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CONSENSUS SEEKING OF DELAYED HIGH-ORDER MULTI-AGENT SYSTEMS WITH PREDICTOR-BASED ALGORITHM

Cheng-Lin Liu and Fei Liu

This paper investigates the high-order consensus problem for the multi-agent systems with agent's dynamics described by high-order integrator, and adopts a general consensus algorithm composed of the states' coordination control. Under communication delay, consensus algorithm in usual asynchronously-coupled form just can make the agents achieve a stationary consensus, and sufficient consensus condition is obtained based on frequency-domain analysis. Besides, a predictor-based consensus algorithm is constructed via multiplying the delayed neighboring agents' states by a delay-related compensation part. In our proposed algorithm, a compensating delay is introduced to match the communication delay. Specially, the original high-order consensus is regained when the compensating delay equals to the communication delay, but cannot be achieved if the compensating delay is not equivalent to the communication delay. Moreover, sufficient consensus convergence conditions are also obtained for the agents under our predictor-based algorithm with different compensating delay. Numerical studies for multiple quadrotors illustrate the correctness of our results.

Keywords: high-order multi-agent system, consensus, communication delay, predictor-based consensus algorithm, multiple quadrotors

Classification: 93A14, 93C85

1. INTRODUCTION

Collective behavior caused by the distributed coordination control mechanism of multiple autonomous agents has stimulated more and more researchers' interests in various fields, e. g., biology, physics, and engineering, etc [26]. As the simplest cooperative collective behavior, consensus means that the outputs of several autonomous agents reach a common value, and has attracted more and more attentions in recent years for its broad applications in clock synchronization, sensor network, formation control of unmanned systems, flocking and swarming, etc. So far, different consensus algorithms have been proposed for multi-agent systems with agents' dynamics modeled by single integrators [19], double integrators [9], high-order integrators [5, 15, 20, 22, 23, 34, 40] and fractional-order dynamics [35] respectively, and consensus criteria have been obtained for the agents reaching asymptotic consensuses under fixed and switching interconnection topologies.

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Information transmission in a multi-agent network brings non-negligible communication delay between neighboring agents, and consensus algorithms subjected to communication delay are usually divided into synchronously-coupled and asynchronously-coupled forms. In synchronously-coupled algorithm, self-delays introduced for each agent in the coordination part equal the corresponding communication delays, while asynchronously-coupled algorithm requires each agent to use its delayed state with the delay different from the corresponding communication delay, or use its current state to compare with its delayed neighboring agents' states. Consensus convergence of synchronously-coupled algorithm depends on the communication delay strictly for the multi-agent systems under fixed [19, 29, 33, 37] or switched topologies [4, 6, 17, 24, 30, 32, 38]. With proper control parameters, differently, the stationary consensus algorithms in asynchronously-coupled form is convergent without any relationship to the communication delay value for the first-order, second-order and high-order agents [10, 11, 12, 21, 27]. However, dynamical consensus algorithm in asynchronously-coupled form just drives second-order or high-order dynamic agents to reach the stationary consensus asymptotically [2, 14, 18, 25, 31], and the consensus convergence is strictly dependent on the communication delay.

For the second-order multi-agent systems with dynamical consensus algorithm, some compensation-based consensus algorithms, which are in the asynchronously-coupled form accompanied with delayed state compensations, have been designed to regain the original dynamical consensus [13, 16, 39]. Compared with usual synchronously-coupled algorithm, interestingly, the compensation-based consensus algorithms in asynchronously-coupled form tolerate higher communication delay [13]. Nevertheless, how to retrieve the original dynamical consensus state of high-order multi-agent system with asynchronously-coupled consensus algorithm has attracted little attention. For the high-order heterogeneous multi-agent systems, Tian and Zhang [25] modified the usual asynchronously-coupled consensus algorithm by introducing self-delays to match the different communication delays, and necessary and sufficient consensus condition implied that high-order consensus convergence did not require the self-delay of each agent to equal the corresponding communication delay.

In this paper, a general consensus algorithm, which just consists of the state coordination control parts, is proposed to solve the high-order consensus problem of multi-agent systems composed of high-order integrators. With communication delay, we adopt the usual asynchronously-coupled consensus algorithm and construct a predictor-based form via multiplying the neighbor's delayed state by a compensating delay-related matrix. Based on frequency-domain analysis, final consensus behaviors and consensus convergence conditions are analyzed for usual asynchronously-coupled consensus algorithm and our predictor-based consensus algorithm respectively. Under the predictor-based algorithm with the compensating delay different from the communication delay, the agents just achieve an asymptotic stationary consensus. Furthermore, our proposed algorithm can drive the agents to reach the original high-order dynamical consensus asymptotically if the compensating delay equals the communication delay.

