Kybernetika

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Kybernetika, Vol. 54 (2018), No. 1, 96–109

Persistent URL: http://dml.cz/dmlcz/147153

KYBERNETIKA — VOLUME *54* (2018), NUMBER 1, PAGES 96–109

CONTROLLING THE STOCHASTIC SENSITIVITY IN THERMOCHEMICAL SYSTEMS UNDER INCOMPLETE INFORMATION

Irina Bashkirtseva

Complex dynamic regimes connected with the noise-induced mixed-mode oscillations in the thermochemical model of flow reactor are studied. It is revealed that the underlying reason of such excitability is in the high stochastic sensitivity of the equilibrium. The problem of stabilization of the excitable equilibrium regimes is investigated. We develop the control approach using feedback regulators which reduce the stochastic sensitivity and keep the randomly forced system near the stable equilibrium. We consider also a case when the information about system state is incomplete. Our new mathematical technique is applied to the stabilization of operating modes in the flow chemical reactors forced by random disturbances.

Keywords: stabilization, stochastic sensitivity, flow reactor, incomplete information

Classification: 60H10, 93E20

1. INTRODUCTION

The variety and complexity of the dynamic regimes in real processes is related to their nonlinearity and stochasticity. Interplay of these factors can cause unexpected phenomena which have no analogues in the initial deterministic models, such as noise-induced transitions [11], stochastic resonance [8, 2], noise-induced complexity [21, 23], stochastic excitability [15], etc. These complex phenomena are observed in various models of neural activity, lasers, population dynamics, and electronics. Recently, mathematical models of flow thermochemical reactors with a wide variety of nonlinear dynamic regimes attracted attention of researchers [12, 13].

Traditionally, a deterministic stability of the equilibrium mode is considered as a condition of the proper operation of the chemical reactor. However, such stability can be insufficient, especially in nonlinear stochastic systems. In excitable systems, even weak noise can destroy a stable operating mode, and cause unacceptable stochastic oscillations. In the study of the probabilistic mechanisms of the noise-induced phenomena, the stochastic sensitivity analysis [5, 16] can be a useful tool. It was shown that the noise-induced large-amplitude oscillations and stochastic transformations are a consequence of the high stochastic sensitivity of initial deterministic attractors. To ensure a proper

DOI: 10.14736/kyb-2018-1-0096

operating mode in such excitable systems, it is important to develop adequate control procedures.

Control problems of nonlinear stochastic systems attract attention of many researchers (see, for instance, [3, 9, 10, 14, 18, 20, 22] and bibliography therein). A theory of the control based on the stochastic sensitivity synthesis was developed in [4, 6].

In control procedures, feedback regulators are frequently used. These regulators form a control input as a function of the system states. However, in practice, the available information on the current system state usually is not complete. So, for nonlinear stochastic systems, a development of the control theory with incomplete information is a subject of a high importance [17]. A problem of the synthesis of the stochastic sensitivity in the case of incomplete information when the observations of the system states contain random errors was studied in [7]. In the present paper, we consider a problem of the synthesis of the stochastic sensitivity in the case of incomplete information when only some coordinates of the system state are observable.

This problem is solved in Section 2 for the general randomly forced system. A design of the feedback static regulator providing the required stochastic sensitivity of the equilibrium is reduced to the solution of the corresponding matrix equation. Here, an attainability of the required sensitivity is discussed, and explicit formulas for the regulator parameters are derived.

The aim of the current paper is to show how this new mathematical technique can be applied to the solution of the important engineering problem of the stabilization of operating modes in the flow chemical reactors forced by random disturbances. Here, we consider a conceptual dynamic model of the flow reactor proposed by Volter and Salnikov [19].

In Section 3, an influence of the random noise on this model is studied in different mono- and bistability parametric zones. In the bistability zone where the system exhibits a coexistence of the equilibrium and limit cycle, noise generates an intermittency of small- and large-amplitude oscillations. A stochastic excitability in a monostability zone of stable equilibria is demonstrated. It is shown that the reason of this excitability is in the high level of the stochastic sensitivity of the equilibrium.

In Section 4, for the stabilization of the equilibrium mode in stochastic flow reactor, a general mathematical approach from Section 2 is applied. It is shown how to synthesize the appropriate regulator which reduces the stochastic sensitivity and suppresses unaccepted large-amplitude stochastic oscillations in the randomly forced flow reactor. Cases of complete and incomplete information about system states are covered.

