Kybernetika

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Kybernetika, Vol. 56 (2020), No. 4, 810–820

Persistent URL: http://dml.cz/dmlcz/148385

KYBERNETIKA — VOLUME 56 (2020), NUMBER 4, PAGES 810–820

ROBUST PI-D CONTROLLER DESIGN FOR DESCRIPTOR SYSTEMS USING REGIONAL POLE PLACEMENT AND/OR H_2 PERFORMANCE

Vojtech Veselý and Ladislav Körösi

The paper deals with the problem of obtaining a robust PI-D controller design procedure for linear time invariant descriptor uncertain polytopic systems using the regional pole placement and/or H_2 criterion approach in the form of a quadratic cost function with the state, derivative state and plant input (QSR). In the frame of Lyapunov Linear Matrix Inequality (LMI) regional pole placement approach and/or H_2 quadratic cost function based on Bellman-Lyapunov equation, the designed novel design procedure guarantees the robust properties of closed-loop system with parameter dependent quadratic stability/quadratic stability. In the obtained design procedure the designer could use controller with different structures such as P, PI, PID, PI-D. For the PI-D's D-part of controller feedback the designer could choose any available output/state derivative variables of descriptor systems. Obtained design procedure is in the form of Bilinear Matrix Inequality (BMI). The effectiveness of the obtained results is demonstrated on two examples.

Keywords: descriptor system, robust PI-D controller, state derivative feedback, output feedback, pole placement

Classification: 93B51, 93B52, 93B55, 93B60

1. INTRODUCTION

A way to guarantee satisfactory transients of closed-loop linear systems is to place the closed-loop eigenvalues in a suitable region, defined as a regional pole region or LMI region in the complex plane. Regional pole assignment is considered in [8, 9, 12, 18] and others. The descriptor systems have received attention over the past two decades due to their ability to describe such systems as power systems, chemical processes, robotic and economic systems [19] and so on. Specifically, the descriptor systems explicitly describe some static constraints on physical variables. The stability of such system has been studied in [3, 5, 7, 20]. In the above papers the authors have obtained the asymptotic (robust) stability conditions based on Lyapunov stability theory. Design of robust controllers for descriptor systems are given in papers [1, 2, 5, 6, 10, 17]. The conditions of C(complete)-controllability and/or C- observability descriptor systems are obtained in [16]. To the best of the authors' knowledge, there are no publications to

DOI: 10.14736/kyb-2020-4-0810

design output feedback robust controller for uncertain polytopic descriptor systems using regional pole placement approach. The above approach belongs to the class of N-P hard (Non-deterministic Polynomial-time Hardness) [4]. There are following difficulties to robust controller design for linear continuous time systems with output feedback and regional pole placement:

- strict pole placement design procedure holds if conditions [11] are verified,
- non strict pole placement (regional pole placement) design procedure holds if the uncertain plant satisfies conditions given in [13] and for descriptor systems [16]. Method of Lyapunov stability theory leads to regional pole placement using LMI-D-stability region.

For linear uncertain descriptor systems in [16] it is shown that for the system interval matrices (E, A, B, C) the robust controllability and observability could be solved in terms of the structured singular value of defined fixed matrices. In this paper we left open the problem of controllability and observability of uncertain linear descriptor systems to the design of robust PI-D controller.

In this paper the novel design procedure is obtained to design the robust PI-D controller for uncertain polytopic linear descriptor systems based on regional pole placement and/or minimizing H_2 quadratic cost function, known as QSR (state, derivative state and input) performance. For feedback of D part of the controller the designer could use any /or all states/outputs derivative of descriptor systems.

We would like to stress that in the paper [21] for the time variant descriptor system the time variant controller -gain scheduled controller has been designed and in the proposed paper, for the time invariant descriptor system the original design procedure is obtained to design PID robust time invariant controller.

This paper is organized as follows. Section 2. provides mathematical model of uncertain polytopic descriptor system, preliminaries and a short survey of regional pole placement approach with H_2 performance. Finally, the problem formulation is given. In Section 3. the main results are given. Examples and Conclusion are given in the last two Sections.

Notations. The notations used in this paper are standard in the field of robust controller design. The relation for symmetric matrices $A > B$ means that matrix $A - B$ is positive definite. The transcript "T" stands for transposition, $A \in R^{m\times n}$ denotes a set of real $m \times n$ matrices, I_n is $n \times n$ identity matrix, 1_d is the vector of dimension "d" with all entries "1", $\otimes$ denotes the Kronecker product.