2. PROBLEM DESCRIPTION

2.1. High-order agents and interconnection topology

Investigate the high-order dynamic agents given by

$$\begin{aligned}\dot{\xi}_{i0}(t) &= \xi_{i1}(t),\\ \dot{\xi}_{i1}(t) &= \xi_{i2}(t),\\ &\vdots\\ \dot{\xi}_{i(l-1)}(t) &= u_i(t),\ i = 1,2,\ldots,n,\end{aligned} \tag{1}$$

where $\xi_i = [\xi_{i0}, \xi_{i1}, \ldots, \xi_{i(l-1)}]^T \in R^l$ and $u_i \in R$ are the state and input of the agent i respectively. The system (1) can be expressed in a multi-variable form as

$$\dot{\xi}_i = A\xi_i + bu_i,\ i = 1,2,\ldots,n, \tag{2}$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0\\ 0 & 0 & 1 & \cdots & 0\\ \vdots & \cdots & & \ddots & 0\\ 0 & 0 & \cdots & 0 & 1\\ 0 & 0 & & \cdots & 0 \end{bmatrix},\quad b = \begin{bmatrix} 0\\ 0\\ \vdots\\ 0\\ 1 \end{bmatrix}.$$

Usually, a weighted digraph can describe the interconnection structure of the multi-agent network (2), in which agents correspond to the nodes and information flow corresponds to a directed edge. A digraph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ consists of a set of vertices $\mathcal{V} = \{1, \ldots, n\}$, a set of edges $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ and a weighted adjacency matrix $\mathcal{A} = [a_{ij}] \in R^{n\times n}$ with $a_{ij} \geq 0$. A finite index set $\mathcal{I} = \{1, 2, \ldots, n\}$ represents the node indexes. A directed edge from i to j in $\mathcal{G}$ is denoted by $e_{ij} = (i, j) \in \mathcal{E}$. Assume $a_{ij} > 0 \Leftrightarrow e_{ij} \in \mathcal{E}$ and $a_{ii} = 0$ for all $i \in \mathcal{I}$. $N_i = \{j \in \mathcal{V} : (i, j) \in \mathcal{E}\}$ expresses the set of node i's neighbors. The degree matrix is defined as $\mathcal{D} = \text{diag}\{\sum_{j=1}^{n} a_{ij}, i = 1, 2, \ldots, n\}$, and the Laplacian matrix is defined as $\mathcal{L} = \mathcal{D} - \mathcal{A}$.

In $\mathcal{G}$, the node j is said to be *reachable* from the node i if there exists a directed path from i to j. Otherwise, j is not reachable from i. A globally reachable node is defined as the node which is reachable from every other node in the digraph.

Assumption 2.1. The interconnection topology of the multi-agent systems (2) has a globally reachable node, and each node has at least one neighboring node.

Under Assumption 2.1, 0 is a simple eigenvalue of $\mathcal{L}$, i. e., $\text{rank}(\mathcal{L}) = n - 1$, and the other eigenvalues all have positive real parts [8]. In this paper, the eigenvalues of $\mathcal{D}^{-1}\mathcal{L}$ are denoted as $\lambda_i, i = 1, \ldots, n$, and we assume $\lambda_1 = 0$. According to Greshgorin disk theorem, $\text{Re}(\lambda_i) > 0$ and $|\lambda_i - 1| \leq 1, i = 2, \ldots, n$ hold from the definitions of $\mathcal{D}$ and $\mathcal{L}$.

2.2. Dynamical consensus algorithm

For high-order multi-agent systems (2), we are going to focus on the following consensus algorithm without stabilization control part [22, 34]

$$u_i = \frac{1}{d_i} K \sum_{j \in N_i} a_{ij}(\xi_j - \xi_i), \; i \in \mathcal{I}, \tag{3}$$

where $K = [\kappa_0, \kappa_1, \dots, \kappa_{l-1}]$ with $\kappa_m > 0, m = 0, 1, \dots, l-1$, N_i is the set of agent i's neighbors, $a_{ij} > 0$, $j \in N_i$ is the coupling weight corresponding to the adjacency element of $\mathcal{A}$ in the digraph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$, and $d_i = \sum_{j \in N_i} a_{ij}$. It is obvious that the dynamical algorithm (3) is reasonable under Assumption 2.1.

With the algorithm (3), the agents (2) can converge to a l**th-order consensus** defined as

$$\lim_{t \to \infty} (\xi_{im}(t) - \xi_{jm}(t)) = 0, \quad i, j \in \mathcal{I}, \quad m = 0, 1, \dots, l-1,$$

and

$$\lim_{t \to \infty} \xi_{im}(t) \neq 0, \quad i \in \mathcal{I}, \quad m = 0, 1, \dots, l-1.$$

Subjected to non-negligible communication delay, the algorithm (3) becomes

$$u_i(t) = \frac{1}{d_i} K \sum_{j \in N_i} a_{ij}(\xi_j(t-\tau) - \xi_i(t)), \quad i \in \mathcal{I}, \tag{4}$$

where $\tau > 0$ is the communication delay.

Remark 2.2. Based on current literature [14, 18, 25, 31], the asynchronously-coupled form (4) change the original lth-order consensus behavior of the high-order agents (2) without communication delay, and may brings rigorous collective behaviors including stationary consensus seeking, periodic synchronous oscillation, etc.