2. CONTROLLING THE STOCHASTIC SENSITIVITY

Consider a general stochastic system with control

$$\dot{x} = f(x, u) + \varepsilon\sigma(x)\xi(t), \tag{1}$$

where x is an n-dimensional state, u is an l-dimensional control, $f(x, u)$ is an n-vector-function, $\xi(t)$ is an q-dimensional standard Gaussian process with parameters $\mathrm{E}\xi(t) = 0$, $\mathrm{E}\xi(t)\xi^\top(\tau) = \delta(t-\tau)I$ (I is the identity matrix), and ε is the noise intensity. The $n\times q$-matrix-function $\sigma(x)$ characterizes a dependence of random disturbances on the system state.

It is supposed that the unforced and uncontrolled system (1) (with $\varepsilon = 0$, $u = 0$) has an equilibrium $\bar{x}$. The stability of $\bar{x}$ is undetermined.

For the synthesis of the control input, one needs some information about states x of system (1). In the ideal case, we know all states exactly. But in practice, the available information on the current state $x(t)$ is usually incomplete.

In the present paper, we consider a case that the measurement m-vector $y(t)$ is connected with the state $x(t)$ by the following relation:

$$y(t) = g(x(t)).$$

In this case, the following static regulator of the fixed structure will be considered:

$$u = K(y - \bar{y}), \qquad \bar{y} = g(\bar{x}). \tag{2}$$

Here, K is a constant $l \times m$-matrix.

Denote by $\mathbf{K}$ a set of matrices K which provide an exponential stability of the equilibrium $\bar{x}$ for the corresponding closed-loop deterministic system

$$\dot{x} = f(x, K(g(x) - g(\bar{x}))) \tag{3}$$

in some neighbourhood of $\bar{x}$.

For deviations $v(t) = x(t) - \bar{x}$ of states $x(t)$ of system (3) from the equilibrium $\bar{x}$, consider the following first approximation system:

$$\dot{v} = (F + BKC)v, \qquad F = \frac{\partial f}{\partial x}(\bar{x}, 0), \quad B = \frac{\partial f}{\partial u}(\bar{x}, 0), \quad C = \frac{\partial g}{\partial x}(\bar{x}). \tag{4}$$

So, the set $\mathbf{K}$ is defined as

$$\mathbf{K} = \{K |\ Re\lambda_i(F + BKC) < 0\}.$$

Here, $\lambda_i(F + BKC)$ are the eigenvalues of the matrix $F + BKC$. It is supposed that the set $\mathbf{K}$ is not empty.

Consider now the corresponding closed-loop stochastic system

$$\dot{x} = f(x, K(g(x) - g(\bar{x}))) + \varepsilon\sigma(x)\xi(t). \tag{5}$$

The vector $z(t) = \left.\frac{\partial x^\varepsilon(t)}{\partial \varepsilon}\right|_{\varepsilon=0}$ of the sensitivity of the solution $x^\varepsilon(t)$ of the stochastic system (5) satisfies the following system

$$\dot{z} = (F + BKC)z + G\xi(t), \quad G = \sigma(\bar{x}). \tag{6}$$

For the covariance matrix $V(t) = \mathrm{cov}(z(t), z(t))$, one can write the equation

$$\dot{V} = (F + BKC)V + V(F + BKC)^\top + S, \ \ S = GG^\top. \tag{7}$$

For any $K \in \mathbf{K}$, this equation has a unique stable stationary solution W satisfying the following algebraic equation

$$(F + BKC)W + W(F + BKC)^\top + S = 0. \tag{8}$$

For non-singular noises ($\det S \neq 0$), the solution W of equation (8) is positive definite. The matrix W is called a *stochastic sensitivity matrix* of the equilibrium $\bar{x}$. For small noise, this matrix allows us to find the first approximation of the covariance matrix of the stationary distributed solutions $\bar{x}^\varepsilon(t)$ of system (5):

$$\mathrm{cov}(\bar{x}^\varepsilon(t), \bar{x}^\varepsilon(t)) \approx \varepsilon^2 W.$$

Thus, the control of the dispersion of random states around the equilibrium $\bar{x}$ can be reduced to a synthesis of the assigned stochastic sensitivity matrix W by the appropriate regulator (2).

For any $K \in \mathbf{K}$, the regulator (2) provides an exponential stability of the equilibrium $\bar{x}$ for the deterministic system (3), and forms the corresponding stochastic sensitivity matrix W_K of this equilibrium in the stochastic system (5).

Denote by $\mathbf{M}$ a set of symmetric and positive definite $n \times n$-matrices. For the assigned matrix $W \in \mathbf{M}$, it is necessary to find a matrix $K \in \mathbf{K}$ guaranteeing the equality $W_K = W$, where W_K is a solution of equation (8).

