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EVENT-BASED MULTI-OBJECTIVE FILTERING FOR MULTI-RATE TIME-VARYING SYSTEMS WITH RANDOM SENSOR SATURATION

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This paper focuses on the multi-objective filtering of multirate time-varying systems with random sensor saturations, where both the variance-constrained index and the H_∞ index are employed to evaluate the filtering performance. According to address issues, the high-frequency period of the internal state of the system is nondestructively converted to the low-frequency period, which determined by the measurement devices. Then the saturated output of multiple sensors is modeled as a sector bounded nonlinearity. At the same time, in order to reduce the communication frequency between sensors and filters, a communication scheduling rule is designed by the utilization of an event-triggered mechanism. By means of random analysis technology, the sufficient conditions are given to guarantee the preset H_∞ performance and variance constraint performance indexes of the system, and then the solution of the desired filter is obtained by using linear matrix inequalities. Finally, the validity and effectiveness of the proposed filter scheme are verified by numerical simulation.

Keywords: multi-rate time-varying system, stochastic saturation, H_∞ filtering, variance-constraints, event-triggered scheme

Classification: 93Cxx

1. INTRODUCTION

In practical networked control systems, sensor saturation may be one of the most common phenomena due to physical or security limitations. At present, many research results have been published on the filtering problem of networked system under sensor saturation [1, 3, 9, 15, 22, 23, 24, 26, 28, 32]. Aiming at the saturation phenomenon in discrete systems, a dynamic filter has been designed to make sure that the system has satisfactory filtering performance in both linear and nonlinear ranges in references [2, 23]. [24] has studied the problem of set-membership filtering for discrete time-varying systems with sensor saturation under unknown but bounded noise. By modeling the saturation and unknown bounded process into an ellipsoid, the filtering gain is obtained by convex optimization method. Nevertheless, the randomness of sensor saturation is not taken into account. In the actual network environment, sensor saturation may be affected by some uncertain factors, such as variable saturation level caused by sensor damage,

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intermittent saturation caused by random sensor failure and environment change. In consequence, it is more practical to model the saturation of sensors as a random model. Considering the random saturation, the H_∞ filtering problem has been researched in reference [22].

Many studies have shown that in the networked control system, if the signals of sensors, controllers, actuators and other nodes in the system can be updated at a faster frequency, the filtering performance of the system will be improved [6, 7, 8, 16, 17, 27]. On the other hand, as the networked control system has limited network bandwidth, computing capacity and other resources, it is obviously unrealistic to adopt the infinite fast update frequency. In order to balance the contradiction between limited resources and system performance, the multi-rate sampling method has attracted the attention of many researchers [10, 11, 20, 25, 29, 30, 31, 32]. Meanwhile, in the research of time-invariant multi-rate systems, lots of achievements have been made in recent years. Dealing with multi-rate problems, lifting technique is used in most literature to transform multi-rate problem into single-rate problem. In literature [10], the state estimation problem of minimum mean square error under four rates about state updating rate, measurement sampling rate, estimated updating rate and estimated output rate has been studied. The state estimation problem of multi-rate system is solved by transforming multi-rate form into single-rate form via lifting technique. Literature [11] has studied the multi-sensor filtering problem in the case of packet loss where the sensor signals with different sampling periods are converted to the same period by lifting technique, and the solution of observer-based filter is given after considering unknown input, packet loss and multi-rate.

In literature [20], multi-rate fusion estimation has been studied for statistic nonlinear systems with colored measurement noise. The multi-rate sampling system is transformed into a single-rate system by lifting technique, and a fusion estimation scheme is proposed for local state estimation. Literature [29] has studied the state estimation problem of multi-rate systems with probability sensor failure and measurement quantification. Under the premise of ensuring the stability of the system, an estimator scheme that satisfies the preset H_∞ performance index and variance constraint performance index is given by using LMI. Also, literature [30] has studied the finite-horizon H_∞ filtering problem for a class of multi-rate networked systems with fading channels. By means of random analysis, the boundedness of the estimate error and the given H_∞ performance are guaranteed.

As we all know, the bandwidth of communication network is also limited. Simultaneous transmission of multiple packets will cause data collision. To avoid this problem, various communication protocols have been proposed. For instant, different kinds of protocols are used in this area [4, 12, 13, 18, 21, 32]. Literature [12] has studied the finite time domain H_∞ filtering problem of multi-rate time-varying systems with quantization effect by using stochastic communication protocol (SCP). The multi-rate system is converted into a single-rate system with the same slow sampling rate by lifting technique. The solution of time-varying filter satisfying H_∞ performance index is given by using the completely flat method and Riccati difference equation (RDE) technique. In reference [32], H_∞ filtering problem of a class of nonlinear dynamic time-delay systems with high-speed transmission channel has been studied under the SCP. Most of the above papers on multi-rate networked systems only consider time-invariant conditions [10, 11, 20, 29, 30]. Due to the complexity and uncertainty of network environment, time-varying networked system model is more close to reality. A few time-varying multi-rate networked systems [12, 32] consider only one H_∞ constraint performance index, while in practical projects there are often multiple performance indexes [5, 14, 19]. It is worth pointing out that most results of the variance constraint theory focus on the steady-state behavior of time-invariant systems over an infinite time range, while in a real system, all of the parameters change over time. Therefore, it is more meaningful to consider the variance problem of time-varying systems in a finite time range in order to provide better transient performance.

In this paper, the study is mainly to continue to improve the research of multi-rate time-varying networked system on the basis of predecessors. The main challenges of this study are how to design appropriate event trigger for multi-rate time-varying systems and how to design filter under the scheduling of event-triggered protocol. In this paper, we will focus on solving these two challenges. Specifically, the main contributions of this paper lie in the following aspects:

(i) We focus on the finite time domain filtering problem for a kind of multi-rate time-varying networked system with variance constraint and H_∞ performance index.

(ii) After transforming the multi-rate system into a single-rate system through lifting technology, the H_∞ performance of the multi-rate system is discussed, and sufficient conditions to meet the preset system performance index are given.

(iii) The filter gain matrices are obtained by solving a set of recursive linear matrix inequalities (LMIs).

The rest of this paper is organized as follows. Section 2 formulates the variance-constrained H_∞ filtering problem for the time-varying multi-rate systems under stochastic sensor saturation. The main results are given in Section 3. Section 4 includes a simulation example to verify the proposed filtering scheme. Our conclusion is presented in Section 5.

Notation: The notations used throughout the paper are fairly standard except where otherwise stated. $\mathbb{R}^{n\times m}$ and $\mathbb{R}^n$ denote, respectively, the set of all $n \times m$ real matrix space and the n-dimensional Euclidean space. For given matrix X, $\mathrm{tr}\{X\}$ and X^T represent the operation of the trace and the transpose, respectively. $\mathbb{E}\{x\}$ denotes the expectation of stochastic variable x, and $\mathbb{P}\{x\}$ represents the occurrence probability of event x. $\mathrm{diag}_{1,n}\{A_i\}$ and $\mathrm{col}_{1,n}\{b_i\}$ stand for $\mathrm{diag}\{A_1, A_2, \ldots, A_n\}$ and the vector $[b_1^T\ b_2^T\ \ldots b_n^T]^T$, respectively. They are further simplified as $\mathrm{diag}_n\{A\}$ and $\mathrm{col}_n\{b\}$ if all A_i or b_i (written as A or b) are the same.

2. STATEMENT OF THE PROBLEM

Consider the following class of discrete-time systems

$$x(T_{k+1}) = A(T_k)\,x(T_k) + B(T_k)\,\nu(T_k) \tag{1}$$
$$z(T_k) = L(T_k)\,x(T_k) \tag{2}$$

measured by m sensors subject to randomly occurring sensor saturations

$$y_i(t_k) = \alpha_i(t_k)\,\sigma(C_i(t_k)\,x(t_k)) + (1-\alpha_i(t_k))\,C_i(t_k)\,x(t_k) + D_i(t_k)\,\omega_i(t_k) \quad (3)$$

for $i = 1, 2, \ldots, m$, where T_k $(k = 0, 1, 2, \ldots)$ is the sampling instant, $x(T_k) \in \mathbb{R}^{n_x}$ represents the state vector, $y_i(t_k) \in \mathbb{R}$ is the measurement received by sensor i, $z(T_k) \in \mathbb{R}^{n_z}$ is the signal to be estimated, $\nu(T_k) \in \mathbb{R}^{n_\nu}$ is a zero mean Gaussian white noise sequence with covariance $V > 0$, and $\omega_i(t_k) \in \mathbb{R}$ is a zero mean Gaussian white noise sequence with covariance $W_i > 0$. $A(T_k)$, $B(T_k)$, $C_i(t_k)$, $D_i(t_k)$, $L(T_k)$ are known matrices with appropriate dimensions. Gaussian noise is a kind of noises existing widely in some practical systems. For example, the thermal noise in most electronic devices is a typical Gaussian white noise.

The variable $\alpha_i(t_k)$, which governs the randomly occurring sensor saturation phenomenon, is a Bernoulli distributed white sequence taking value on 0 or 1 with

$$\text{Prob}\{\alpha_i(t_k) = 1\} = \mathbb{E}\{\alpha_i(t_k)\} = \bar{\alpha}_i,\ \text{Prob}\{\alpha_i(t_k) = 0\} = 1 - \bar{\alpha}_i$$

where $\bar{\alpha}_i \in [0, 1]$ is a known constant.

The saturation function $\sigma(\cdot) : \mathbb{R} \mapsto \mathbb{R}$ is defined as

$$\sigma(\vartheta) = \text{sign}(\vartheta) \min\{1, |\vartheta|\} \quad (4)$$

where $\text{sign}(\cdot)$ denotes the signum function. Without loss of generality, the saturation level is taken as unity here.

Let U_1 and U_2 be some real matrices with $U \triangleq U_2 - U_1 > 0$. A nonlinearity $\Phi(\cdot)$ is said to satisfy the sector condition with respect to U_1 and U_2 if

$$(\Phi(y) - U_1 y)^T (\Phi(y) - U_2 y) \leq 0$$

In this case, the sector-bounded nonlinearity $\Phi(\cdot)$ is said to belong to the sector $[U_1, U_2]$.

Note that, if there exist diagonal matrices $\underline{K}_i$ and $\bar{K}_i$ such that $0 \leq \underline{K}_i < I \leq \bar{K}_i$, then the saturation function $\sigma(C_i(t_k)\,x(t_k))$ in (2) can be written as follows

$$\sigma(C_i(t_k)\,x(t_k)) = \underline{K}_i C_i(t_k)\,x(t_k) + \Phi(C_i(t_k)\,x(t_k)) \quad (5)$$

where $\Phi(C_i(t_k)\,x(t_k))$ is a nonlinear vector-valued function satisfying the sector condition with $U_1 = 0$ and $U_2 = K_i \triangleq \bar{K}_i - \underline{K}_i$. In other words, $\Phi(C_i(t_k)\,x(t_k))$ satisfies the following inequality

$$\Phi^T(C_i(t_k)\,x(t_k))\,(\Phi(C_i(t_k)\,x(t_k)) - K_i C_i(t_k)\,x(t_k)) \leq 0. \quad (6)$$

For convenience of later analysis, we denote

$$\begin{gathered} y(t_k) = \text{col}_{1,m}\{y_i(t_k)\},\ \omega(t_k) = \text{col}_{1,m}\{\omega_i(t_k)\}, \\ C(t_k) = \text{col}_{1,m}\{C_i(t_k)\},\ D(t_k) = \text{diag}_{1,m}\{D_i(t_k)\},\ \Lambda(t_k) = \text{diag}_{1,m}\{\alpha_i(t_k)\}. \end{gathered}$$

Then, the sensor model (3) can be expressed in the following compact form

$$y(t_k) = \Lambda(t_k)\,\kappa(C(t_k)\,x(t_k)) + (I - \Lambda(t_k))\,C(t_k)\,x(t_k) + D(t_k)\,\xi(t_k) \quad (7)$$

where $\kappa\left(C\left(t_k\right)x\left(t_k\right)\right)\triangleq\left[\sigma^T\left(C_1\left(t_k\right)x\left(t_k\right)\right),\quad\ldots,\quad\sigma^T\left(C_m\left(t_k\right)x\left(t_k\right)\right)\right]^T$.