Inspired by the compensation-based algorithms in [13, 16, 39], we modify the consensus algorithm (4) into a predictor-based form as follows

$$u_i(t) = \frac{1}{d_i} K \sum_{j \in N_i} a_{ij}(\mathrm{e}^{A\tau_0}\xi_j(t-\tau) - \xi_i(t)), i \in \mathcal{I}, \tag{5}$$

where the constant $\tau_0 > 0$ named compensating delay matches the communication delay τ, and the predictive factor $\mathrm{e}^{A\tau_0}$ is

$$\mathrm{e}^{A\tau_0} = \begin{bmatrix} 1 & \tau_0 & \frac{1}{2}\tau_0^2 & \cdots & \frac{1}{(l-1)!}\tau_0^{l-1} \\ 0 & 1 & \tau_0 & \cdots & \frac{1}{(l-2)!}\tau_0^{l-2} \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 & \tau_0 \\ 0 & 0 & \cdots & 0 & 1 \end{bmatrix}.$$

Remark 2.3. For the general linear time-invariant system with input delay, predictor feedback controllers have been extensively used since the system's closed-loop response is idealized as if there were no input delay [1], and the related results can be extended to the linear multi-agent systems with input delay. Unlike the usual predictor feedback controller, we simply introduce a predictor-based factor into the general asynchronously-coupled consensus algorithm to obtain better delay robustness than the synchronously-coupled consensus algorithm in this paper.

3. MAIN RESULTS

Firstly, we consider the consensus algorithm (3) without communication delay, and the closed-loop form of the agents (2) driven by (3) is

$$\dot{\xi}_i = A\xi_i + \frac{1}{d_i} bK \sum_{j \in N_i} a_{ij}(\xi_j - \xi_i), i \in \mathcal{I}. \tag{6}$$

The following consensus criterion is gained from some existing works [22, 34].

Theorem 3.1. (Ren et al. [22], Yu et al. [34]) Under Assumption 2.1, the multi-agent system (6) achieves a lth-order consensus if and only if $A - \lambda_i bK, \forall i \in \{2, \ldots, n\}$ is a Hurwitz matrix.

Now, we take the closed-loop form of the agents (2) and our proposed algorithm (5) into account.

$$\dot{\xi}_i(t) = A\xi_i(t) + \frac{1}{d_i} bK \sum_{j \in N_i} a_{ij}(\mathrm{e}^{A\tau_0}\xi_j(t-\tau) - \xi_i(t)), i \in \mathcal{I}. \tag{7}$$

It is apparent that the system (7) with $\tau_0 = 0$ corresponds to the agents (2) with the usual asynchronously-coupled consensus algorithm (4).

By taking the Laplace transforms of the above system, we get the characteristic equation of the system (7) about $\xi^0(t) = [\xi_{10}(t), \xi_{20}(t), \ldots, \xi_{n0}(t)]^T$ as

$$\det(s^l I - K\mathrm{e}^{A\tau_0}\Gamma(s)\mathcal{D}^{-1}\mathcal{A}\mathrm{e}^{-s\tau} + K\Gamma(s)I) = 0, \tag{8}$$

where $\Gamma(s) = [1, s, \ldots, s^{l-2}, s^{l-1}]^T$. The equation (8) is rewritten as

$$\det(s^l I + K\Gamma(s)\mathcal{D}^{-1}\mathcal{L} + (K\Gamma(s) - K\mathrm{e}^{A\tau_0}\Gamma(s)\mathrm{e}^{-s\tau})\mathcal{D}^{-1}\mathcal{A}) = 0,$$

which equals

$$\prod_{i=1}^{n}(s^l + \lambda_i K\Gamma(s) + (1-\lambda_i)(K\Gamma(s) - K\mathrm{e}^{A\tau_0}\Gamma(s)\mathrm{e}^{-s\tau})) = 0.$$

We investigate the roots of the following equation

$$s^l + \lambda_i K\Gamma(s) + (1-\lambda_i)(K\Gamma(s) - K\mathrm{e}^{A\tau_0}\Gamma(s)\mathrm{e}^{-s\tau}) = 0 \tag{9}$$

with $i = 1, 2, \ldots, n$.

From the consensus analysis in [13], the final consensus behavior of agents (7) is determined by the equation (9) with $\lambda_1 = 0$, while the equation (9) with $\lambda_i = 2, \ldots, n$ determines whether the agents (7) could achieve an asymptotic consensus.

3.1. Final collective behavior under predictor-based algorithm

When $\lambda_1 = 0$, the equation (9) becomes

$$s^l + K\Gamma(s) - K\mathrm{e}^{A\tau_0}\Gamma(s)\mathrm{e}^{-s\tau} = 0. \tag{10}$$

By computing, we obtain

$$\mathrm{e}^{A\tau_0}\Gamma(s) = \begin{bmatrix} 1 + \tau_0 s + \cdots + \frac{\tau_0^{l-1}}{(l-1)!}s^{l-1} \\ s + \tau_0 s^2 + \cdots + \frac{\tau_0^{l-2}}{(l-2)!}s^{l-1} \\ \vdots \\ s^{l-2} + \tau_0 s^{l-1} \\ s^{l-1} \end{bmatrix}, \tag{11}$$

so the equation (10) is rewritten as

$$s^l + K(s) \begin{bmatrix} \mathrm{e}^{s\tau} - (1 + \tau_0 s + \cdots + \frac{\tau_0^{l-1}}{(l-1)!}s^{l-1}) \\ \mathrm{e}^{s\tau} - (1 + \tau_0 s + \cdots + \frac{\tau_0^{l-2}}{(l-2)!}s^{l-2}) \\ \vdots \\ \mathrm{e}^{s\tau} - (1 + \tau_0 s) \\ \mathrm{e}^{s\tau} - 1 \end{bmatrix} \mathrm{e}^{-s\tau} = 0, \tag{12}$$

where $K(s) = [\kappa_0, \kappa_1 s, \cdots, \kappa_{l-2}s^{l-2}, \kappa_{l-1}s^{l-1}]$.