2. PRELIMINARIES AND PROBLEM FORMULATION

Consider the following uncertain polytopic descriptor LTI system in the form

$$E_n\dot{x}_n = A(\xi)_n x_n + B(\xi)_n u,\ y_n = C_n x_n \tag{1}$$

where we assume that system matrices $A(\xi)_n, B(\xi)_n$ are affine with respect to uncertain parameter ξ, that is

$$(A(\xi)_n, B(\xi)_n) = \sum_{j=1}^{N}(A_{nj}, B_{nj})\xi_j, \tag{2}$$

$$\sum_{j=1}^{N} \xi_j = 1, \ \sum_{j=1}^{N} \dot{\xi}_j = 0, \ \xi_j \geq 0.$$

Uncertainty $\xi_j \in \Omega_\xi, \ j = 1, \dots, N; \ \dot{\xi}_j \in \Omega_t$ are constant or time varying parameters; the sets Ω_ξ and Ω_t are defined by (2) as a sum of corresponding variables, that is

$$\xi \in \Omega_\xi = \left\{ \xi \in R^N : \xi_j \geq 0, \sum_{j=1}^{N} \xi_j = 1 \right\}$$

$$\dot{\xi} \in \Omega_t = \left\{ \dot{\xi} \in R^N : \sum_{j=1}^{N} \dot{\xi} = 0 \right\}$$

matrices A_{nj}, B_{nj}, C are constant matrices of corresponding dimensions; for matrix E_n with constant entries it holds that $rank(E_n) \leq rank(A(\xi)_n)$. For PI controller design the plant state vector has to be expanded such that the part of new state variable includes the integral plant outputs [22]. Let $\dot{z} = y_n = C_n x_n$. The following plant model is obtained

$$E\dot{x} = E \begin{bmatrix} \dot{x}_n \\ \dot{z} \end{bmatrix} = A(\xi)x + B(\xi)u, \ y = Cx \tag{3}$$

where

$$A(\xi) = \begin{bmatrix} A(\xi)_n & 0 \\ C_n & 0 \end{bmatrix} \in R^{n \times n}, \ B(\xi) = \begin{bmatrix} B_n(\xi) \\ 0 \end{bmatrix} \in R^{n \times m},$$
$$C = \begin{bmatrix} C_n & 0 \\ 0 & I \end{bmatrix} \in R^{l \times n}, \ C_d = \begin{bmatrix} C_{dn} & 0 \\ 0 & 0 \end{bmatrix}, \ E = \begin{bmatrix} E_n & 0 \\ 0 & I \end{bmatrix}$$

where C_{dn} is the output matrix for derivative controller feedback. Let H_2 be the quadratic (QSR) cost function with matrices

$$J_c = \int_{t_o}^{\infty} J \, \mathrm{d}t \tag{4}$$

$$J = (x^T Q x + \dot{x}^T E^T S E \dot{x} + u^T R u, \ Q, S \geq 0, \ R > 0.$$

For system (3) if $det(\lambda E - A(\xi)) \neq 0$ for some $\lambda \in C$ then it is regular, if all eigenvalues of $det(\lambda E - A(\xi))$ lie in the left half of the complex plane, the system is asymptotically stable. The system is complete controllable if for any $t_1 > 0$, $x(0) \in R^n$ and $w \in R^n$ there exists a control input such that $x(t_1) = w$. The following theorem, and definition play an important role.

Theorem 2.1. (Chen and Liu [7]) The equilibrium $x = 0$ of a system (3) is asymptotically stable, if $n \times n$ matrix $P(\xi) > 0$ exists, such that along the solution of (3), the time derivative of Lyapunov function $V(Ex) = (Ex)^T P(Ex)$ is negative definite for all variate of Ex.

Definition 2.2. (Pea [18]) An LMI region is any subset of a complex plane that can be defined as

$$D = \left\{ z \in C : L + zM + \overline{z}M^T < 0 \right\} \tag{5}$$

where $L = L^T \in R^{d\times d}$ are real symmetric matrices and $M \in R^{d\times d}$ is a real matrix,

$$f_d(z) = L + zM + \overline{z}M^T \tag{6}$$

is the characteristic function of D-LMI region and $d \geq 1$ characterizes the complexity of D-LMI region.