On the other hand, it follows from (5) and (6) that

$$\kappa\left(C\left(t_k\right)x\left(t_k\right)\right)=\underline{K}C\left(t_k\right)x\left(t_k\right)+\psi\left(C\left(t_k\right)x\left(t_k\right)\right)\tag{8}$$

with

$$\psi^T\left(C\left(t_k\right)x\left(t_k\right)\right)\left(\psi\left(C\left(t_k\right)x\left(t_k\right)\right)-\hat{K}C\left(t_k\right)x\left(t_k\right)\right)\leq 0\tag{9}$$

where

$$\begin{aligned}&\underline{K}=\operatorname{diag}\left\{\underline{K_1},\underline{K_2},\ldots\underline{K_m}\right\},\ \hat{K}=\operatorname{diag}\left\{K_1,K_2,\ldots,K_m\right\},\\&\psi\left(C\left(t_k\right)x\left(t_k\right)\right)=\left[\Phi^T\left(C_1\left(t_k\right)x\left(t_k\right)\right),\quad\ldots,\Phi^T\left(C_m\left(t_k\right)x\left(t_k\right)\right)\right]^T.\end{aligned}$$

To analyze the effect from the sampling periods, we make the following assumptions about (1), (2) and (7).

Assumption 2.1. The system state $x(T_k)$ is updated at instants T_k, and the sampling period satisfies $T_{k+1}-T_k=h$ for any $k=0,1,2,\ldots$.

Assumption 2.2. The measurement $y(t_k)$ from the system (1) is sampled at instants t_k satisfying $t_{k+1}-t_k=bh$ where b is a positive integer.

The event generator function $f\left(\cdot,\cdot\right)$ is defined as follows

$$f\left(\varphi\left(t_k\right),\delta\right)=\varphi^T\left(t_k\right)\varphi\left(t_k\right)-\delta y^T\left(t_k\right)y\left(t_k\right)$$

where $\varphi\left(t_k\right)\triangleq y\left(t_{k_i}\right)-y\left(t_k\right)$ with $y\left(t_{k_i}\right)$ being the measurement at the latest event time t_{k_i} and $y\left(t_k\right)$ is the current measurement. Here, $\delta\in[0\ 1)$ is the threshold.

The transmission of the measurement output to the filter is triggered as long as the condition

$$f\left(\varphi\left(t_k\right),\delta\right)>0\tag{10}$$

is satisfied. Therefore, the sequence of event-triggered instants $0\leq t_{k_0}\leq t_{k_1}\leq\ldots\leq t_{k_i}\leq\ldots$ is determined iteratively by

$$t_{k_{i+1}}=\inf\left\{t_k\in\mathbb{N}\left|t_k>t_{k_i},f\left(\varphi\left(t_k\right),\delta\right)>0\right.\right\}.$$

In practical application, if we need a better filter performance, we will set a smaller parameter. If you need a lower communication frequency, set a larger parameter.

By applying the relation (1) and (2) recursively, one obtains the following equations with time scale t_k

$$\left\{\begin{aligned}x\left(t_{k+1}\right)&=A_b\left(t_k\right)x\left(t_k\right)+B_1\left(t_k\right)\bar{\nu}\left(t_k\right)\\x\left(t_{k+1}-h\right)&=A_{b-1}\left(t_k\right)x\left(t_k\right)+B_2\left(t_k\right)\bar{\nu}\left(t_k\right)\\&\vdots\\x\left(t_{k+1}-(b-1)h\right)&=A_1\left(t_k\right)x\left(t_k\right)+B_b\left(t_k\right)\bar{\nu}\left(t_k\right)\\z\left(t_k-ih\right)&=L\left(t_k-ih\right)x\left(t_k-ih\right),\ (i=0,1,2,\ldots,b-1)\end{aligned}\right.\tag{11}$$

where

$$\begin{aligned}
A_n(t_k) &= \prod_{i=1}^{n} A(t_k + (n-i)h), \\
A_n^s(t_k) &= \begin{cases} I, & s = n \\ \prod_{i=1}^{n-s} A(t_k + (n-i)h), & s < n \end{cases} \\
B_n^s(t_k) &= A_n^s(t_k) B(t_k + (s-1)h), \\
B_1(t_k) &= \left[B_b^1(t_k)\ B_b^2(t_k)\ \ldots\ B_b^{b-1}(t_k)\ B_b^b(t_k)\right], \\
B_2(t_k) &= \left[B_{b-1}^1(t_k)\ B_{b-1}^2(t_k)\ \ldots\ B_{b-1}^{b-1}(t_k)\ 0\right], \\
&\ldots\ldots \\
B_{b-1}(t_k) &= \left[B_2^1(t_k)\ B_2^2(t_k)\ \mathrm{col}_{b-2}^T\{0\}\right], \quad B_b(t_k) = \left[B_1^1(t_k)\ \mathrm{col}_{b-1}^T\{0\}\right], \\
\bar{\nu}(t_k) &= \mathrm{col}\left\{\nu(t_k), \nu(t_k+h), \ldots, \nu(t_k + (b-1)h)\right\}.
\end{aligned}$$

Based on the system (11), the following filter is constructed in this paper

$$\begin{cases}
\hat{x}(t_{k+1}) = G_1(t_k)\hat{x}(t_k) + H_1(t_k) y(t_{k_i}) \\
\hat{x}(t_{k+1} - h) = G_2(t_k)\hat{x}(t_k) + H_2(t_k) y(t_{k_i}) \\
\vdots \\
\hat{x}(t_{k+1} - (b-1)h) = G_b(t_k)\hat{x}(t_k) + H_b(t_k) y(t_{k_i}) \\
\hat{z}(t_k - ih) = L\hat{x}(t_k - ih), (i = 0, 1, \ldots, b-1)
\end{cases} \tag{12}$$

where $\hat{x}(t_k - ih) \in \mathbb{R}^{n_x}$ and $\hat{z}(t_k - ih) \in \mathbb{R}^{n_z}$ $(i = 0, 1, 2, \ldots, b-1)$ are, respectively, the estimated state and the estimated output. $G_\varrho(t_k)$ and $H_\varrho(t_k)$ $(\varrho = 1, 2, \ldots, b)$ are the filter gains to be designed.

Denote the following vectors and matrices

$$\begin{aligned}
\bar{x}(t_k) &= \mathrm{col}\left\{x(t_k), x(t_k - h), \ldots, x(t_k - (b-1)h)\right\}, \\
\hat{\bar{x}}(t_k) &= \mathrm{col}\left\{\hat{x}(t_k), \hat{x}(t_k - h), \ldots, \hat{x}(t_k - (b-1)h)\right\}, \\
\bar{z}(t_k) &= \mathrm{col}\left\{z(t_k), z(t_k - h), \ldots, z(t_k - (b-1)h)\right\}, \\
\hat{\bar{z}}(t_k) &= \mathrm{col}\left\{\hat{z}(t_k), \hat{z}(t_k - h), \ldots, \hat{z}(t_k - (b-1)h)\right\}, \\
\tilde{\alpha}_i(t_k) &= \alpha_i(t_k) - \bar{\alpha}_i,\ (i = 1, 2, \ldots, m), \\
E_i &= \mathrm{diag}\left\{\mathrm{diag}_{i-1}\{0\}, 1, \mathrm{diag}_{m-i}\{0\}\right\}, \\
\bar{A}(t_k) &= \mathrm{col}\left\{A_b(t_k), A_{b-1}(t_k), \ldots, A(t_k)\right\}, \\
\hat{\bar{A}}(t_k) &= \left[\bar{A}(t_k) \quad \mathrm{col}_{b-1}^{\mathrm{T}}\{0\}\right], \quad \bar{H}(t_k) = \mathrm{col}_{1,b}\{H_i(t_k)\}, \\
\bar{C}_i(t_k) &= -\bar{H}(t_k) E_i C(t_k), \quad \bar{\Lambda}(t_k) = \bar{H}(t_k)\Lambda, \\
\bar{\bar{A}}(t_k) &= \bar{H}(t_k) C(t_k) - \bar{H}(t_k)\Lambda C(t_k), \\
\hat{A}(t_k) &= \left[\bar{\bar{A}}(t_k) \quad \mathrm{col}_{b-1}^{\mathrm{T}}\{0\}\right], \quad \bar{D}(t_k) = \bar{H}(t_k) D(t_k), \\
\bar{G}(t_k) &= \mathrm{col}_{1,b}\{G_i(t_k)\}, \quad \hat{\bar{G}}(t_k) = \left[\bar{G}(t_k) \quad \mathrm{col}_{b-1}^{\mathrm{T}}\{0\}\right], \\
\hat{\bar{B}}(t_k) &= \mathrm{col}_{1,b}\{B_i(t_k)\}, \quad \hat{L}(t_k) = \mathrm{diag}_b\{L(t_k)\}.
\end{aligned}$$

Then, the system of (11) and (12) can be, respectively, compacted as

$$\begin{cases} \bar{x}(t_{k+1}) = \hat{\bar{A}}(t_k)\bar{x}(t_k) + \hat{\bar{B}}(t_k)\bar{\nu}(t_k) \\ \bar{z}(t_k) = \hat{L}(t_k)\bar{x}(t_k) \end{cases} \tag{13}$$

and

$$\begin{cases} \hat{\bar{x}}(t_{k+1}) = \hat{A}(t_k)\bar{x}(t_k) + \sum_{i=1}^{m}\tilde{\alpha}_i(t_k)\bar{C}_i(t_k)\bar{x}(t_k) + \hat{\bar{G}}(t_k)\hat{\bar{x}}(t_k) \\ \qquad + \bar{H}(t_k)\varphi(t_k) + \bar{D}(t_k)\omega(t_k) + \bar{\Lambda}(t_k)\kappa(C(t_k)x(t_k)) \\ \qquad + \sum_{i=1}^{m}\tilde{\alpha}_i(t_k)\bar{H}(t_k)E_i\kappa(C(t_k)x(t_k)) \\ \hat{\bar{z}}(t_k) = \hat{L}(t_k)\hat{\bar{x}}(t_k). \end{cases} \tag{14}$$

Letting $\eta(t_k) \triangleq \left[\bar{x}^T(t_k) \;\; \hat{\bar{x}}^T(t_k)\right]^T$ and $e_z(t_k) \triangleq \bar{z}(t_k) - \bar{z}(t_k)$, we can obtain the following augmented system from (13) and (14)

$$\begin{cases} \eta(t_{k+1}) = \tilde{A}_1(t_k)\eta(t_k) + \sum_{i=1}^{m}\tilde{\alpha}_i(t_k)\tilde{A}_{2i}(t_k)\eta(t_k) \\ \qquad + \tilde{H}(t_k)\varphi(t_k) + \tilde{\Lambda}(t_k)\kappa\left(\tilde{C}(t_k)\eta(t_k)\right) \\ \qquad + \sum_{i=1}^{m}\tilde{\alpha}_i(t_k)\tilde{H}_i(t_k)\kappa\left(\tilde{C}(t_k)\eta(t_k)\right) + \tilde{B}(t_k)d(t_k) \\ e_z(t_k) = \tilde{L}(t_k)\eta(t_k) \end{cases} \tag{15}$$

where

$$\begin{aligned} &\tilde{A}_1(t_k) = \begin{bmatrix} \hat{\bar{A}}(t_k) & 0 \\ \hat{A}(t_k) & \hat{\bar{G}}(t_k) \end{bmatrix}, \quad \tilde{A}_{2i}(t_k) = \begin{bmatrix} 0 & 0 \\ \bar{C}_i(t_k) & 0 \end{bmatrix}, \\ &\tilde{B}(t_k) = \mathrm{diag}\{\hat{\bar{B}}(t_k), \bar{D}(t_k)\}, \quad \tilde{\Lambda}(t_k) = \left[\begin{array}{cc} 0 & \bar{\Lambda}^T(t_k) \end{array}\right]^T, \\ &\tilde{C}(t_k) = \left[C(t_k) \quad \mathrm{col}_{2b-1}^T\{0\}\right], \quad d(t_k) = \left[\begin{array}{cc} \bar{\nu}^T(t_k) & \omega^T(t_k) \end{array}\right]^T, \\ &\tilde{H}(t_k) = \left[\begin{array}{cc} 0 & \bar{H}^T(t_k) \end{array}\right]^T, \quad \tilde{H}_i(t_k) = \left[\begin{array}{cc} 0 & E_i^T\bar{H}^T(t_k) \end{array}\right]^T \\ &\tilde{L}(t_k) = \left[\hat{L}(t_k) \quad -\hat{L}(t_k)\right], \quad \tilde{D}(t_k) = \left[0 \quad D(t_k)\right], \end{aligned}$$

and $d(t_k)$ satisfies the following relationship

$$\begin{aligned} &\mathbb{E}\{d(t_k)\} = 0, \quad \mathbb{E}\left\{d(t_k)d^T(t_s)\right\} = 0 \;\; (k \neq s), \\ &\mathbb{E}\left\{d(t_k)d^T(t_k)\right\} = \mathrm{diag}\Big\{\underbrace{V,\ldots,V}_{b}, W\Big\} = R. \end{aligned} \tag{16}$$

The main purpose of this paper is to design the filter in the form of (12) such that the following requirements are satisfied simultaneously.