Based on the Taylor series of $\mathrm{e}^{s\tau}$ at $s = 0$, the equation (12) can be reformulated as follows.

$$s(s^{l-1}+K(s) \begin{bmatrix} (\tau - \tau_0) + \cdots + \frac{\tau^{l-1}-\tau_0^{l-1}}{(l-1)!}s^{l-2} + \frac{1}{l!}\tau^l s^{l-1} + \frac{1}{(l+1)!}\tau^{l+1}s^l + \cdots \\ (\tau - \tau_0) + \cdots + \frac{\tau^{l-2}-\tau_0^{l-2}}{(l-2)!}s^{l-3} + \frac{1}{(l-1)!}\tau^{l-1}s^{l-2} + \frac{1}{l!}\tau^l s^{l-1} + \cdots \\ \vdots \\ (\tau - \tau_0) + \frac{1}{2!}\tau^2 s + \frac{1}{3!}\tau^3 s^2 + \cdots \\ \tau + \frac{1}{2!}\tau^2 s + \cdots \end{bmatrix} \mathrm{e}^{-s\tau}) = 0, \tag{13}$$

Remark 3.2. Obviously, the equation (13) has only one root at $s = 0$ when $\tau_0 \neq \tau$. When $\tau_0 = \tau$, the equation (13) becomes

$$s^l(1 + K \begin{bmatrix} \frac{1}{l!}\tau^l + \frac{1}{(l+1)!}\tau^{l+1}s + \cdots \\ \frac{1}{(l-1)!}\tau^{l-1} + \frac{1}{l!}\tau^l s + \cdots \\ \vdots \\ \frac{1}{2!}\tau^2 + \frac{1}{3!}\tau^3 s + \cdots \\ \tau + \frac{1}{2!}\tau^2 s + \cdots \end{bmatrix} \mathrm{e}^{-s\tau}) = 0. \tag{14}$$

Evidently, the above equation (14) has only l roots at $s = 0$ with $\tau > 0$. It should be pointed out that we just consider the case $\tau_0 \leq \tau$ in our paper.

Now, we present sufficient conditions, which are relatively conservative but easy to calculate proper delay bounds numerically, for checking the final consensus behavior.

Proposition 3.3. With $\tau_0 = 0$ and $\kappa_m > 0, m = 0, 1, \ldots, l-1$, the roots of (10) all lie on the open left half complex plane except for one root at $s = 0$, i. e., the agents (7) just can converge to a stationary consensus asymptotically if possible, if $\frac{(1-\sigma-\mathrm{e}^{-\mathrm{j}\omega\tau})n(\mathrm{j}\omega)}{(\mathrm{j}\omega)^l+\sigma n(\mathrm{j}\omega)}$ does not enclose the point $(-1, \mathrm{j}0)$ with $\omega \in R$ except for $\omega = 0$, where $n(s) = K\Gamma(s) = \kappa_0 + \kappa_1 s + \cdots + \kappa_{l-1}s^{l-1}$, and $s^l + \sigma n(s)$ is Hurwitz for some $\sigma \in C$.

Proof. The equation (10) with $\tau_0 = 0$ is rewritten as

$$s^l + n(s)(1 - \mathrm{e}^{-s\tau}) = 0, \tag{15}$$

which equals

$$1 + \frac{(1 - \sigma - \mathrm{e}^{-s\tau})n(s)}{s^l + \sigma n(s)} = 0.$$

According to the above analysis, the equation (15) has one root at $s = 0$, so Proposition 3.3 holds in the light of the Nyquist stability criterion. □

In Proposition 3.3, the parameter σ is chosen arbitrarily to guarantee that $s^l + \sigma n(s)$ is Hurwitz. In addition, the analysis method for Proposition 3.3 can be applied into the equation (10) with $0 < \tau_0 \neq \tau$ and $\tau_0 = \tau$.

Proposition 3.4. The roots of (10) with $0 < \tau_0 \neq \tau$ and $\kappa_m > 0,\ m = 0, 1, \ldots, l-1$ all lie on the open left half complex plane except for one root at $s = 0$, i. e., the agents (7) just can converge to a stationary consensus asymptotically if possible, if $\frac{(1-\sigma)n(\mathrm{j}\omega)-(n(\mathrm{j}\omega)+\Delta n(s))\mathrm{e}^{-\mathrm{j}\omega\tau}}{(\mathrm{j}\omega)^l+\sigma n(\mathrm{j}\omega)}$ does not enclose the point $(-1, \mathrm{j}0)$ with $\omega \in R$ except for $\omega = 0$, where $s^l + \sigma n(s)$ is Hurwitz for some $\sigma \in C$, and $\Delta n(s) = \kappa_0\tau_0 s + (\frac{1}{2}\kappa_0\tau_0^2 + \kappa_1\tau_0)s^2 + \cdots + (\frac{1}{(l-1)!}\kappa_0\tau_0^{l-1} + \frac{1}{(l-2)!}\kappa_1\tau_0^{l-2} + \cdots + \kappa_{l-2}\tau_0)s^{l-1}$.