The problem studied in this paper is to design a robust PI-D controller such that all closed-loop eigenvalues lying in the prescribed D-LMI region and/or cost function (4) have minimal value. Control algorithm of PI-D controller is as follows

$$\begin{gathered} u = K_p y_n + K_i z + K_d C_{dn} E_n \dot{x}_n \\ = \begin{bmatrix} K_p C_n & K_i \end{bmatrix} x + \begin{bmatrix} K_d C_{dn} E_n & 0 \end{bmatrix} \dot{x} = Kx + K_D C_d E\dot{x} \\ K = \begin{bmatrix} K_p C_n & K_i \end{bmatrix},\ K_D = \begin{bmatrix} K_d & 0 \end{bmatrix}. \end{gathered} \tag{7}$$

To obtain the minimal value of cost function (4) the well-known Bellman-Lyapunov equation should be used.

Lemma 2.3. (Kuncevic and Lycak [14]) Consider the system (3) and control algorithm (7). Control algorithm (7) is the guaranteed cost control law for the closed-loop system $E\dot{x} = (I - B(\xi)K_D C_d)^{-1}\ (A(\xi) + B(\xi)K)x = A_c(\xi)x$ if and only if there exists a Lyapunov function $V(Ex) = (Ex)^T P(\xi)\ (Ex)$ such that the following condition holds

$$B_e = \max_u \left\{ \frac{\mathrm{d}V(\cdot)}{\mathrm{d}t} + J \right\} \leq 0. \tag{8}$$

Uncertain system (3) with control algorithm (7) conforming to Lemma 2.3. is called robust stable with guaranteed cost. For concrete structure of $V(\cdot)$ "if and only if" may to be reduced to "if".

3. ROBUST PI-D CONTROLLER DESIGN IN LMI - D-REGION

Using Theorem 2.2 from [8] it follows

Lemma 3.1. The uncertain system $A_c(\xi)$ is robustly $D-$stable if positive definite symmetric matrix $P(\xi)$ exists such that

$$L \otimes P(\xi) + M \otimes (P(\xi)A_c(\xi)) + M^T \otimes (A_c(\xi)^T P(\xi)) < 0 \tag{9}$$

for $P(\xi) > 0,\ \xi \in \Omega_\xi,\ \dot{\xi} \in \Omega_t$.

Multiplying (9) by x^T and x and using the closed-loop equation with $E = I$, one can obtain in matrix form

$$\frac{\mathrm{d}V_{ex}}{\mathrm{d}t} = v^T \begin{bmatrix} L \otimes P(\xi) & M \otimes P(\xi) & 0 \\ M^T \otimes P(\xi) & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} v < 0 \tag{10}$$

where $v^T = \left[(1_d \otimes x)^T \quad (1_d \otimes \dot{x})^T \quad (1_d \otimes u)^T\right]$.

Lyapunov function for the descriptor systems is in the form $V(Ex) = (Ex)^T P(\xi)(Ex)$ and for its first time derivative one obtains in matrix form

$$\frac{\mathrm{d}V(\cdot)}{\mathrm{d}t} = \begin{bmatrix} x^T & \dot{x}^T \end{bmatrix} \begin{bmatrix} E^T P(\dot{\xi})E & E^T P(\xi)E \\ E^T P(\xi)E & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} < 0. \tag{11}$$

If one takes into account that in our case ξ is time varying and (9) has been obtained for the case of $E = I$, then due to (11), (10) could be rewritten for descriptor systems as follows

$$\frac{\mathrm{d}V_{ex}}{\mathrm{d}t} = v^T \begin{bmatrix} L \otimes E^T(P(\xi) + P(\dot{\xi}))E & M \otimes E^T P(\xi)E & 0 \\ M^T \otimes E^T(P(\xi)E & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} v < 0. \tag{12}$$

Note that (11) and characteristic function (6) are related by the following substitution

$$\left\{E^T P(\xi)E, E^T P(\xi), P(\xi)E\right\} \leftrightarrow \left\{1, z, \overline{z}\right\}.$$

Inequality (9) for descriptor system and time varying ξ is as follows

$$L \otimes (E^T(P(\xi) + P(\dot{\xi})E) + M \otimes (E^T P(\xi)A_c(\xi)) + M^T \otimes ((E^T P(\xi)A_c(\xi))^T) < 0. \tag{13}$$