R1) For the given positive scalar γ, positive definite weighted matrices $U1$, $U2$, S and the initial state $x(t_0)$, the H_∞ performance index is satisfied

$$J_1 \triangleq \mathbb{E}\left\{\sum_{k=0}^{N-1} \|e_z(t_k)\|^2 - \gamma^2\|d(t_k)\|_U^2\right\} - \gamma^2\|x(t_0)\|_S^2 < 0 \tag{17}$$

where $\|x(t_0)\|_S^2 = x^T(t_0)Sx(t_0)$ and $\|d(t_k)\|_U^2 = d^T(t_k)Ud(t_k)$ with $U = \operatorname{diag}\{U1, U2\}$.

R2) The state estimation error covariances satisfy the following constraints

$$J_2 \triangleq \mho(t_k) = \mathbb{E}\left\{e_j(t_k)e_j^T(t_k)\right\} \le \Theta(t_k)\,(j = 0, 1, \ldots, b-1) \tag{18}$$

where $e_j(t_k) = x(t_k - jh) - \hat{x}(t_k - jh)$ and $\{\Theta(t_k)\}$ is a sequence of given matrices specifying the acceptable covariance upper bounds obtained from the engineering requirements.

3. MAIN RESULTS

3.1. H_∞ performance analysis

Theorem 3.1. Let the disturbance attenuation level $\gamma > 0$, the positive definite weighted matrices $U > 0$, $S > 0$, the scalar $\delta \in [0\ 1)$ and the gain matrices $\{G_\varrho(t_k)\}_{0\le k\le N-1}$ and $\{H_\varrho(t_k)\}_{0\le k\le N-1}$ ($\varrho = 1, 2, \ldots, b$) be given. For the augmented system (15), the performance criterion (17) is guaranteed for all nonzero $d(t_k)$ if there exist families of positive scalars $\{\lambda(t_k)\}_{0\le k\le N-1}$ and positive definite matrices $\{P(t_k)\}_{0\le k\le N}$ with the initial condition

$$P(t_0) - \gamma^2\bar{S} < 0 \tag{19}$$

guaranteeing

$$\Upsilon(t_k) = \begin{bmatrix} \Upsilon_{11}(t_k) & \Upsilon_{12}(t_k) & \Upsilon_{13}(t_k) & \Upsilon_{14}(t_k) \\ * & \Upsilon_{22}(t_k) & \Upsilon_{23}(t_k) & \Upsilon_{24}(t_k) \\ * & * & \Upsilon_{33}(t_k) & \Upsilon_{34}(t_k) \\ * & * & * & \Upsilon_{44}(t_k) \end{bmatrix} < 0 \tag{20}$$

where

$$\begin{aligned}
\bar{S} &= \operatorname{diag}\Big\{S, \operatorname{diag}_{2b-1}\{0\}\Big\}, \quad \rho_i = (1-\bar{\alpha}_i)\,\bar{\alpha}_i,\ (i = 1, 2, \ldots, m) \\
\Upsilon_{11}(t_k) &= \big(\tilde{A}_1(t_k) + \tilde{\Lambda}(t_k)\underline{K}\tilde{C}(t_k)\big)^T P(t_{k+1})\big(\tilde{A}_1(t_k) + \tilde{\Lambda}(t_k)\underline{K}\tilde{C}(t_k)\big) \\
&\quad + \delta\sum_{i=1}^{m}\rho_i\big(E_i\underline{K}\tilde{C}(t_k) - E_i\tilde{C}(t_k)\big)^T\big(E_i\underline{K}\tilde{C}(t_k) - E_i\tilde{C}(t_k)\big) \\
&\quad + \delta\big(\Lambda\underline{K}\tilde{C}(t_k) + (I-\Lambda)\tilde{C}(t_k)\big)^T\big(\Lambda\underline{K}\tilde{C}(t_k) + (I-\Lambda)\tilde{C}(t_k)\big) \\
&\quad + \sum_{i=1}^{m}\rho_i\big(\tilde{A}_{2i}(t_k) + \tilde{H}_i(t_k)\underline{K}\tilde{C}(t_k)\big)^T P(t_{k+1})\big(\tilde{A}_{2i}(t_k) \\
&\quad + \tilde{H}_i(t_k)\underline{K}\tilde{C}(t_k)\big) + \tilde{L}^T(t_k)\tilde{L}(t_k) - P(t_k), \\
\Upsilon_{12}(t_k) &= \big(\tilde{A}_1(t_k) + \tilde{\Lambda}(t_k)\underline{K}\tilde{C}(t_k)\big)^T P(t_{k+1})\tilde{H}(t_k),
\end{aligned}$$

$$
\begin{aligned}
\Upsilon_{13}(t_k) ={}& \big(\tilde{A}_1(t_k) + \tilde{\Lambda}(t_k)\underline{K}\tilde{C}(t_k)\big)^T P(t_{k+1})\tilde{\Lambda}(t_k) \\
&+ \sum_{i=1}^{m}\rho_i\big(\tilde{A}_{2i}(t_k) + \tilde{H}_i(t_k)\underline{K}\tilde{C}(t_k)\big)^T P(t_{k+1})\tilde{H}_i(t_k) \\
&+ \delta\big(\Lambda\underline{K}\tilde{C}(t_k) + (I-\Lambda)\tilde{C}(t_k)\big)^T\Lambda \\
&+ \delta\sum_{i=1}^{m}\rho_i\big(E_i\underline{K}\tilde{C}(t_k) - E_i\tilde{C}(t_k)\big)^T E_i + \frac{\lambda(t_k)}{2}\tilde{C}^T(t_k)\hat{K} \\
\Upsilon_{14}(t_k) ={}& \big(\tilde{A}_1(t_k) + \tilde{\Lambda}(t_k)\underline{K}\tilde{C}(t_k)\big)^T P(t_{k+1})\tilde{B}(t_k) \\
&+ \delta\big(\Lambda\underline{K}\tilde{C}(t_k) + (I-\Lambda)\tilde{C}(t_k)\big)^T\tilde{D}(t_k) \\
\Upsilon_{22}(t_k) ={}& \tilde{H}^T(t_k)P(t_{k+1})\tilde{H}(t_k) - I, \\
\Upsilon_{23}(t_k) ={}& \tilde{H}^T(t_k)P(t_{k+1})\tilde{\Lambda}(t_k), \quad \Upsilon_{24}(t_k) = \tilde{H}^T(t_k)P(t_{k+1})\tilde{B}(t_k), \\
\Upsilon_{33}(t_k) ={}& \tilde{\Lambda}^T(t_k)P(t_{k+1})\tilde{\Lambda}(t_k) + \sum_{i=1}^{m}\rho_i\tilde{H}_i^T(t_k)P(t_{k+1})\tilde{H}_i(t_k) \\
&+ \delta\sum_{i=1}^{m}\rho_i E_i^2 + \delta\Lambda^2 - \lambda(t_k)I, \\
\Upsilon_{34}(t_k) ={}& \tilde{\Lambda}^T(t_k)P(t_{k+1})\tilde{B}(t_k) + \delta\Lambda\tilde{D}(t_k), \\
\Upsilon_{44}(t_k) ={}& \tilde{B}^T(t_k)P(t_{k+1})\tilde{B}(t_k) + \delta\tilde{D}^T(t_k)\tilde{D}(t_k) - \gamma^2 U.
\end{aligned}
$$

P r o o f. For the purpose of performance analysis, select the Lyapunov function

$$V(\eta(t_k)) = \eta^T(t_k)P(t_k)\eta(t_k).$$

Then, its difference is defined as follows

$$\Delta V(\eta(t_k)) = \eta^T(t_{k+1})P(t_{k+1})\eta(t_{k+1}) - \eta^T(t_k)P(t_k)\eta(t_k)$$

In what follows, along the dynamics of the system (15), we have

$$
\begin{aligned}
&\mathbb{E}\{\triangle V(t_k)\} \\
={}& \mathbb{E}\Big\{\eta^T(t_k)\Big(\tilde{A}_1(t_k)^T P(t_{k+1})\tilde{A}_1(t_k) \\
&+ \sum_{i=1}^{m}\rho_i\tilde{A}_{2i}^T(t_k)P(t_{k+1})\tilde{A}_{2i}(t_k) - P(t_k)\Big)\eta(t_k) \\
&+ \varphi^T(t_k)\tilde{H}^T(t_k)P(t_{k+1})\tilde{H}(t_k)\varphi(t_k) \\
&+ \kappa^T\Big(\tilde{C}(t_k)\eta(t_k)\Big)\Big(\tilde{\Lambda}^T(t_k)P(t_{k+1})\tilde{\Lambda}(t_k) \\
&+ \sum_{i=1}^{m}\rho_i\tilde{H}_i^T(t_k)P(t_{k+1})\tilde{H}_i(t_k)\Big)\kappa\Big(\tilde{C}(t_k)\eta(t_k)\Big) \\
&+ d^T(t_k)\tilde{B}^T(t_k)P(t_{k+1})\tilde{B}(t_k)d(t_k)
\end{aligned}
$$

$$
\begin{aligned}
&+2\eta^T(t_k)\tilde{A}_1^T(t_k)P(t_{k+1})\tilde{H}(t_k)\varphi(t_k)\\
&+2\eta^T(t_k)\Big(\tilde{A}_1^T(t_k)P(t_{k+1})\tilde{\Lambda}(t_k)\\
&+\sum_{i=1}^{m}\rho_i\tilde{A}_{2i}^T(t_k)P(t_{k+1})\tilde{H}_i(t_k)\Big)\kappa\Big(\tilde{C}(t_k)\eta(t_k)\Big)\\
&+2\kappa^T\Big(\tilde{C}(t_k)\eta(t_k)\Big)\tilde{\Lambda}^T(t_k)P(t_{k+1})\tilde{B}(t_k)d(t_k)\\
&+2\varphi^T(t_k)\tilde{H}^T(t_k)P(t_{k+1})\tilde{B}(t_k)d(t_k)\\
&+2\eta^T(t_k)\tilde{A}_1^T(t_k)P(t_{k+1})\tilde{B}(t_k)d(t_k)\\
&+2\varphi^T(t_k)\tilde{H}^T(t_k)P(t_{k+1})\tilde{\Lambda}(t_k)\kappa\Big(\tilde{C}(t_k)\eta(t_k)\Big)\Big\}.
\end{aligned}
\tag{21}
$$