Proof. From (11), the equation (10) with $0 < \tau_0 \neq \tau$ is reformulated as

$$s^l + n(s) - (n(s) + \Delta n(s))\mathrm{e}^{-s\tau} = 0,$$

which equals

$$1 + \frac{(1 - \sigma)n(s) - (n(s) + \Delta n(s))\mathrm{e}^{-s\tau}}{s^l + \sigma n(s)} = 0.$$

Proposition 3.4 holds obviously. □

Proposition 3.5. The roots of (10) with $\tau_0 = \tau$ and $\kappa_m > 0, m = 0, 1, \ldots, l-1$ all lie on the open left half complex plane except for l roots at $s = 0$, i. e., the agents (7) just can achieve an asymptotic lth-order consensus if possible, if $\frac{(1-\sigma)n(\mathrm{j}\omega)-(n(\mathrm{j}\omega)+\Delta\hat{n}(\mathrm{j}\omega))\mathrm{e}^{-\mathrm{j}\omega\tau}}{(\mathrm{j}\omega)^l+\sigma n(\mathrm{j}\omega)}$ does not enclose the point $(-1, \mathrm{j}0)$ with $\omega \in R$ except for $\omega = 0$, where $s^l + \sigma n(s)$ is Hurwitz for some $\sigma \in C$, and $\Delta\hat{n}(s) = \kappa_0\tau s + (\frac{1}{2}\kappa_0\tau^2 + \kappa_1\tau)s^2 + \cdots + (\frac{1}{(l-1)!}\kappa_0\tau^{l-1} + \frac{1}{(l-2)!}\kappa_1\tau^{l-2} + \cdots + \kappa_{l-2}\tau)s^{l-1}$.

Proposition 3.5 is easily proved analogous to Proposition 3.4.

Remark 3.6. To regain the original high-order consensus behavior under our predictor-based algorithm, the compensating delay τ_0 must equal the communication delay, so the concrete value of communication delay should be known. When the delay value is unknown, hence, we will get the estimation of the communication delay by designing an adaptive delay estimator in our future work.

3.2. Consensus convergence criteria

In this section, we will mainly obtain the consensus criteria for the multi-agent systems (7) by analyzing the roots of the equation (9) for $i = 2, \dots, n$.

The equation (9) is rewritten as

$$s^l + \lambda_i n(s) + (1 - \lambda_i)(n(s) - (n(s) + \Delta n(s))\mathrm{e}^{-s\tau}) = 0, i = 2, \dots, n. \tag{16}$$

To continue the consensus analysis, we make the following assumption.

Assumption 3.7. Under Assumption 2.1, the sufficient and necessary consensus conditions in Theorem 3.1 hold.

Theorem 3.8. With $\tau_0 = 0$, Proposition 3.3 and Assumption 3.7 hold. Define

$$M_i(s) = s\frac{(1 - \lambda_i)n(s)}{s^l + \lambda_i n(s)}, i = 2, \dots, n. \tag{17}$$

The agents (7) achieve an asymptotic stationary consensus, if

$$\tau |M_i(\mathrm{j}\omega)| < 1, i = 2, \dots, n \tag{18}$$

holds for $\omega \in R$.

Proof. Since Assumption 3.7 establishes, we obtain that the zeros of $s^l + \lambda_i n(s)$, $i = 2, \dots, n$ all lie on the open left half complex plane. With $\tau_0 = 0$, the equation (16) equals

$$1 + M_i(s)\frac{(1 - \mathrm{e}^{-s\tau})}{s} = 0, i = 2, \dots, n. \tag{19}$$

Obviously, $\frac{(1-\mathrm{e}^{-s\tau})}{s}$ and $M_i(s)$ both have no poles in the open right half complex plane. For $\max_{\omega \in [0,\infty)} |\frac{\mathrm{e}^{-\mathrm{j}\omega\tau}-1}{\mathrm{j}\omega}| < \tau$, we achieve that

$$\begin{aligned} |M_i(\mathrm{j}\omega)\frac{(1 - \mathrm{e}^{-\mathrm{j}\omega\tau})}{\mathrm{j}\omega}| &= |M_i(\mathrm{j}\omega)||\frac{(1 - \mathrm{e}^{-\mathrm{j}\omega\tau})}{\mathrm{j}\omega}| \\ &< \tau |M_i(\mathrm{j}\omega)| \\ &< 1 \end{aligned}$$

holds for $\omega \in R$ based on (18). Thus, the Nyquist curve $M_i(\mathrm{j}\omega)\frac{(1-\mathrm{e}^{-\mathrm{j}\omega\tau})}{\mathrm{j}\omega}$ does not enclose the point $(-1, \mathrm{j}0)$, i. e., the roots of (19) all lie on the open left half complex plane. From Proposition 3.3, thus, the agents in the system (7) converge to a stationary consensus asymptotically. Theorem 3.8 is proved. □

Theorem 3.9. With $0 < \tau_0 \neq \tau$, Proposition 3.4 and Assumption 3.7 hold. Let

$$\bar{M}_i(s) = \frac{(1-\lambda_i)s^l}{s^l + \lambda_i n(s)},\ i = 2, \dots, n, \tag{20}$$

and the agents (7) converge to a stationary consensus asymptotically, if

$$\left|\frac{n(\mathrm{j}\omega) - (n(\mathrm{j}\omega) + \Delta n(\mathrm{j}\omega))\mathrm{e}^{-\mathrm{j}\omega\tau}}{(\mathrm{j}\omega)^l}\right| < \frac{1}{|\bar{M}_i(\mathrm{j}\omega)|},\ i = 2, \dots, n \tag{21}$$

holds for $\omega \in R$.