To split system matrices from Lyapunov matrices and to obtain closed-loop matrix convex with respect to uncertain parameter ξ, one could introduce the auxiliary matrices $N_i,\ i = 1, 2, \ldots, 6$ in the following way

$$\begin{gathered} v_2^T \begin{bmatrix} 2N_1^T \\ 2N_2^T \\ 2N_3^T \end{bmatrix} \begin{bmatrix} -A(\xi) & E & -B(\xi) \end{bmatrix} v_2 = 0 \\ v_2^T = \begin{bmatrix} x^T & (\dot{x})^T & u^T \end{bmatrix} \\ v_2^T \begin{bmatrix} 2N_4^T \\ 2N_5^T \\ 2N_6^T \end{bmatrix} \begin{bmatrix} -K & -K_D C_d E & I_m \end{bmatrix} v_2 = 0. \end{gathered} \tag{14}$$

Summarizing modified (14) and (12) one can obtain for the first derivative of extended Lyapunov function the following inequality

$$\frac{\mathrm{d}V_{ex}(\cdot)}{\mathrm{d}t} = \sum_{j=1}^{N} v_2^T W_j v_2 < 0 \tag{15}$$

where $W_j = \{w_{jik}\}_{3\times 3}$

$$
\begin{aligned}
w_{j11} &= L \otimes \left(E^T P_j + \sum_{k=1}^{N} P_k \dot{\xi}_k E \right) - N_1^T (I_d \otimes A_j) - (I_d \otimes A_j)^T N_1 - N_4^T (I_d \otimes K) \\
&\quad - (I_d \otimes K)^T N_4 \\
w_{j12} &= M \otimes (E^T P_j E) + N_1^T (I_d \otimes E) - (I_d \otimes A_j)^T N_2 - N_4^T (I_d \otimes K_D C d E) \\
&\quad - (I_d \otimes K)^T N_5 \\
w_{j13} &= -N_1^T (I_d \otimes B_j) - (I_d \otimes A_j)^T N_3 + N_4^T - (I_d \otimes K)^T N_6 \\
w_{j23} &= -N_2^T (I_d \otimes B_j) + (I_d \otimes E)^T N_3 + N_5^T - (I_d \otimes K_D C_d E)^T N_6 \\
w_{j22} &= N_2^T (I_d \otimes E) + (I_d \otimes E)^T N_2 - N_5^T (I_d \otimes K_D C_d E) - (I_d \otimes K_D C_d E)^T N_5 \\
w_{j33} &= -N_3^T (I_d \otimes B_j) - (I_d \otimes B_j)^T N_3 + N_6^T + N_6
\end{aligned}
$$

where without change of notation one has $N_i = N_{inew} = (I_d \otimes N_{iold}),\ i = 1, 2, \ldots, 6$, $Q_e = I_d \otimes Q,\ S_e = (I_d \otimes S),\ R_e = (I_d \otimes R)$. For H_2 performance robust controller design with quadratic cost function and to obtain the guaranteed cost control law due to (8) one needs to change the following variables w_{jik} as follows

$$
\begin{aligned}
w_{j11new} &= w_{j11} + Q_e, \\
w_{j22new} &= w_{j22} + (I_d \otimes E)^T S_e (I_d \otimes E) \\
w_{j33new} &= w_{j33} + R_e \\
w_{j12new} &= w_{j12} \\
w_{j13new} &= w_{j13} \\
w_{j23new} &= w_{j23}.
\end{aligned}
$$

Condition (15) ensures the robust properties of polytopic closed-loop descriptor system with parameter dependent quadratic stability. The demanded performance of uncertain closed-loop descriptor systems to be guaranteed by moving all vertices of uncertain descriptor system the closed-loop eigenvalues to the prescribed LMI region and/or minimization of H_2 quadratic cost function (4). Slightly modifying of the (15) one could obtain the robust controller design procedure to the polytopic descriptor systems for the design of robust controller with structures P, PI, PID, PI-D.