Substituting (8) into (21) with $C(t_k)x(t_k)=\tilde{C}(t_k)\eta(t_k)$, one has

$$
\begin{aligned}
&\mathbb{E}\{\triangle V(t_k)\}\\
=\;&\mathbb{E}\Big\{\eta^T(t_k)\Big(\tilde{A}_1^T(t_k)P(t_{k+1})\tilde{A}_1(t_k)\\
&+\sum_{i=1}^{m}\rho_i\tilde{A}_{2i}^T(t_k)P(t_{k+1})\tilde{A}_{2i}(t_k)-P(t_k)\Big)\eta(t_k)\\
&+\varphi^T(t_k)\tilde{H}^T(t_k)P(t_{k+1})\tilde{H}(t_k)\varphi(t_k)\\
&+\Big(\underline{K}\tilde{C}(t_k)\eta(t_k)+\psi\Big(\tilde{C}(t_k)\eta(t_k)\Big)\Big)^T\\
&\times\Big(\tilde{\Lambda}^T(t_k)P(t_{k+1})\tilde{\Lambda}(t_k)+\sum_{i=1}^{m}\rho_i\tilde{H}_i^T(t_k)P(t_{k+1})\\
&\times\tilde{H}_i(t_k)\Big)\Big(\underline{K}\tilde{C}(t_k)\eta(t_k)+\psi\Big(\tilde{C}(t_k)\eta(t_k)\Big)\Big)\\
&+d^T(t_k)\tilde{B}^T(t_k)P(t_{k+1})\tilde{B}(t_k)d(t_k)\\
&+2\eta^T(t_k)\tilde{A}_1^T(t_k)P(t_{k+1})\tilde{H}(t_k)\varphi(t_k)\\
&+2\eta^T(t_k)\Big(\tilde{A}_1^T(t_k)P(t_{k+1})\tilde{\Lambda}(t_k)+\sum_{i=1}^{m}\rho_i\tilde{A}_{2i}^T(t_k)P(t_{k+1})\\
&\times\tilde{H}_i(t_k)\Big)\Big(\underline{K}\tilde{C}(t_k)\eta(t_k)+\psi\Big(\tilde{C}(t_k)\eta(t_k)\Big)\Big)\\
&+2\Big(\underline{K}\tilde{C}(t_k)\eta(t_k)+\psi\Big(\tilde{C}(t_k)\eta(t_k)\Big)\Big)^T\tilde{\Lambda}^T(t_k)P(t_{k+1})\\
&\times\tilde{B}(t_k)d(t_k)+2\varphi^T(t_k)\tilde{H}^T(t_{k+1})P(t_{k+1})\tilde{B}(t_k)d(t_k)\\
&+2\eta^T(t_k)\tilde{A}_1^T(t_k)P(t_{k+1})\tilde{B}(t_k)d(t_k)+2\varphi^T(t_k)\tilde{H}^T\\
&\times(t_k)P(t_{k+1})\tilde{\Lambda}(t_k)\Big(\underline{K}\tilde{C}(t_k)\eta(t_k)+\psi\Big(\tilde{C}(t_k)\eta(t_k)\Big)\Big)\Big\}\\
=\;&\mathbb{E}\left\{\tilde{\eta}^T(t_k)\bar{\Upsilon}(t_k)\tilde{\eta}(t_k)\right\}
\end{aligned}
\tag{22}
$$

where

$$\begin{aligned}
\tilde{\eta}(t_k) &= \begin{bmatrix}\eta^T(t_k) & \varphi^T(t_k) & \psi^T\left(\tilde{C}(t_k)\eta(t_k)\right) & d^T(t_k)\end{bmatrix}^T,\\
\bar{\Upsilon}(t_k) &= \begin{bmatrix}\bar{\Upsilon}_{11}(t_k) & \Upsilon_{12}(t_k) & \bar{\Upsilon}_{13}(t_k) & \bar{\Upsilon}_{14}(t_k)\\ * & \bar{\Upsilon}_{22}(t_k) & \Upsilon_{23}(t_k) & \Upsilon_{24}(t_k)\\ * & * & \bar{\Upsilon}_{33}(t_k) & \bar{\Upsilon}_{34}(t_k)\\ * & * & * & \bar{\Upsilon}_{44}(t_k)\end{bmatrix},\\
\bar{\Upsilon}_{11}(t_k) &= \left(\tilde{A}_1(t_k) + \tilde{\Lambda}(t_k)\underline{K}\tilde{C}(t_k)\right)^T P(t_{k+1})\left(\tilde{A}_1(t_k) + \tilde{\Lambda}(t_k)\underline{K}\tilde{C}(t_k)\right)\\
&\quad - P(t_k) + \sum_{i=1}^{m}\rho_i\left(\tilde{A}_{2i}(t_k) + \tilde{H}_i(t_k)\underline{K}\tilde{C}(t_k)\right)^T\\
&\quad \times P(t_{k+1})\left(\tilde{A}_{2i}(t_k) + \tilde{H}_i(t_k)\underline{K}\tilde{C}(t_k)\right),\\
\bar{\Upsilon}_{13}(t_k) &= \left(\tilde{A}_1(t_k) + \tilde{\Lambda}(t_k)\underline{K}\tilde{C}(t_k)\right)^T P(t_{k+1})\tilde{\Lambda}(t_k)\\
&\quad \times \sum_{i=1}^{m}\rho_i\left(\tilde{A}_{2i}(t_k) + \tilde{H}_i(t_k)\underline{K}\tilde{C}(t_k)\right)^T P(t_{k+1})\tilde{H}_i(t_k),\\
\bar{\Upsilon}_{14}(t_k) &= \left(\tilde{A}_1(t_k) + \tilde{\Lambda}(t_k)\underline{K}\tilde{C}(t_k)\right)^T P(t_{k+1})\tilde{B}(t_k),\\
\bar{\Upsilon}_{22}(t_k) &= \tilde{H}^T(t_k)P(t_{k+1})\tilde{H}(t_k),\\
\bar{\Upsilon}_{33}(t_k) &= \tilde{\Lambda}^T(t_k)P(t_{k+1})\tilde{\Lambda}(t_k) + \sum_{i=1}^{m}\rho_i\tilde{H}_i^T(t_k)P(t_{k+1})\tilde{H}_i(t_k),\\
\bar{\Upsilon}_{34}(t_k) &= \tilde{\Lambda}^T(t_k)P(t_{k+1})\tilde{B}(t_k), \quad \bar{\Upsilon}_{44}(t_k) = \tilde{B}^T(t_k)P(t_{k+1})\tilde{B}(t_k).
\end{aligned}$$

Considering the event condition (10) and the nonlinear function inequality (9), we have

$$\begin{aligned}
\mathbb{E}\{\triangle V(t_k)\} \leq\ & \mathbb{E}\left\{\tilde{\eta}^T(t_k)\Upsilon(t_k)\tilde{\eta}(t_k)\right\}\\
& -\mathbb{E}\left\{\left(\varphi^T(t_k)\varphi(t_k) - \delta y^T(t_k)y(t_k)\right)\right\}\\
& -\mathbb{E}\left\{\lambda(t_k)\psi^T(t_k)\left(\psi(t_k) - \hat{K}\tilde{C}(t_k)\eta(t_k)\right).\right\}
\end{aligned} \tag{23}$$

Furthermore, for the convenience of derivation, we rewrite the (7) as follows:

$$\begin{aligned}
y(t_k) =\ & \left(\sum_{i=1}^{m}\tilde{\alpha}_i(t_k)E_i + \Lambda\right)\kappa\left(\tilde{C}(t_k)\eta(t_k)\right)\\
& + \left((I - \Lambda) - \sum_{i=1}^{m}\tilde{\alpha}_i(t_k)E_i\right)\tilde{C}(t_k)\eta(t_k) + \tilde{D}(t_k)d(t_k).
\end{aligned} \tag{24}$$

In order to introduce H_∞ performance, we add the following zero terms to (23)

$$e_z^T(t_k)e_z(t_k) - \gamma^2 d^T(t_k)Ud(t_k) - \left(e_z^T(t_k)e_z(t_k) - \gamma^2 d^T(t_k)Ud(t_k)\right) \tag{25}$$

Combining (24) and (25), one has

$$\begin{aligned}
&\mathbb{E}\left\{\triangle V\left(t_k\right)\right\}\\
\leq&\mathbb{E}\left\{\tilde{\eta}^T\left(t_k\right)\varUpsilon\left(t_k\right)\tilde{\eta}\left(t_k\right)\right\}\\
&-\mathbb{E}\left\{\left(\varphi^T\left(t_k\right)\varphi\left(t_k\right)-\delta y^T\left(t_k\right)y\left(t_k\right)\right)\right\}\\
&-\mathbb{E}\left\{\lambda\left(t_k\right)\psi^T\left(t_k\right)\left(\psi\left(t_k\right)-\hat{K}\tilde{C}\left(t_k\right)\eta\left(t_k\right)\right)\right\}\\
&+e_z^T\left(t_k\right)e_z\left(t_k\right)-\gamma^2d^T\left(t_k\right)Ud\left(t_k\right)\\
&-\left(e_z^T\left(t_k\right)e_z\left(t_k\right)-\gamma^2d^T\left(t_k\right)Ud\left(t_k\right)\right)\\
=&\mathbb{E}\left\{\tilde{\eta}^T\left(t_k\right)\varUpsilon\left(t_k\right)\tilde{\eta}\left(t_k\right)\right\}-\mathbb{E}\left\{e_z^T\left(t_k\right)e_z\left(t_k\right)-\gamma^2d^T\left(t_k\right)Ud\left(t_k\right)\right\}.
\end{aligned}\tag{26}$$

Summing (26) on both sides from 0 to $N-1$ with respect to k results in

$$\begin{aligned}
\sum_{k=0}^{N-1}\triangle V\left(t_k\right)\leq&\sum_{k=0}^{N-1}\mathbb{E}\left\{\tilde{\eta}^T\left(t_k\right)\varUpsilon\left(t_k\right)\tilde{\eta}\left(t_k\right)\right\}\\
&-\sum_{k=0}^{N-1}\mathbb{E}\left\{e_z^T\left(t_k\right)e_z\left(t_k\right)-\gamma^2d^T\left(t_k\right)Ud\left(t_k\right)\right\}.
\end{aligned}\tag{27}$$

By shifting items, one has

$$\begin{aligned}
J_1\ \leq\ &\mathbb{E}\left\{\sum_{k=0}^{N-1}\tilde{\eta}^T\left(t_k\right)\varUpsilon\left(t_k\right)\tilde{\eta}\left(t_k\right)\right\}-\mathbb{E}\left\{V\left(t_N\right)-V\left(t_0\right)\right\}-\gamma^2\|x\left(t_0\right)\|_S^2\\
=\ &\mathbb{E}\left\{\sum_{k=0}^{N-1}\tilde{\eta}^T\left(t_k\right)\varUpsilon\left(t_k\right)\tilde{\eta}\left(t_k\right)\right\}-\eta^T\left(t_N\right)P\left(t_N\right)\eta\left(t_N\right)\\
&+\eta^T\left(t_0\right)\left(P\left(t_0\right)-\gamma^2\bar{S}\right)\eta\left(t_0\right).
\end{aligned}\tag{28}$$

Since $P\left(t_N\right)>0$, $\varUpsilon\left(t_k\right)<0$ and $P\left(t_0\right)-\gamma^2\bar{S}<0$, we have $J_1\leq 0$ and the proof is now complete. □

3.2. Variance analysis

In order to facilitate the proof and derivation later in this subsection, we define

$$\mathscr{E}\left(t_k\right)\triangleq\mathbb{E}\left\{\eta\left(t_k\right)\eta^T\left(t_k\right)\right\}.\tag{29}$$

Theorem 3.2. Let the scalar $\delta\in[0\ 1)$ and the gain matrices $\{G_\varrho\left(t_k\right)\}_{0\leq k\leq N-1}$ and $\{H_\varrho\left(t_k\right)\}_{0\leq k\leq N-1}$ ($\varrho=1,2,\ldots,b$) be given. For the augmented system (15), we have $\mho(t_k)\leq\Theta(t_k)$ with the initial condition $\mathscr{E}\left(t_0\right)=Q\left(t_0\right)$ if there exist families of positive matrices $\{Q\left(t_k\right)\}_{1\leq k\leq N+1}$ satisfying

$$Q\left(t_{k+1}\right)\geq\mathscr{F}\left(Q\left(t_k\right)\right),\quad \Xi_\varrho\mathscr{E}(t_{k+1})\Xi_\varrho^T\leq\quad Q(t_{k+1}),\tag{30}$$

where

$$\begin{aligned}\mathscr{F}\left(Q\left(t_k\right)\right)= & 3\tilde{A}_1\left(t_k\right) Q\left(t_k\right) \tilde{A}_1^T\left(t_k\right)+2 \sum_{i=1}^m \rho_i \tilde{A}_{2i}\left(t_k\right) Q\left(t_k\right) \tilde{A}_{2i}^T\left(t_k\right) \\ & +3 m \tilde{\Lambda}\left(t_k\right) \tilde{\Lambda}^T\left(t_k\right)+2 m \sum_{i=1}^m \rho_i \tilde{H}_i\left(t_k\right) \tilde{H}_i^T\left(t_k\right) \\ & +\tilde{B}\left(t_k\right) R \tilde{B}^T\left(t_k\right)+3 \delta \operatorname{tr}(\Im) \tilde{H}\left(t_k\right) \tilde{H}^T\left(t_k\right)\end{aligned} \tag{31}$$

with

$$\begin{aligned}\Xi_\varrho= & {\left[\begin{array}{llllll}\operatorname{col}_{\varrho-1}^T(0) & I & \operatorname{col}_{b-\varrho}^T(0) & \operatorname{col}_{\varrho-1}^T(0) & -I & \operatorname{col}_{b-\varrho}^T(0)\end{array}\right], } \\ \Im= & 2 m \sum_{i=1}^m \rho_i E_i+2 m \Lambda^2+2 \tilde{C}^T\left(t_k\right)(I-\Lambda)^2 \tilde{C}\left(t_k\right) Q\left(t_k\right) \\ & +2 \sum_{i=1}^m \rho_i \tilde{C}^T\left(t_k\right) E_i \tilde{C}\left(t_k\right) Q\left(t_k\right)+\tilde{D}^T\left(t_k\right) \tilde{D}\left(t_k\right) R .\end{aligned}$$