Proof. Analogous to the proof in Theorem 3.8, Assumption 3.7 ensures that the polynomial $s^l + \lambda_i n(s)$ has all its zeros on the open left half complex plane. Moreover, Proposition 3.4 guarantees that the agents (7) can only converge to a stationary consensus asymptotically.

Next, reformulate the equation (16) as

$$1 + \bar{M}_i(s)\frac{n(s) - (n(s) + \Delta n(s))\mathrm{e}^{-s\tau}}{s^l} = 0, i = 2, \dots, n. \tag{22}$$

If the condition (21) holds, the roots of (22) all lie on the open left half complex plane. Therefore, the agents in the system (7) converge to a stationary consensus asymptotically. Theorem 3.9 is proved. □

In addition, the results in Theorem 3.9 can be extended to the case $\tau_0 = \tau$ directly.

Theorem 3.10. With $\tau_0 = \tau$, Proposition 3.5 and Assumption 3.7 hold. The agents (7) achieve a lth-order consensus asymptotically, if

$$\left|\frac{n(\mathrm{j}\omega) - (n(\mathrm{j}\omega) + \Delta \hat{n}(\mathrm{j}\omega))\mathrm{e}^{-\mathrm{j}\omega\tau}}{(\mathrm{j}\omega)^l}\right| < \frac{1}{|\bar{M}_i(\mathrm{j}\omega)|},\ i = 2, \dots, n \tag{23}$$

holds for $\omega \in R$, where $\bar{M}_i(s)$ is defined in (20).

Remark 3.11. Intuitively, introducing compensating delay reduces the communication delay robustness from the conditions (21) and (23) in Theorem 3.9 and Theorem 3.10 respectively, i. e., the predictor-based consensus algorithm (5) tolerate smaller communication delay than the usual asynchronously-coupled algorithm (4), and we will demonstrate this property in the following numerical studies.

Remark 3.12. In this paper, we just consider the predictor-based consensus algorithm in continuous-time form (5), so we will make some further investigation on our proposed algorithm with sampled date in the light of the consensus analysis of second-order multi-agent systems with sampled data [36, 7].

4. NUMERICAL SIMULATION FOR QUADROTORS

In this section, we take the quadrotor (see Figure 1) as the investigation object, and extend the above theoretical results to the consensus problem of multiple quadrotors with communication delay.

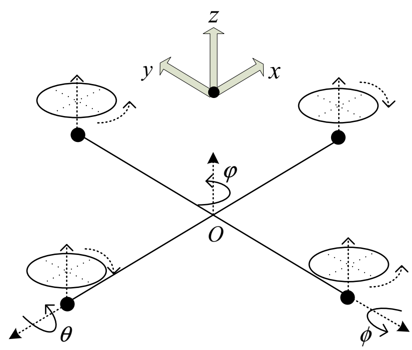

Fig. 1. Quadrotor aircraft.

The quadrotor's dynamics are given by [3, 28]

$$\begin{aligned}
\ddot{x}_i &= u_{i1}(\cos\phi_i \sin\theta_i \cos\psi_i + \sin\phi_i \sin\psi_i), \\
\ddot{y}_i &= u_{i1}(\cos\phi_i \sin\theta_i \sin\psi_i - \sin\phi_i \cos\psi_i), \\
\ddot{z}_i &= u_{i1}(\cos\phi_i \cos\theta_i) - g, \\
\ddot{\theta}_i &= u_{i2}, \quad \ddot{\phi} = u_{i3}, \quad \ddot{\psi}_i = u_{i4},
\end{aligned} \tag{24}$$

where x_i, y_i and z_i denote the position of the quadrotor center of gravity in the earth-frame, θ_i, ϕ_i and ψ_i are the pitch, roll and yaw angles, and g is the gravity constant. u_{i1} is the linear acceleration applied to the quadrotor in the z direction, u_{i2}, u_{i3} and u_{i4} are respectively the angular accelerations for θ_i, ϕ_i and ψ_i respectively.

The quadrotor model (24) can be simplified as follows [3]. To decouple the z_i and $x_i - y_i$ axes, one can assume that in hovering condition, $u_{i1} \approx g$ in the x_i and y_i directions. Furthermore, by assuming small angles θ_i and ϕ_i and a constant yaw angle ψ_i (for instance $\psi_i = 0$), (24) can be rewritten as:

$$\dot{\xi}_{ix} = A\xi_{ix} + b\hat{u}_{i2}, \tag{25}$$

$$\dot{\xi}_{iy} = A\xi_{iy} + b\hat{u}_{i3}, \tag{26}$$

in which

$$\begin{aligned}
\xi_{ix} &= [x_i, v_{ix}, \hat{\theta}_i, \omega_{i\theta}]^T, \ \hat{\theta}_i = g\theta_i, \ \hat{u}_{i2} = gu_{i2}, \\
\xi_{iy} &= [y_i, v_{iy}, \hat{\phi}_i, \omega_{i\phi}]^T, \ \hat{\phi}_i = -g\phi_i, \ \hat{u}_{i3} = -gu_{i3}, \\
A &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \ b = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}.
\end{aligned}$$

Thus, $\hat{u}_{i2}$ and $\hat{u}_{i3}$ are designed as the consensus algorithms (3), (4) and (5) respectively, and we get $u_{i2} = \frac{1}{g}\hat{u}_{i2}$ and $u_{i3} = -\frac{1}{g}\hat{u}_{i3}$ from the definitions. Since the coordinates of x and y have the same dynamics, we just investigate the consensus control of the coordinate x in this section and the consensus problem of y can be studied in the same way.