4. EXAMPLES

Example 4.1. The first example has been borrowed from [15]. This system is tested open-loop as *not-regular and unstable*. The parameters of the system plant model are as follows

$$
E = \begin{bmatrix} 1 & 0 & 0.5 \\ 2 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix},\quad A = \begin{bmatrix} 2.4 & 0.2 & 1.2 \\ 4 & 1.5 & 2 \\ 0 & 0 & 0 \end{bmatrix},\quad B = \begin{bmatrix} 4 & 1 \\ 1 & 1 \\ 1 & 2 \end{bmatrix}.
$$

Output matrices

$$C = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \ C_d = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}.$$

The problem is to design decentralized PI-D output feedback controller using regional pole placement and H_2 cost function in the quadratic form (4) with $Q = I * q_o,\ q_o = 0.01;\ S = s_o * I,\ s_o = 1;\ R = r_o * I,\ r_o = 1$. Let the LMI region be a disk of radius r and center $(-q, 0)$ with characteristic function

$$f_D(z) = \begin{bmatrix} -r & q+z \\ q+\overline{z} & -r \end{bmatrix} < 0.$$

With designed PI-D controller we would like to move closed-loop eigenvalues to defined disk type LMI regions. Disk type LMI regions for three cases are given as follows.

a. $q = 10,\ r = 9.9$

b. $q = 10,\ r = 9.7$ (case b.1) and $q = 40,\ r = 39.7$ (case b.2)

c. $q = 60,\ r = 59.6$.

For three cases with calculation we obtain the following parameters of decentralized PI-D controllers and closed-loop eigenvalues as follows.

Case a. The following two decentralized PI-D controllers and closed-loop eigenvalues are obtained

$$R_1(s) = \left(-15.0582 - \frac{7.8751}{s}\right) y_n + 0.8692\dot{x}_{n1}$$
$$R_2(s) = \left(-0.8742 - \frac{0.2531}{s}\right) y_n - 4.3345\dot{x}_{n2}$$

Closed-loop eigenvalues of descriptor systems with obtained controllers are

$$Eigclosedloop = \{-0.3202; -1.856 \pm 0.2756i; -0.6263\}$$

All closed-loop eigenvalues are lying in the prescribed LMI region.

Case b.1 BMI failed.

Case b.2 The obtained controllers are

$$R_1(s) = \left(-167.6385 - \frac{80}{s}\right) y_n + 11.3955\dot{x}_{n1}$$
$$R_2(s) = \left(-9.1885 - \frac{3.3605}{s}\right) y_n - 48.723\dot{x}_{n2}$$

Closed-loop eigenvalues

$$Eigclosedloop = \{-0.4021 \pm 0.3065i; -1.4384; -2.1703\}$$

Case c. The obtained controllers for parameters $q = 60,\ r = 59.6$ are

$$R_1(s) = \left(-342.7597 - \frac{185.036}{s}\right) y_n + 11.3955\dot{x}_{n1}$$
$$R_2(s) = \left(-62.6302 - \frac{24.9924}{s}\right) y_n - 317.6562\dot{x}_{n2}$$

For the Case c. the closed-loop eigenvalues are

$$Eigclosedloop = \{-0.428 \pm 0.2531i; -1.1229; -1.9857\}.$$

From above simple example one could observe that moving the closed-loop eigenvalues to the left-side of complex plane the controller gains rapidly increases.

Example 4.2. For the second example we have taken the uncertain modified descriptor system [23] with *two vertices, unstable system and with maximal rate value of system parameter changes* $\max \dot{\xi} = 0.01/sec$. The parameters of controlled descriptor systems are as follows

$$E = \begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 1 \\ 2 & 0 & 1 \end{bmatrix},\ C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix},\ C_d = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$
$$A_1 = \begin{bmatrix} 1.5 & 0.5 & 1 \\ -1 & 0 & 1 \\ 0.5 & 0 & 1 \end{bmatrix},\ B_1 = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}$$
$$A_2 = \begin{bmatrix} 1.3 & 0.8 & 0.7 \\ -0.8 & -0.1 & 0.8 \\ 0.4 & 0 & 0.75 \end{bmatrix},\ B_1 = \begin{bmatrix} 0.8 & 0.75 \\ 0.75 & 0 \\ 0.75 & 1 \end{bmatrix}.$$

LMI region for the first is defined as disk with $q = 20,\ r = 19.8$. The obtained robust decentralized PI-D controllers have the following parameters

$$R_1(s) = \left(-2.7698 - \frac{0.645}{s}\right) y_n + 0.28523\dot{x}_{n1}$$
$$R_2(s) = \left(1.39 + \frac{0.3718}{s}\right) y_n + 1.6879\dot{x}_{n3}.$$