P r o o f. From (15) and (29), we have

$$\begin{aligned}\mathscr{E}\left(t_{k+1}\right) = & \mathbb{E}\left\{\eta\left(t_{k+1}\right) \eta^T\left(t_{k+1}\right)\right\} \\ = & \mathbb{E}\Big\{\tilde{A}_1\left(t_k\right) \mathscr{E}\left(t_k\right) \tilde{A}_1^T\left(t_k\right)+\sum_{i=1}^m \rho_i \tilde{A}_{2i}\left(t_k\right) \mathscr{E}\left(t_k\right) \tilde{A}_{2i}^T\left(t_k\right) \\ & +\tilde{\Lambda}\left(t_k\right) \kappa\Big(\tilde{C}\left(t_k\right) \eta\left(t_k\right)\Big) \kappa^T\Big(\tilde{C}\left(t_k\right) \eta\left(t_k\right)\Big) \tilde{\Lambda}^T\left(t_k\right) \\ & +\sum_{i=1}^m \rho_i \tilde{H}_i\left(t_k\right) \kappa\Big(\tilde{C}\left(t_k\right) \eta\left(t_k\right)\Big) \kappa^T\Big(\tilde{C}\left(t_k\right) \eta\left(t_k\right)\Big) \tilde{H}_i^T\left(t_k\right) \\ & +\tilde{A}_1\left(t_k\right) \eta\left(t_k\right) \varphi^T\left(t_k\right) \tilde{H}^T\left(t_k\right)+\tilde{H}\left(t_k\right) \varphi\left(t_k\right) \eta^T\left(t_k\right) \tilde{A}_1^T\left(t_k\right) \\ & +\tilde{A}_1\left(t_k\right) \eta\left(t_k\right) \kappa^T\Big(\tilde{C}\left(t_k\right) \eta\left(t_k\right)\Big) \tilde{\Lambda}^T\left(t_k\right) \\ & +\tilde{\Lambda}\left(t_k\right) \kappa\Big(\tilde{C}\left(t_k\right) \eta\left(t_k\right)\Big) \eta^T\left(t_k\right) \tilde{A}_1^T\left(t_k\right) \\ & +\sum_{i=1}^m \rho_i \tilde{A}_{2i}\left(t_k\right) \eta\left(t_k\right) \kappa^T\Big(\tilde{C}\left(t_k\right) \eta\left(t_k\right)\Big) \tilde{H}_i^T\left(t_k\right) \\ & +\sum_{i=1}^m \rho_i \tilde{H}_i\left(t_k\right) \kappa\Big(\tilde{C}\left(t_k\right) \eta\left(t_k\right)\Big) \eta^T\left(t_k\right) \tilde{A}_{2i}^T\left(t_k\right) \\ & +\tilde{H}\left(t_k\right) \varphi\left(t_k\right) \kappa^T\Big(\tilde{C}\left(t_k\right) \eta\left(t_k\right)\Big) \tilde{\Lambda}^T\left(t_k\right) \\ & +\tilde{\Lambda}\left(t_k\right) \kappa\Big(\tilde{C}\left(t_k\right) \eta\left(t_k\right)\Big) \varphi^T\left(t_k\right) \tilde{H}^T\left(t_k\right) \\ & +\tilde{H}\left(t_k\right) \varphi\left(t_k\right) \varphi^T\left(t_k\right) \tilde{H}^T\left(t_k\right)+\tilde{B}\left(t_k\right) R \tilde{B}^T\left(t_k\right)\Big\} .\end{aligned} \tag{32}$$

Applying the elementary inequality, one has

$$\begin{aligned}\mathscr{E}(t_{k+1}) \leq\ & 3\tilde{A}_1(t_k)\mathscr{E}(t_k)\tilde{A}_1^T(t_k) + 2\sum_{i=1}^{m}\rho_i\tilde{A}_{2i}(t_k)\mathscr{E}(t_k)\tilde{A}_{2i}^T(t_k) \\ & +3\tilde{H}(t_k)\varphi(t_k)\varphi^T(t_k)\tilde{H}^T(t_k) \\ & +3\tilde{\Lambda}(t_k)\kappa\left(\tilde{C}(t_k)\eta(t_k)\right)\kappa^T\left(\tilde{C}(t_k)\eta(t_k)\right)\tilde{\Lambda}^T(t_k) \\ & +2\sum_{i=1}^{m}\rho_i\tilde{H}_i(t_k)\kappa\left(\tilde{C}(t_k)\eta(t_k)\right)\kappa^T\left(\tilde{C}(t_k)\eta(t_k)\right)\tilde{H}_i^T(t_k) \\ & +\tilde{B}(t_k)R\tilde{B}^T(t_k).\end{aligned} \tag{33}$$

Considering the saturation function defined in (4), we have

$$\sigma(\vartheta)\sigma^T(\vartheta) \leq 1.$$

Furthermore, one has

$$\kappa\left(\tilde{C}(t_k)\eta(t_k)\right)\kappa^T\left(\tilde{C}(t_k)\eta(t_k)\right) \leq mI. \tag{34}$$

Combining the event condition (10) and the saturation constraint (34) results in

$$\begin{aligned}\mathscr{E}(t_{k+1}) \leq\ & 3\tilde{A}_1(t_k)\mathscr{E}(t_k)\tilde{A}_1^T(t_k) + 2\sum_{i=1}^{m}\rho_i\tilde{A}_{2i}(t_k)\mathscr{E}(t_k)\tilde{A}_{2i}^T(t_k) \\ & + 3m\tilde{\Lambda}(t_k)\tilde{\Lambda}^T(t_k) + 2m\sum_{i=1}^{m}\rho_i\tilde{H}_i(t_k)\tilde{H}_i^T(t_k) \\ & + 3\mathrm{tr}\left(\mathbb{E}\left\{y(t_k)y^T(t_k)\right\}\right)\delta\tilde{H}(t_k)\tilde{H}^T(t_k) + \tilde{B}(t_k)R\tilde{B}^T(t_k).\end{aligned} \tag{35}$$

Substituting (24) into (35) and then considering the saturation constraint (34) yield

$$\mathscr{E}(t_{k+1}) \leq \mathscr{F}(\mathscr{E}(t_k)). \tag{36}$$

The subsequent proof is dealt with via the mathematical induction. First, one has $Q(t_0) \geq \mathscr{E}(t_0)$. Then, suppose $Q(t_k) \geq \mathscr{E}(t_k)$ is true. Now, let us show that t_{k+1} also holds:

$$Q(t_{k+1}) \geq \mathscr{F}(Q(t_k)) \geq \mathscr{F}(\mathscr{E}(t_k)) \geq \mathscr{E}(t_{k+1}).$$

Finally, since $\mathscr{E}(t_k) \leq Q(t_k)$, we have

$$\mho(t_k) \leq \Xi_\varrho Q(t_k)\Xi_\varrho^T. \tag{37}$$

Combining (37) and (30), we have $\mho(t_k) \leq \Theta(t_k)$, which is equivalent to (18). Now the proof is complete. □

3.3. Performance Synthesization

According to Theorem 3.1 and Theorem 3.2 can be easily accessed.

Theorem 3.3. Let the disturbance attenuation level $\gamma > 0$, the positive definite weighted matrices $U > 0$, $S > 0$, the scalar $\delta \in [0\ 1)$ and the gain matrices $\{G_\varrho(t_k)\}_{0\le k\le N-1}$ and $\{H_\varrho(t_k)\}_{0\le k\le N-1}$ $(\varrho = 1, 2, \ldots, b)$ be given. For the augmented system (15), the performance criterions (17) and (18) are guaranteed if there exist families of positive scalars $\{\lambda(t_k)\}_{0\le k\le N}$ and positive definite matrices $\{P(t_k), Q(t_k)\}_{1\le k\le N+1}$ with the initial conditions

$$P(t_0) - \gamma^2 \bar{S} < 0\ Q(t_0) = \mathscr{E}(t_0) \tag{38}$$

satisfying the following recursive matrix inequalities

$$\begin{bmatrix} \hat{\Upsilon}_{11}(t_k) & * & * & * & * & * \\ \hat{\Upsilon}_{21}(t_k) & \hat{\Upsilon}_{22}(t_k) & * & * & * & * \\ \hat{\Upsilon}_{31}(t_k) & \hat{\Upsilon}_{32}(t_k) & \hat{\Upsilon}_{33}(t_k) & * & * & * \\ \hat{\Upsilon}_{41}(t_k) & \hat{\Upsilon}_{42}(t_k) & 0 & \hat{\Upsilon}_{44}(t_k) & * & * \\ \hat{\Upsilon}_{51}(t_k) & \hat{\Upsilon}_{52}(t_k) & 0 & 0 & \hat{\Upsilon}_{55}(t_k) & * \\ \hat{\Upsilon}_{61}(t_k) & \hat{\Upsilon}_{62}(t_k) & 0 & 0 & 0 & \hat{\Upsilon}_{66}(t_k) \end{bmatrix} < 0 \tag{39}$$

$$\begin{bmatrix} -Q(t_{k+1}) & \Omega_{12}(t_k) & \Omega_{13}(t_k) & \Omega_{14}(t_k) & \sqrt{3\delta}\tilde{H}(t_k) \\ * & \Omega_{22}(t_k) & 0 & 0 & 0 \\ * & * & \Omega_{33}(t_k) & 0 & 0 \\ * & * & * & \Omega_{44}(t_k) & 0 \\ * & * & * & * & \Omega_{55}(t_k) \end{bmatrix} < 0 \tag{40}$$

$$\Xi_\varrho \mathscr{E}(t_{k+1}) \Xi_\varrho^T \le Q(t_{k+1}), \quad (\varrho = 1, 2, \ldots, b) \tag{41}$$

where

$$\begin{aligned}
\hat{\Upsilon}_{11}(t_k) =& \operatorname{diag}\{-P(t_k), -I\}, \\
\hat{\Upsilon}_{21}(t_k) =& \operatorname{diag}\left\{0.5\lambda(t_k)\hat{K}\tilde{C}(t_k), 0\right\}, \\
\hat{\Upsilon}_{22}(t_k) =& \operatorname{diag}\{-\lambda(t_k) I, -\gamma^2 U\}, \\
\hat{\Upsilon}_{31}(t_k) =& \left[\tilde{A}_1(t_k) + \tilde{\Lambda}(t_k)\underline{K}\tilde{C}(t_k) \quad \tilde{H}(t_k)\right], \\
\hat{\Upsilon}_{32}(t_k) =& \left[\tilde{\Lambda}(t_k) \quad \tilde{B}(t_k)\right], \quad \hat{\Upsilon}_{61}(t_k) = \left[\hat{\Upsilon}_{611}(t_k) \quad 0\right], \\
\hat{\Upsilon}_{41}(t_k) =& \left[\hat{\Upsilon}_{411}(t_k) \quad 0\right], \quad \hat{\Upsilon}_{51}(t_k) = \left[\hat{\Upsilon}_{511}(t_k) \quad 0\right], \\
\hat{\Upsilon}_{42}(t_k) =& \left[\hat{\Upsilon}_{421}(t_k) \quad 0\right], \quad \hat{\Upsilon}_{52}(t_k) = \left[\hat{\Upsilon}_{521}(t_k) \quad 0\right], \\
\hat{\Upsilon}_{421}(t_k) =& \operatorname{col}_{1,m}\{\sqrt{\rho_i}\tilde{H}_i(t_k)\}, \quad \hat{\Upsilon}_{521}(t_k) = \operatorname{col}_{1,m}\{\sqrt{\delta\rho_i}E_i\}, \\
\hat{\Upsilon}_{411}(t_k) =& \operatorname{col}_{1,m}\{\sqrt{\rho_i}\phi_i(t_k)\}, \quad \hat{\Upsilon}_{511}(t_k) = \operatorname{col}_{1,m}\{\sqrt{\delta\rho_i}\Gamma_i(t_k)\}, \\
\hat{\Upsilon}_{611}(t_k) =& \operatorname{col}\left\{\sqrt{\delta}\left(\Lambda\underline{K}\tilde{C}(t_k) + (I - \Lambda)\tilde{C}(t_k)\right), \tilde{L}(t_k)\right\},
\end{aligned}$$