Now, investigate a multi-agent network with six quadrotors given by (25) and the interconnection topology is shown in Figure 2. Obviously, each agent has at least one neighboring agent and the set of globally reachable nodes is {1,2,3,4,5,6}.

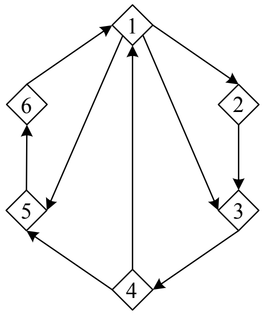

Fig. 2. Interconnection topology of six agents.

The values of coupling weights determine the eigenvalues of the Laplacian matrix, and affect the consensus convergence rate according to Theorem 3.1 [22, 34]. In this paper, we mainly study the delay effect on the consensus convergence, and donot analyze how the different coupling weights and control parameters affect the consensus performance. Then, we choose a set of positive weights as: $a_{12} = 0.7, a_{13} = 0.3, a_{15} = 0.5, a_{23} = 0.3, a_{34} = 0.6, a_{41} = 0.4, a_{45} = 1.0, a_{56} = 0.2, a_{61} = 0.8$, and we get the eigenvalues of $\mathcal{D}^{-1}\mathcal{L}$ as $\lambda_1 = 0, \lambda_2 = 1.7238, \lambda_3 = 1.4506 + \mathrm{j}0.7659, \lambda_4 = 1.4506 - \mathrm{j}0.7659, \lambda_5 = 0.6853 + \mathrm{j}0.6933, \lambda_6 = 0.6853 - \mathrm{j}0.6933$. For simplicity, the control parameter is chosen as $K = \kappa * [1, 4, 6, 4]^T$, in which κ guarantees that the condition in Theorem 3.1 hold.

By choosing $\kappa = 3$, i. e., $K = 3 * [1, 4, 6, 4]^T$, the high-order agents converge to the 4rd-consensus asymptotically (see Figure 3).

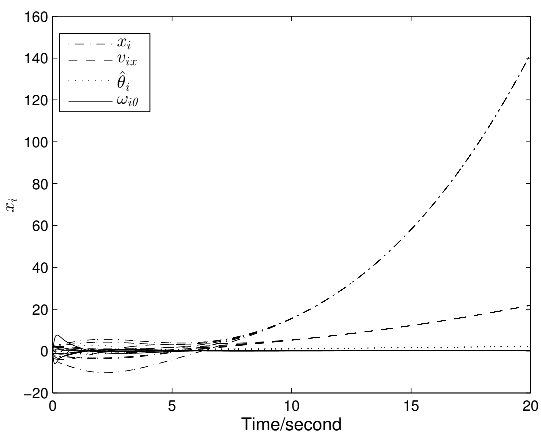

Fig. 3. Consensus convergence without communication delay.

Under non-negligible communication delay, we get $\tau < 0.0684$(s) from Proposition 3.3 and the condition (18) in Theorem 3.8 with $K = 3 * [1, 4, 6, 4]^T$. Then, the agents (25) with asynchronously-coupled consensus algorithm (4) converge to a stationary consensus (see Figure 4). By simulation, the largest communication delay that the system can tolerate is $\tau_{\max} = 0.96$(s).

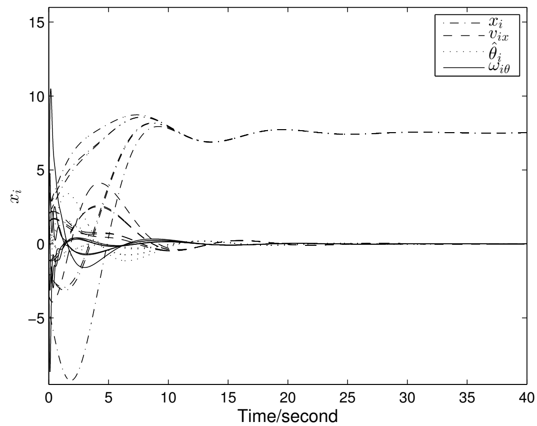

Fig. 4. Asynchronously-coupled consensus convergence with communication delay.

Investigating the agents (25) under the predictor-based consensus algorithm (5) with $0 < \tau_0 \neq \tau$, we choose $\tau_0 = 0.05$(s) and obtain $\tau < 0.0675$(s) from Proposition 3.4 and the condition (21) in Theorem 3.9 with $K = 3 * [1, 4, 6, 4]^T$, i. e., the agents (25) converge to a stationary consensus asymptotically (see Figure 5). To show the effect of introducing the delay-dependent compensation part, we consider the relationship between the communication delay and the compensating delay. With the control gain $K = 3 * [1, 4, 6, 4]$, the largest communication delay τ that the agents (25) with (5) can tolerate decreases as the compensating delay increases by numerical simulation (see Figure 6), i. e., there is a trade-off between the communication delay and the compensating delay.