Closed-loop eigenvalues of uncertain descriptor system with obtained controllers for the case of the first vertex are

$$Eigclosedloop_1 = \{-2.8844 \pm 1.2936i; -0.9156 \pm 1.1052i\}$$

and for the second vertex

$$Eigclosedloop_2 = \{-3.8629; -1.7555 \pm 1.4013i; -0.3378\}.$$

For the same uncertain system parameters let LMI region be *defined as conic a sector[as conic sector] with apex at the origin and inner angle* 2η, $\eta = 1.25rad$ Characteristic function of conic sector is as follows

$$f_D(z) = \begin{bmatrix} sin\eta(z+\overline{z}) & cos\eta(z-\overline{z}) \\ cos\eta(\overline{z}-z) & sin\eta(z+\overline{z}) \end{bmatrix} < 0.$$

The obtained two robust PI-D controllers are

$$R_1(s) = \left(-0.4 - \frac{0.0414}{s}\right) y_n + 0.9451\dot{x}_{n1}$$

$$R_2(s) = \left(-13.4204 - \frac{2.145}{s}\right) y_n - 3./8372\dot{x}_{n3}.$$

Closed-loop eigenvalues for uncertain descriptor system in the first vertex are

$$Eigclosedloop_1 = \{-31.8479; -2.22 \pm 0.9392i; -1.7725\}$$

and in the second one are

$$Eigclosedloop_2 = \{-28.6215; -3.2409 \pm 1.7679i; -0.1489\}.$$

From above examples one could observe that for the given descriptor system parameters moving the closed-loop eigenvalues to the left of complex plane simultaneously the controller parameters are increasing too. We finally get all closed-loop eigenvalues to the strong demanded LMI region but due to high gains especially of D-part of controller, real input constraints and system disturbances the performance may be unpracticall. Note that to obtain satisfactory results the uncertain descriptor system needs to meet the condition of C-controllability and C-observability [16]. Examples show, that all closed-loop eigenvalues lying inside the prescribed LMI region which with H_2 performance for controller designer open new possibilities to obtain the demanded quality of uncertain descriptor closed-loop systems.

5. CONCLUSIONS

The paper deals with the problem to obtain the novel design procedure for linear uncertain polytopic descriptor system to design of robust PI-D controller. Designed controller ensures to closed-loop uncertain system robust properties with parameter dependent quadratic stability/quadratic stability. The demanded closed-loop performance is guaranteed by moving the closed-loop uncertain system eigenvalues to defined regional pole region-LMI region and/or minimizing the H_2 quadratic cost function. Obtained design procedure is in the form of BMI. To obtain the less conservative results, to split the system and Lyapunov matrices and to ensure that uncertain parameters are convex we have introduced to the design procedure six auxiliary matrices. Examples show that all closed-loop eigenvalues lying in the prescribed LMI region which prove the effectiveness of proposed method. Above novel design procedure for controller designer opens the new possibilities to obtain the demanded closed-loop system quality. The chosen controller structure may be as P, PI, PID, and PI-D where for last cases for D-part of controller one may use any output/state derivative of descriptor system, instead of descriptor states. In the future we will follow the proposed idea using LMI region for a more broadly class of uncertain continuous and discrete-time systems.

ACKNOWLEDGEMENT

The paper was supported by the Slovak Scientific Grant Agency VEGA, Grant No. 1/0049/20.

(Received February 7, 2019)

REFERENCES

[1] G. J. Bara: Robust analysis and control of parameter-dependent uncertaintain descriptor systems. Systems Control Lett. *60* (2011), 356–364. DOI:10.1016/j.sysconle.2011.03.001

[2] M. Darouch, F. Ameto, and M. Alma: Functional observers design for descriptor systems via LMI: Continuous and Discrete-time cases. Automatika *86* (2017), 216–219. DOI:10.1016/j.automatica.2017.08.016

[3] D. Debeljkovic, N. Visnjic, and M. Pjescic: The stability of linear continuous singular systems in the sense of Lyapunov: An overview. Sci. Techn. Rev. *LVII* (2007), 1, 51–64.