$$
\begin{aligned}
\hat{\varUpsilon}_{62}(t_k) &= \begin{bmatrix}\hat{\varUpsilon}_{621}(t_k) & \hat{\varUpsilon}_{622}(t_k)\end{bmatrix}, \quad \hat{\varUpsilon}_{33}(t_k) = -P^{-1}(t_{k+1}),\\
\hat{\varUpsilon}_{44}(t_k) &= \mathrm{diag}_m\{-P^{-1}(t_{k+1})\}, \quad \hat{\varUpsilon}_{55}(t_k) = \mathrm{diag}_m\{I\},\\
\hat{\varUpsilon}_{66}(t_k) &= \mathrm{diag}_2\{I\}, \quad \hat{\varUpsilon}_{621}(t_k) = \mathrm{col}\left\{\sqrt{\delta}\varLambda, 0\right\},\\
\hat{\varUpsilon}_{622}(t_k) &= \mathrm{col}\left\{\sqrt{\delta}\tilde{D}(t_k), 0\right\}, \quad \varGamma_i(t_k) = E_i\underline{K}\tilde{C}(t_k) - E_i\tilde{C}(t_k),\\
\phi_i(t_k) &= \tilde{A}_{2i}(t_k) + \tilde{H}_i(t_k)\underline{K}\tilde{C}(t_k),\\
\varOmega_{12}(t_k) &= \begin{bmatrix}\sqrt{3}\tilde{A}_1(t_k)Q(t_k) & \sqrt{3m}\tilde{\varLambda}(t_k) & \tilde{B}(t_k)\end{bmatrix},\\
\varOmega_{22}(t_k) &= \mathrm{diag}\left\{-Q(t_k), -I, -R^{-1}\right\},\\
\varOmega_{13}(t_k) &= \begin{bmatrix}\sqrt{2\rho_1}\tilde{A}_{21}(t_k)Q(t_k) & \dots & \sqrt{2\rho_m}\tilde{A}_{2m}(t_k)Q(t_k)\end{bmatrix},\\
\varOmega_{14}(t_k) &= \begin{bmatrix}\sqrt{2m\rho_1}\tilde{H}_1(t_k) & \dots & \sqrt{2m\rho_m}\tilde{H}_m(t_k)\end{bmatrix},\\
\varOmega_{33}(t_k) &= -\,\mathrm{diag}_m\{Q(t_k)\}, \quad \varOmega_{44}(t_k) = -\mathrm{diag}_m\{I\},\\
\varOmega_{55}(t_k) &= -\left(\mathrm{tr}\left[\varOmega_{551}(t_k)\right] + \mathrm{tr}\left[\varOmega_{552}(t_k)\right] + \mathrm{tr}\left[\varOmega_{553}(t_k)\right]\right)^{-1} I,\\
\varOmega_{551}(t_k) &= 2m\sum_{i=1}^{m}\rho_i E_i + 2m\varLambda^2, \varOmega_{553}(t_k) = \tilde{D}^T(t_k)\tilde{D}(t_k)R,\\
\varOmega_{552}(t_k) &= 2\tilde{C}^T(t_k)(I-\varLambda)^2\tilde{C}(t_k)Q(t_k) + 2\sum_{i=1}^{m}\rho_i\tilde{C}^T(t_k)E_i\tilde{C}(t_k)Q(t_k).
\end{aligned}
$$

P r o o f. By the Schur complement lemma, the inequality (39) is equivalent to (21), and the inequality (40) is equivalent to (30). Thus, according to Theorem 3.1 and Theorem 3.2, the H_∞ index defined in (17) satisfies $J_1 < 0$ and, at the same time, the covariance of system (15) achieves $\mho(t_k) \le \Theta(t_k)$. The proof is now complete. □

3.4. Finite horizon filter design

The following theorem provides the desired filter gains.

Theorem 3.4. Let the disturbance attenuation level $\gamma > 0$, the positive definite weighted matrices $U > 0$, $S > 0$, the scalar $\delta \in [0\ 1)$ and a sequence of prespecified variance upper bounds $\{\Theta(t_k)\}_{0\le k\le N}$ be given. For the augmented system (15), the performance criterions (17) and (18) are guaranteed if there exist families of positive scalars $\{\lambda(t_k)\}_{0\le k\le N-1}$ and positive definite matrices $\{\mathcal{P}_\varrho(t_k), Q_\varrho(t_k)\}_{1\le k\le N}$, ($\varrho = 1, 2, \dots, 2b$), families of matrices $\{G_\varrho(t_k), H_\varrho(t_k)\}_{0\le k\le N-1}$ ($\varrho = 1, 2, \dots, b$) satisfying the following recursive matrix inequalities

$$
Q_\varrho(t_{k+1}) + Q_{b+\varrho}(t_{k+1}) < \Theta(t_{k+1}) \tag{42}
$$

$$\begin{bmatrix} \tilde{\Upsilon}_{11}(t_k) & * & * & * & * & * & * & * \\ \tilde{\Upsilon}_{21}(t_k) & \tilde{\Upsilon}_{22}(t_k) & * & * & * & * & * & * \\ \tilde{\Upsilon}_{31}(t_k) & \tilde{\Upsilon}_{32}(t_k) & \tilde{\Upsilon}_{33}(t_k) & * & * & * & * & * \\ \tilde{\Upsilon}_{41}(t_k) & \tilde{\Upsilon}_{42}(t_k) & 0 & \tilde{\Upsilon}_{44}(t_k) & * & * & * & * \\ \tilde{\Upsilon}_{51}(t_k) & \tilde{\Upsilon}_{52}(t_k) & 0 & 0 & \tilde{\Upsilon}_{55}(t_k) & * & * & * \\ \tilde{\Upsilon}_{61}(t_k) & \tilde{\Upsilon}_{62}(t_k) & 0 & 0 & 0 & \tilde{\Upsilon}_{66}(t_k) & * & * \\ \tilde{\Upsilon}_{71}(t_k) & \tilde{\Upsilon}_{72}(t_k) & 0 & 0 & 0 & 0 & -I & * \\ \tilde{\Upsilon}_{81}(t_k) & 0 & 0 & 0 & 0 & 0 & 0 & \tilde{\Upsilon}_{88}(t_k) \end{bmatrix} < 0 \quad (43)$$

$$\begin{bmatrix} \tilde{\Omega}_{11}(t_k) & 0 & \tilde{\Omega}_{13}(t_k) & \tilde{\Omega}_{14}(t_k) & 0 & 0 & 0 \\ * & \tilde{\Omega}_{22}(t_k) & \tilde{\Omega}_{23}(t_k) & \tilde{\Omega}_{24}(t_k) & \tilde{\Omega}_{25}(t_k) & \tilde{\Omega}_{26}(t_k) & \sqrt{3\delta}\bar{H}(t_k) \\ * & * & \tilde{\Omega}_{33}(t_k) & 0 & 0 & 0 & 0 \\ * & * & * & -R^{-1} & 0 & 0 & 0 \\ * & * & * & * & \tilde{\Omega}_{55}(t_k) & 0 & 0 \\ * & * & * & * & * & \tilde{\Omega}_{66}(t_k) & 0 \\ * & * & * & * & * & * & \tilde{\Omega}_{77}(t_k) \end{bmatrix} < 0 \quad (44)$$

with the initial condition

$$\begin{cases} P_1(t_0) \le \gamma^2 S, \quad P_t(t_0) = 0, (t = 2, 3, \ldots, 2b) \\ \mathscr{E}(t_0) = Q(t_0), \quad Q_\varrho(t_0) + Q_{b+\varrho}(t_0) \le \Theta(t_0) \end{cases} \quad (45)$$

and updated parameters

$$P_\varrho(t_{k+1}) = \mathcal{P}_\varrho^{-1}(t_{k+1})$$

where

$$\begin{aligned}
&\Upsilon_{11}(t_k) = \operatorname{diag}\{-P_1(t_k), -P_2(t_k), \ldots, -P_{2b}(t_k), -I\} \\
&\tilde{\Upsilon}_{21}(t_k) = \operatorname{diag}\left\{0.5\lambda(t_k)\hat{K}C(t_k), \operatorname{col}_{2b}^{\mathrm{T}}\{0\}\right\}, \quad \tilde{\Upsilon}_{31}(t_k) = \begin{bmatrix}\bar{A}(t_k) & \operatorname{col}_{2b}^{\mathrm{T}}\{0\}\end{bmatrix} \\
&\tilde{\Upsilon}_{41}(t_k) = \begin{bmatrix}\bar{H}(t_k)(I - \Lambda + \Lambda\underline{K})C(t_k) & \operatorname{col}_{b-1}^{\mathrm{T}}\{0\} & \bar{G}(t_k) & \operatorname{col}_{b-1}^{\mathrm{T}}\{0\}) & \bar{H}(t_k)\end{bmatrix} \\
&\tilde{\Upsilon}_{51i}(t_k) = \begin{bmatrix} 0 & \operatorname{col}_{2b}^{\mathrm{T}}\{0\} \\ \bar{H}(t_k)E_i(\underline{K} - I)C(t_k) & \operatorname{col}_{2b}^{\mathrm{T}}\{0\}\end{bmatrix}, (i = 1, 2, \ldots, m) \\
&\tilde{\Upsilon}_{61i}(t_k) = \begin{bmatrix}E_i(\underline{K} - I)C(t_k) & \operatorname{col}_{2b}^{\mathrm{T}}\{0\}\end{bmatrix}, (i = 1, 2, \ldots, m) \\
&\tilde{\Upsilon}_{51}(t_k) = \operatorname{col}_{1,m}\{\tilde{\Upsilon}_{51i}(t_k)\}, \quad \tilde{\Upsilon}_{61}(t_k) = \operatorname{col}_{1,m}\{\tilde{\Upsilon}_{61i}(t_k)\} \\
&\tilde{\Upsilon}_{71}(t_k) = \begin{bmatrix}(\Lambda\underline{K} + I - \Lambda)C(t_k) & \operatorname{col}_{2b}^{\mathrm{T}}(0)\end{bmatrix}, \quad \tilde{\Upsilon}_{81}(t_k) = \begin{bmatrix}\tilde{L}(t_k) & 0\end{bmatrix} \\
&\tilde{\Upsilon}_{22}(t_k) = \operatorname{diag}\{-\lambda(t_k)I, -\gamma^2 U1, -\gamma^2 U2\}, \quad \tilde{\Upsilon}_{32}(t_k) = \begin{bmatrix}0 & \hat{\bar{B}}(t_k) & 0\end{bmatrix} \\
&\tilde{\Upsilon}_{42}(t_k) = \begin{bmatrix}\bar{H}(t_k)\Lambda & 0 & \bar{H}(t_k)D(t_k)\end{bmatrix} \\
&\tilde{\Upsilon}_{52i}(t_k) = \begin{bmatrix} 0 & \operatorname{col}_b^{\mathrm{T}}\{0\} & 0 \\ \bar{H}(t_k)E_i & \operatorname{col}_b^{\mathrm{T}}\{0\} & 0\end{bmatrix}, \quad \tilde{\Upsilon}_{62i}(t_k) = \begin{bmatrix}E_i & \operatorname{col}_b^{\mathrm{T}}\{0\} & 0\end{bmatrix} \\
&\tilde{\Upsilon}_{52}(t_k) = \operatorname{col}_{1,m}\{\tilde{\Upsilon}_{52i}(t_k)\}, \quad \tilde{\Upsilon}_{62}(t_k) = \operatorname{col}_{1,m}\{\tilde{\Upsilon}_{62i}(t_k)\} \\
&\tilde{\Upsilon}_{72}(t_k) = \begin{bmatrix}\Lambda & \operatorname{col}_b^{\mathrm{T}}(0) & D(t_k)\end{bmatrix}
\end{aligned}$$

