \in \mathbb{R}_+, \theta \in [-3, 0].$$

Let $\beta = 1, q = \dfrac{3}{2}$. It is easy to verify that (16) and (17) hold for any $t \in \mathbb{R}_+$ Thus, taking Remark 3.5 into account, the system under consideration is exponentially stable by Theorem 3.4.

To support the theoretical analysis which proved that the system is exponentially stable, we undertook a simulation to observe graphs of numerical solutions of the system. Using the Matlab command ddensd, the numerical solution $x(t, \varphi)$ of the system can be calculated for a given φ. The graph of $||x(t, \varphi)||$ with $\varphi(s) := \binom{\sin s}{\cos s}$, $s \in [-3; 0]$ and that of $\zeta = e^{-2t}$ are given in Figure 1.

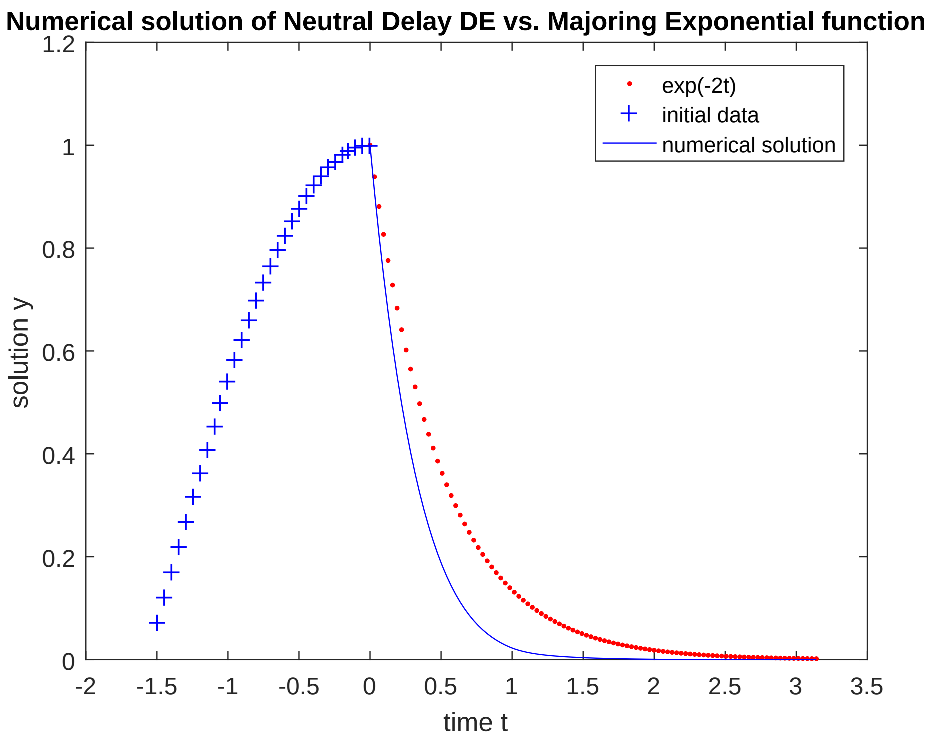

Fig. 1. Numerical solution of a neutral system satisfying the delay-dependent stability criterion (Theorem 3.4).

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Pham Huu Anh Ngoc, Vietnam National University, Ho Chi Minh city, International University, Department of Mathematics, Thu Duc city, Saigon. Vietnam.
e-mail: phangoc@hcmiu.edu.vn

Thai Bao Tran, Faculty of Information Systems, University of Information Technology, Vietnam National University, Ho Chi Minh city, Thu Duc District, Saigon. Vietnam.
e-mail: trantb@uit.edu.vn

Nguyen Dinh Huy, Division of Applied Mathematics, Faculty of Applied Sciences, University of Technology, Vietnam National University, Ho Chi Minh city, Thu Duc District, Saigon. Vietnam.
e-mail: nguyendinhhuy56@hcmut.edu.vn