For the agents (25) with the predictor-based consensus algorithm (5) with $\tau_0 = \tau$, the bound of the communication delay is $\tau \in [0, 0.066]$(s) from Proposition 3.5 and the condition (23) in Theorem 3.10, i. e., the agents (25) with (5) can reach the original 4rd-order consensus asymptotically (see Figure 7). To illustrate the effectiveness of our proposed algorithm compared with that of synchronously-coupled consensus algorithm, let $K = \kappa * [1, 4, 6, 4]$, and Figure 8 shows the largest communication delay that the two algorithms can tolerate with different κ. As expected, predictor-based consensus algorithm bear larger communication delay than synchronously-coupled consensus algorithm.

It should be pointed out that the numerical simulation results above are just for the linearized model of quadrotors (23), but our future research will focus on the practical experiments of the coordination control of multiple quadrotors.

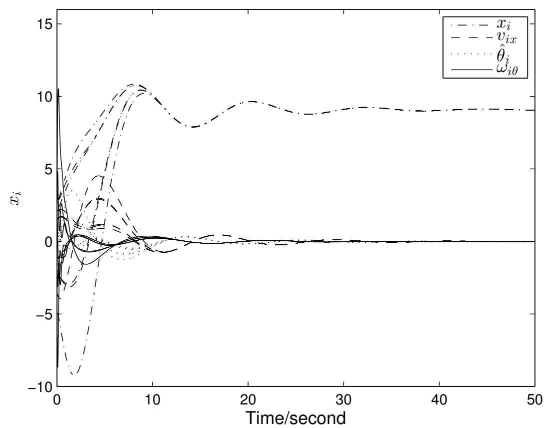

Fig. 5. Stationary consensus convergence of predictor-based algorithm with $0 < \tau_0 \neq \tau$.

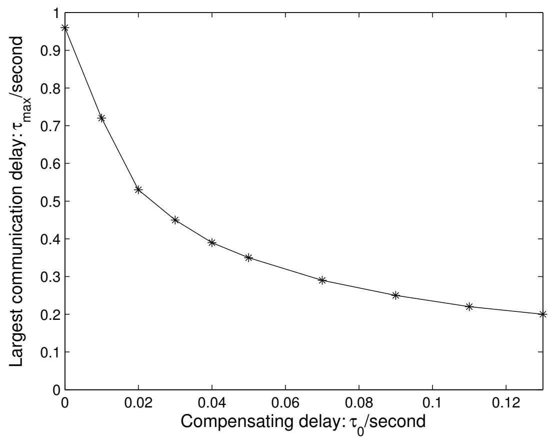

Fig. 6. Trade-off between communication delay τ and compensating delay τ_0.

5. CONCLUSION

For the multi-agent systems composed of high-order integrators, we adopt the normal dynamical consensus algorithm, which just consists of the state coordination control parts without any state stabilization part, to deal with the high-order consensus problem. Subjected to non-negligible communication delay, usual asynchronously-coupled consensus algorithm is adopted, and a predictor-based consensus algorithm is constructed via multiplying the neighbor's delayed state by a compensating delay-related matrix. By using frequency-domain analysis, the high-order agents, which are driven by usual

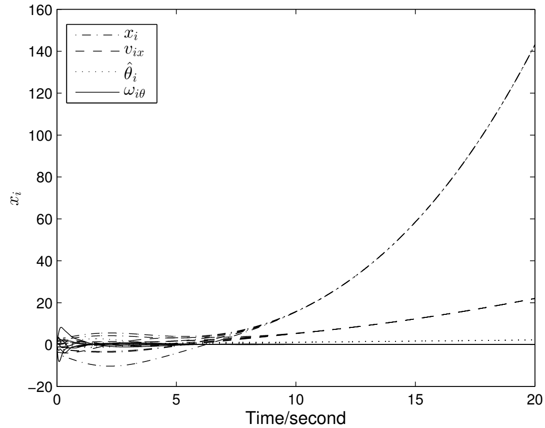

Fig. 7. 4th-order consensus convergence of predictor-based algorithm with $\tau_0 = \tau$.

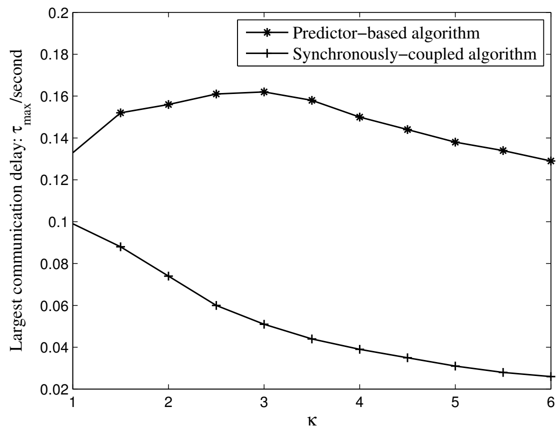

Fig. 8. Delay bound with different κ.

asynchronously-coupled algorithm or the predictor-based algorithm with compensating delay distinct from the communication delay, cannot achieve the original high-order consensus behavior, but reach a stationary consensus asymptotically if possible. Fortunately, the predictor-based algorithm with the compensating delay equalling the communication delay can drive the high-order agents to regain the high-order consensus. In addition, consensus convergence conditions are obtained for usual asynchronously-coupled consensus algorithm and our predictor-based consensus algorithm respectively. When the compensating delay is not equivalent to the communication delay, naturally, there exists a trade-off between the compensating delay and the largest communication delay that the agents can bear. Importantly, the predictor-based consensus algorithm in asynchronously-coupled form can bear larger communication delay than the synchronously-coupled consensus algorithm.

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