$$\begin{aligned}
\tilde{\varUpsilon}_{33}(t_k) &= \mathrm{diag}\left\{-\mathcal{P}_1(t_{k+1}), -\mathcal{P}_2(t_{k+1}), \dots, -\mathcal{P}_b(t_{k+1})\right\} \\
\tilde{\varUpsilon}_{44}(t_k) &= \mathrm{diag}\left\{-\mathcal{P}_{b+1}(t_{k+1}), -\mathcal{P}_{b+2}(t_{k+1}), \dots, -\mathcal{P}_{2b}(t_{k+1})\right\} \\
\tilde{\varUpsilon}_{55}(t_k) &= I_m \otimes \mathrm{diag}\left\{\tilde{\varUpsilon}_{33}(t_k), \tilde{\varUpsilon}_{44}(t_k)\right\} \\
\tilde{\varUpsilon}_{66}(t_k) &= \mathrm{diag}_m\{-I\}, \tilde{\varUpsilon}_{88}(t_k) = \mathrm{diag}_b\{-I\} \\
\tilde{\varOmega}_{11}(t_k) &= -\,\mathrm{diag}_{1,b}\{Q_i(t_{k+1})\}, \quad \tilde{\varOmega}_{22}(t_k) = -\mathrm{diag}_{b+1,2b}\{Q_{b+1}(t_{k+1})\} \\
\tilde{\varOmega}_{13}(t_k) &= \left[\sqrt{3}\bar{A}(t_k) Q_1(t_k) \quad \mathrm{col}_{2b}^{\mathrm{T}}\{0\}\right], \quad \tilde{\varOmega}_{23}(t_k) = \left[\tilde{\varOmega}_{231}(t_k) \quad \tilde{\varOmega}_{232}(t_k)\right] \\
\tilde{\varOmega}_{231}(t_k) &= \left[\sqrt{3}\bar{H}(t_k)\left(C(t_k) - \varLambda C(t_k)\right) Q_1(t_k) \quad \mathrm{col}_{b-1}^{\mathrm{T}}\{0\}\right] \\
\tilde{\varOmega}_{232}(t_k) &= \left[\sqrt{3}\bar{G}(t_k) Q_{b+1}(t_k) \quad \mathrm{col}_{b-1}^{\mathrm{T}}(0) \quad \sqrt{3m}\bar{H}(t_k)\varLambda\right] \\
\tilde{\varOmega}_{33}(t_k) &= \mathrm{diag}\left\{-Q_1(t_k), \dots, -Q_{2b}(t_k), -I\right\} \\
\tilde{\varOmega}_{14}(t_k) &= [\hat{\bar{B}}(t_k) \quad 0], \quad \tilde{\varOmega}_{24}(t_k) = \left[\mathrm{col}_{b-1}^{\mathrm{T}}\{0\} \quad \bar{H}(t_k) D(t_k)\right] \\
\tilde{\varOmega}_{25}(t_k) &= \mathrm{col}_{1,m}^{T}\{\sqrt{2\rho_i} Q_1^T(t_k) C^T(t_k) E_i^T \bar{H}^T(t_k)\} \\
\tilde{\varOmega}_{26}(t_k) &= \mathrm{col}_{1,m}^{T}\{\sqrt{2m\rho_i} E_i^T \bar{H}^T(t_k)\} \\
\tilde{\varOmega}_{55}(t_k) &= -\,\mathrm{diag}_m\{I\}, \quad \tilde{\varOmega}_{66}(t_k) = -\mathrm{diag}_m\{I\} \\
\tilde{\varOmega}_{77}(t_k) &= -\left(\mathrm{tr}\left[\varOmega_{551}(t_k)\right] + \mathrm{tr}\left[\tilde{\varOmega}_{772}(t_k)\right] + \mathrm{tr}\left[\varOmega_{553}(t_k)\right]\right)^{-1} I \\
\tilde{\varOmega}_{772}(t_k) &= 2C^T(t_k)(I - \varLambda)^2 C(t_k) Q_1(t_k) + 2\sum_{i=1}^{m} \rho_i C^T(t_k) E_i C(t_k) Q_1(t_k)
\end{aligned}$$

Proof. First, we select $P(t_k)$ and $Q(t_k)$ with the following structures

$$P(t_k) = \mathrm{diag}_{1,2b}\left\{P_i(t_k)\right\}, \quad Q(t_k) = \mathrm{diag}_{1,2b}\left\{Q_i(t_k)\right\}.$$

Substituting them into (39) and (40), one has the inequality (43) is equivalent to (39), and the inequality (44) is equivalent to (40).

In what follows, based on the proof of Theorem 3.2, we have:

$$\Xi_{\varrho}\mathscr{E}(t_k)\Xi_{\varrho}^T = e_{\varrho-1}(t_k) e_{\varrho-1}^T(t_k) = \mho(t_k). \tag{46}$$

Furthermore, since $\mathscr{E}(t_k) \le Q(t_k)$, from (46), we have

$$\mho(t_k) \le \Xi_{\varrho} Q(t_k) \Xi_{\varrho}^T = Q_{\varrho}(t_k) + Q_{b+\varrho}(t_k). \tag{47}$$

Combining (47) and (42), we have $\mho(t_k) \le \varTheta(t_k)$, which is equivalent to (18).

According to Theorem 3.3, the H_∞ index defined in (17) satisfies $J_1 < 0$ and, at the same time, the covariance of system (15) achieves $\mathscr{E}(t_k) \le Q(t_k)$. Now the proof is complete. □

4. SIMULATION RESULTS

Consider the following discrete-time system with parameters

$$
\begin{aligned}
x(T_{k+1}) &= \begin{bmatrix} 0.15 & 0.2 \\ 0 & 0.4+0.1\sin(0.3T_k) \end{bmatrix} x(T_k) + \begin{bmatrix} 0.02 \\ -0.1\sin(0.1T_k) \end{bmatrix} \nu(T_k), \\
z(T_k) &= \begin{bmatrix} 0.3 & 0.15+0.01\sin(0.2T_k) \end{bmatrix} x(T_k), \\
C_1(t_k) &= \begin{bmatrix} 0.5 & -0.3\sin(t_k) \end{bmatrix}, \quad C_2(t_k) = \begin{bmatrix} -0.5\sin(t_k) & 0.2 \end{bmatrix}, \\
D_1(t_k) &= 0.1, \quad D_2(t_k) = 0.2, \quad \bar{\alpha}_1 = 0.85, \quad \bar{\alpha}_2 = 0.95.
\end{aligned}
$$

The initial state of the discrete-time system and its estimation are respectively

$$
x(t_0) = \begin{bmatrix} 0.8 \\ -0.65 \end{bmatrix}, \hat{x}(t_0) = \begin{bmatrix} -0.75 \\ 0.6 \end{bmatrix}.
$$

The covariances of $\nu(T_k)$, $\omega_1(t_k)$ and $\omega_2(t_k)$ are, respectively, $V = 0.2$, $W_1 = 0.3$ and $W_2 = 0.3$. Correlation coefficients of saturation nonlinear functions are $\underline{K_1} = 0.53$, $\underline{K_2} = 0.62$, $K_1 = 0.87$, and $K_2 = 0.94$. The threshold value in event trigger scheme is $\delta = 0.75$ and the level of H_∞ index is $\gamma = 0.9$. Positive definite weight matrices $S = 0.5I$ and $U = I$, the upper bound of state error variance is set as $\Theta(t_k) = \mathrm{diag}\{0.2, 0.15\}$, and the status update cycle is $h = \frac{1}{b}$ ($b = 1, 2, 3, 4$). From the above values, the following simulation chart can be obtained by using LMI toolbox.

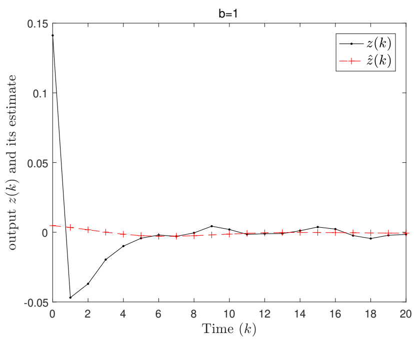
Tables 1-4 represent the list of filter gains required by the system with different b at different times. Figure 1 shows the trajectory of signal $z(t_k)$ and its estimation of $\hat{z}(t_k)$. It can be seen that the filtering performance is improved with the increased b. Figure 4 shows the comparison of the errors between signal $z(t_k)$ and its estimation $\hat{z}(t_k)$ at different b. It can be further seen from the figure that the filtering error converges faster when b is increased. Figs. 2 and 3 describe the variances of the estimated errors under different scalar b. It can be clearly seen that both of them are smaller than the given performance index, and the error variance of each component is improved for the increased b. Figure 5 depicts the time instants when the event occurs. As can be seen from Figure 5 , the event triggering protocol designed in this paper can greatly reduce the rate of data transmission, thus improving the efficiency of energy utilization and reducing the network burden.

t_k	0	1	...	20
G_1	$\begin{bmatrix} 0.6813 & 0.8318 \\ 0.3795 & 0.5028 \end{bmatrix}$	$\begin{bmatrix} 0.1934 & 0.3028 \\ 0.6822 & 0.5417 \end{bmatrix}$	...	$\begin{bmatrix} 0.6408 & 0.8439 \\ 0.1909 & 0.1739 \end{bmatrix}$
H_1	$\begin{bmatrix} 0.7095 & 0.3046 \\ 0.4289 & 0.1897 \end{bmatrix}$	$\begin{bmatrix} 0.1509 & 0.3784 \\ 0.6979 & 0.8600 \end{bmatrix}$	...	$\begin{bmatrix} 0.1708 & 0.4398 \\ 0.9943 & 0.3400 \end{bmatrix}$

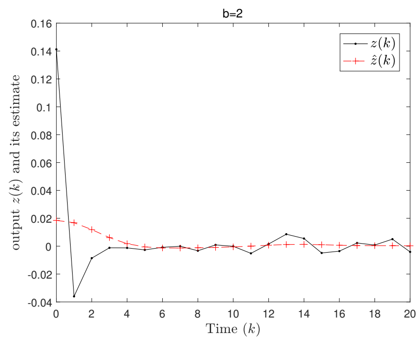
Tab. 1. Estimator gain lists at different times when $b = 1$.

t_k	0	1	$\cdots$	20
G_1	$\begin{bmatrix} 0.4199 & 0.7939 \\ 0.7537 & 0.9200 \end{bmatrix}$	$\begin{bmatrix} 0.6555 & 0.6273 \\ 0.3919 & 0.6991 \end{bmatrix}$	$\cdots$	$\begin{bmatrix} 0.1221 & 0.7218 \\ 0.7627 & 0.6516 \end{bmatrix}$
G_2	$\begin{bmatrix} 0.8447 & 0.6208 \\ 0.3678 & 0.7131 \end{bmatrix}$	$\begin{bmatrix} 0.3972 & 0.6552 \\ 0.4136 & 0.8376 \end{bmatrix}$	$\cdots$	$\begin{bmatrix} 0.7540 & 0.8835 \\ 0.6632 & 0.2722 \end{bmatrix}$
H_1	$\begin{bmatrix} 0.1939 & 0.5682 \\ 0.9048 & 0.6318 \end{bmatrix}$	$\begin{bmatrix} 0.3716 & 0.5947 \\ 0.4253 & 0.5657 \end{bmatrix}$	$\cdots$	$\begin{bmatrix} 0.4194 & 0.0356 \\ 0.2130 & 0.0812 \end{bmatrix}$
H_2	$\begin{bmatrix} 0.2344 & 0.9316 \\ 0.5488 & 0.3352 \end{bmatrix}$	$\begin{bmatrix} 0.7156 & 0.7764 \\ 0.5113 & 0.4983 \end{bmatrix}$	$\cdots$	$\begin{bmatrix} 0.8506 & 0.4662 \\ 0.3402 & 0.9138 \end{bmatrix}$

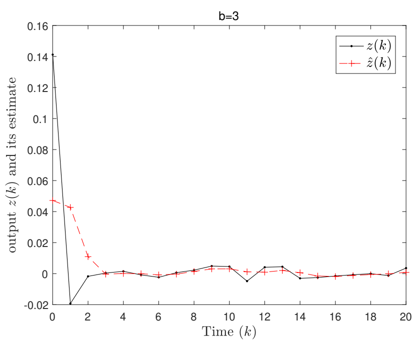
Tab. 2. Estimator gain lists at different times when $b = 2$.