[4] M. Fu: Pole Placement via static output feedback is NP-hard. IEEE Trans Automat. Control *49* (2004), 5, 855–857. DOI:10.1109/tac.2004.828311

[5] L. Van Hien, L. H. Vu, and H. Trinh: Stability of two dimmensional descriptor systems with generalized directional delays. Systems Control Lett. *112* (2018), 42–50. DOI:10.1016/j.sysconle.2017.12.003

[6] M. Chadli, J. Daafouz, and M. Darouch: Stabilization of Singular LPV systems. In: Proc. of 17th IFAC World Congress 2008, pp. 9999–10002. DOI:10.3182/20080706-5-kr-1001.01692

[7] C. Chen and Y. Liu: Lyapunov stability analysis of linear singular dynamical systems. In: Proc. Inter. Conf. on Intelligent Processing Systems, Beijing, 1997. DOI:10.1109/icips.1997.672862

[8] M. Chilali and P. Gahinet: H_∞ design with pole placement constraints: LMI approach. IEEE Trans. Automat. Control *41*, 1996, 3, 358–365. DOI:10.1109/9.486637

[9] M. Chilali, P. Gahinet, and P. Apkarian: Robust pole placement in LMI regions. IEEE Trans. Automat. Control *44* (1999), 12, 2257–2269. DOI:10.1109/9.811208

[10] X. Ji, H. Su, and J. Chu: An LMI approach to robust H_∞ control for uncertain singular time-delay systems. J. Control Theory Appl. *4* (2006), 361–366. DOI:10.1007/s11768-006-5212-2

[11] H. Kimura: A futher result on the problem of pole assignment by output feedback. IEEE Trans. Automa. Control *22* (1977), 3, 458–467. DOI:10.1109/tac.1977.1101520

[12] D. Krokavec and A. Filasova: LMI constraints on system eigenvalues placement in dynamic output control design. IEEE Conference on Control Appllications, 2015. DOI:10.1109/cca.2015.7320862

[13] V. Kucera and C. E. Souza: A necessary and sufficient condition for output feedback stabilizability. Automatika *31* (1995), 9, 1357–1359. DOI:10.1016/0005-1098(95)00048-2

[14] V. M. Kuncevic and M. M. Lycak: Control Systems Design Using Lyapunov Function Approach (In Russian). Nauka, Moscow 1977.

[15] Ch. Lin, B. Chen, P. Shi, and J. P. Yu: Necessary and sufficient conditions of observer-based stabilization for a class of fractional-order descriptor systems. Systems Control Lett. *112* (2018), 31–35. DOI:10.1016/j.sysconle.2017.12.004

[16] Ch. Lin, J. L. Wang, G. H. Yang, and C. B. Soh: Robust C-controllability and/or C-observability for uncertain desriptor systems with interval perturbations in all matrices. Trans. Automat. Control *44* (1999), 9, 1768–1773. DOI:10.1109/9.788550

[17] I. Masabuchi, Y. Kamitane, A. Ohara, and N. Suda: H_∞ control for descriptive systems: A matrix inequalities approach. Automatika *33* (1997), 4, 669–673. DOI:10.1016/s0005-1098(96)00193-8

[18] D. Peaucelle, D. Arzelier, D. Bachelier, and J. Bernussou: A new robust D-stability condition for real convex polytopic uncertainty. Systems Control Lett. *40* (2000), 21–30. DOI:10.1016/s0167-6911(99)00119-x

[19] M. S. Silva and T. P. deLima: Looking for nonnegative solutions of a Leontiev dynamic model. Linear Algebra *364* (2003), 281–316. DOI:10.1016/s0024-3795(02)00569-4

[20] K. Takaba, N. Moriharu, and T. Katayama: A generalized Lyapunov Theorem for descriptive system. Systems Control Lett. *24* (1995), 49–51. DOI:10.1016/0167-6911(94)00041-s

[21] V. Veselý and L. Körösi: Robust PI-D controller design for uncertain linear polytopic systems using LMI regions and H_2 performance. IEEE Trans. Industry Appl. *55* (2019), 5, 5353–5359. DOI:10.1109/tia.2019.2921282

[22] V. Veselý and D. Rosinová: Robust PID-PSD controller design: BMI approach. Asian J. Control *5* (2013), 2, 469–478.

[23] S. Xu, P. Van Dooren, R. Stefan, and J. Lam: Robust stability and stabilization for singular systems with state delay and parameter uncertainty. IEEE Trans. Automat. Control (2002), 1–12. DOI:10.1109/tac.2002.800651

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