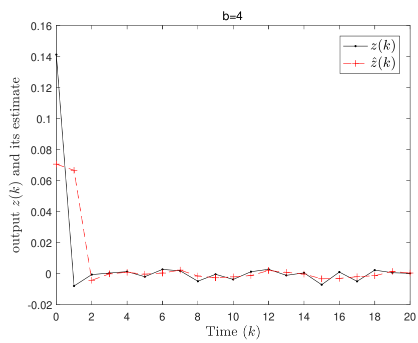
(a) $z(t_k)$ and $\hat{z}(t_k)$ when $b = 1$



(b) $z(t_k)$ and $\hat{z}(t_k)$ when $b = 2$



(c) $z(t_k)$ and $\hat{z}(t_k)$ when $b = 3$



(d) $z(t_k)$ and $\hat{z}(t_k)$ when $b = 4$

Fig. 1. $z(t_k)$ and $\hat{z}(t_k)$ when $b = 1, 2, 3, 4$.

t_k	0	1	…	20
G_1	$\begin{bmatrix} 0.2760 & 0.0129 \\ 0.3685 & 0.8892 \end{bmatrix}$	$\begin{bmatrix} 0.6081 & 0.0020 \\ 0.1760 & 0.7902 \end{bmatrix}$	…	$\begin{bmatrix} 0.9249 & 0.8783 \\ 0.6295 & 0.6417 \end{bmatrix}$
G_2	$\begin{bmatrix} 0.8660 & 0.5695 \\ 0.2542 & 0.1593 \end{bmatrix}$	$\begin{bmatrix} 0.5136 & 0.1034 \\ 0.2132 & 0.1573 \end{bmatrix}$	…	$\begin{bmatrix} 0.7984 & 0.9811 \\ 0.4350 & 0.0960 \end{bmatrix}$
G_3	$\begin{bmatrix} 0.5944 & 0.6586 \\ 0.3311 & 0.8636 \end{bmatrix}$	$\begin{bmatrix} 0.4075 & 0.0527 \\ 0.4078 & 0.9418 \end{bmatrix}$	…	$\begin{bmatrix} 0.5275 & 0.2843 \\ 0.5456 & 0.3708 \end{bmatrix}$
H_1	$\begin{bmatrix} 0.5676 & 0.7918 \\ 0.9805 & 0.1526 \end{bmatrix}$	$\begin{bmatrix} 0.1500 & 0.3111 \\ 0.3844 & 0.1685 \end{bmatrix}$	…	$\begin{bmatrix} 0.0647 & 0.8364 \\ 0.5448 & 0.1453 \end{bmatrix}$
H_2	$\begin{bmatrix} 0.8330 & 0.6390 \\ 0.1919 & 0.6680 \end{bmatrix}$	$\begin{bmatrix} 0.8966 & 0.7340 \\ 0.3227 & 0.4109 \end{bmatrix}$	…	$\begin{bmatrix} 0.1715 & 0.8240 \\ 0.0680 & 0.1340 \end{bmatrix}$
H_3	$\begin{bmatrix} 0.7721 & 0.4416 \\ 0.3798 & 0.4831 \end{bmatrix}$	$\begin{bmatrix} 0.3998 & 0.1693 \\ 0.5055 & 0.5247 \end{bmatrix}$	…	$\begin{bmatrix} 0.8848 & 0.9636 \\ 0.5147 & 0.1205 \end{bmatrix}$

Tab. 3. Estimator gain lists at different times when $b = 3$.

t_k	0	1	…	20
G_1	$\begin{bmatrix} 0.8437 & 0.7000 \\ 0.8815 & 0.7557 \end{bmatrix}$	$\begin{bmatrix} 0.7067 & 0.8137 \\ 0.1684 & 0.4662 \end{bmatrix}$	…	$\begin{bmatrix} 0.8626 & 0.3659 \\ 0.0309 & 0.0938 \end{bmatrix}$
G_2	$\begin{bmatrix} 0.9745 & 0.1313 \\ 0.4022 & 0.7247 \end{bmatrix}$	$\begin{bmatrix} 0.7223 & 0.3625 \\ 0.9949 & 0.7308 \end{bmatrix}$	…	$\begin{bmatrix} 0.3346 & 0.8155 \\ 0.0073 & 0.0250 \end{bmatrix}$
$G3$	$\begin{bmatrix} 0.8995 & 0.0430 \\ 0.1707 & 0.4792 \end{bmatrix}$	$\begin{bmatrix} 0.6497 & 0.0076 \\ 0.6813 & 0.6541 \end{bmatrix}$	…	$\begin{bmatrix} 0.4242 & 0.6767 \\ 0.0338 & 0.3280 \end{bmatrix}$
G_4	$\begin{bmatrix} 0.0939 & 0.9523 \\ 0.6500 & 0.4577 \end{bmatrix}$	$\begin{bmatrix} 0.9452 & 0.7829 \\ 0.6133 & 0.0032 \end{bmatrix}$	…	$\begin{bmatrix} 0.5601 & 0.1501 \\ 0.4248 & 0.0644 \end{bmatrix}$
H_1	$\begin{bmatrix} 0.5369 & 0.4939 \\ 0.0665 & 0.4175 \end{bmatrix}$	$\begin{bmatrix} 0.7970 & 0.1785 \\ 0.6418 & 0.5294 \end{bmatrix}$	…	$\begin{bmatrix} 0.6164 & 0.1766 \\ 0.6290 & 0.4079 \end{bmatrix}$
H_2	$\begin{bmatrix} 0.2923 & 0.7538 \\ 0.2897 & 0.0968 \end{bmatrix}$	$\begin{bmatrix} 0.2187 & 0.0582 \\ 0.5481 & 0.5876 \end{bmatrix}$	…	$\begin{bmatrix} 0.7061 & 0.2109 \\ 0.4298 & 0.5910 \end{bmatrix}$
H_3	$\begin{bmatrix} 0.0769 & 0.7649 \\ 0.7209 & 0.6579 \end{bmatrix}$	$\begin{bmatrix} 0.4161 & 0.0639 \\ 0.1864 & 0.0748 \end{bmatrix}$	…	$\begin{bmatrix} 0.8372 & 0.8856 \\ 0.8883 & 0.9629 \end{bmatrix}$
H_4	$\begin{bmatrix} 0.8104 & 0.3062 \\ 0.3742 & 0.3707 \end{bmatrix}$	$\begin{bmatrix} 0.3100 & 0.9807 \\ 0.9441 & 0.5551 \end{bmatrix}$	…	$\begin{bmatrix} 0.7696 & 0.0681 \\ 0.3679 & 0.1089 \end{bmatrix}$

Tab. 4. Estimator gain lists at different times when $b = 4$.

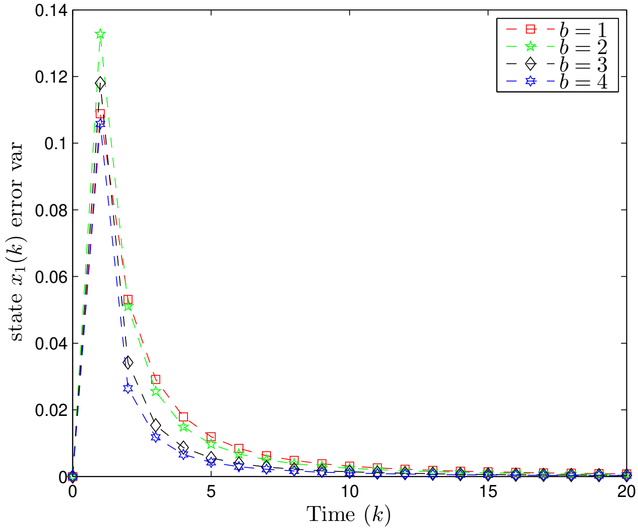


Fig. 2. Variance of $x_1(t_k)$ and $\hat{x}_1(t_k)$ errors at different update frequencies.

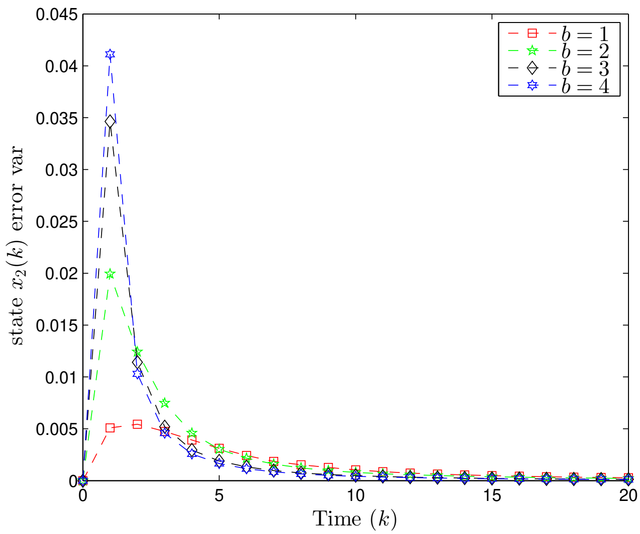


Fig. 3. Variance of $x_2(t_k)$ and $\hat{x}_2(t_k)$ errors at different update frequencies.

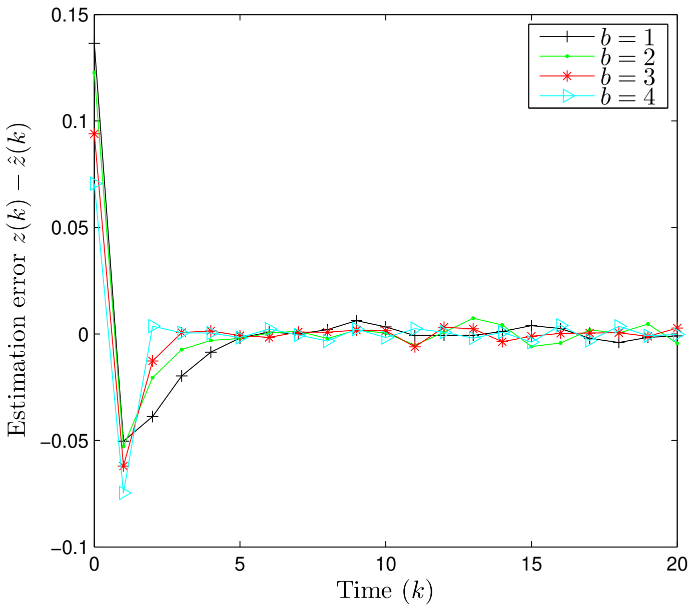


Fig. 4. The error between $z(t_k)$ and $\hat{z}_(t_k)$ at different update frequencies.

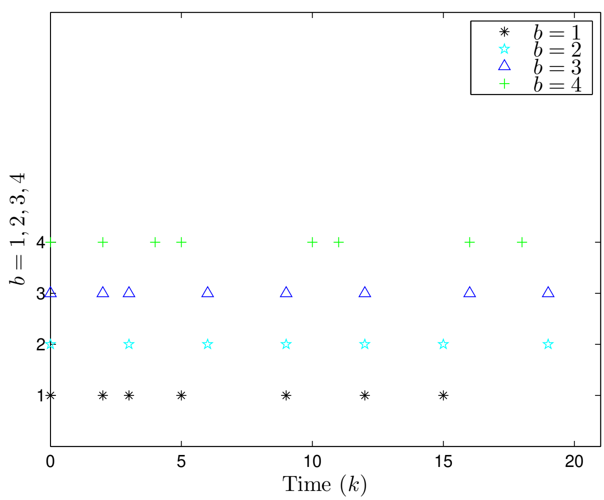


Fig. 5. The event-trigger instants when $b = 1, 2, 3, 4$.

5. CONCLUSION

In this paper, the finite time domain H_∞ filtering problem of a class of multi-rate time-varying networked systems with random sensor saturation is studied. Firstly, a sector bounded model of random saturation was given by means of random analysis technique. Then, after considering the two performance indexes of multi-rate system, namely H_∞ performance and variance constraint, a sufficient condition was given to satisfy the above requirements, and the solution of the filter satisfying the above indexes was obtained by means of LMI. Finally, the validity and effectiveness of the proposed filter scheme were verified by numerical simulation. Next, we will study the filtering problem of multi-rate system under random protocol and RR protocol.

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