Note that not all matrices of the set $\mathbf{M}$ are attainable. The element $W \in \mathbf{M}$ is said to be attainable if the equality $W_K = W$ is true for some $K \in \mathbf{K}$. The attainability set

$$\mathbf{W} = \{W \in \mathbf{M} \mid \exists K \in \mathbf{K} \quad W_K = W\}$$

consists of all attainable elements. We say that the equilibrium $\bar{x}$ is completely stochastic controllable if

$$\forall\, W \in \mathbf{M} \quad \exists\, K \in \mathbf{K} \;:\quad W_K \equiv W.$$

So, the condition of the complete stochastic controllability of the equilibrium $\bar{x}$ can be written as $\mathbf{W} = \mathbf{M}$.

As a result, the problem of the synthesis of the assigned stochastic sensitivity matrix W is reduced to the search of the matrix K satisfying the matrix equation

$$BKCW + WC^\top K^\top B^\top + H(W) = 0, \qquad H(W) = FW + WF^\top + S. \tag{9}$$

Here, the following matrix analysis is needed. The equation (9) is equivalent to the equation

$$BKC = (Q - FW - 0.5S)W^{-1}, \tag{10}$$

where Q is an arbitrary skew-symmetric $n \times n$-matrix. Consider a case when matrices B and C are quadratic and non-singular. Then for any $W \in \mathbf{M}$, the solution of the equation (10) can be found as

$$K = B^{-1}\left[Q - FW - 0.5S\right]W^{-1}C^{-1}. \tag{11}$$

The fact that this matrix K is a solution of equation (9) can be verified by the direct substitution of (11) into (9).

So, the regulator (2) with the feedback matrix (11) can synthesize any assigned stochastic sensitivity matrix. This means that the equilibrium $\bar{x}$ is completely stochastic controllable.

In the general case, the equation (10) can be unsolvable. Based on the pseudo-inversion matrix technique [1], the following necessary and sufficient condition of the solvability of (10) can be derived:

$$(I - BB^+)\,[Q - FW - 0.5S]\,W^{-1} = 0, \quad [Q - FW - 0.5S]\,W^{-1}(I - C^+C) = 0. \quad (12)$$

Here, the sign "+" means a pseudo-inversion.

If the equalities (12) are true, then the equation (10) has a solution

$$K = B^+\,[Q - FW - 0.5S]\,W^{-1}C^+. \quad (13)$$

So, the equalities (12) define the attainability of the matrix W.

Further, we apply this theory to the stabilization of equilibrium mode of the stochastically forced thermochemical system modeling the operation of the flow reactor.

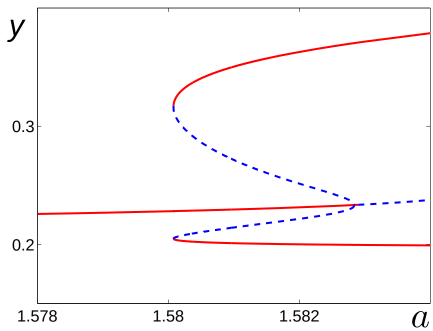

Fig. 1. Bifurcation diagram of the deterministic system (14): stable equilibria and cycles (solid lines), and unstable equilibria and cycles (dashed lines).

3. STOCHASTIC EXCITABILITY IN THE MODEL OF THE FLOW REACTOR

Consider a kinetic model of the thermochemical reaction in the flow reactor [19] of ideal mixing:

$$\begin{aligned} \dot{x} &= -x \exp\left(-\frac{1}{y}\right) + l(a - x), \\ \dot{y} &= x \exp\left(-\frac{1}{y}\right) + m(b - y). \end{aligned} \quad (14)$$

Here, the variable x is the concentration of the reagent, and y is a temperature inside the reactor. Parameters a and b stand for the concentration and temperature at the reactor inlet, and l, m are positive parameters.

Following [19] we fix $l = 0.5, m = 0.25, b = 0.165$. The system (14) has an equilibrium $M(\bar{x}(a), \bar{y}(a))$. Consider how this system dynamics depends on the parameter a

varying in the range $1.578 < a < 1.584$. In this parametric zone, one can observe three dynamic regimes. For $a < a_1 = 1.580079$, the system is monostable with a single stable equilibrium M; for $a_1 < a < a_2 = 1.582843$, the system is bistable with coexisting equilibrium and limit cycle; and for $a > a_2$, the system is monostable with a limit cycle as a single attractor. The critical point a_1 marks the saddle-node bifurcation, and a_2 corresponds to the subcritical Hopf bifurcation. In the bifurcation diagram (see Figure 1), y-coordinates of attractors and repellers of system (14) are presented.

To study an impact of noise on the thermochemical processes in the reactor, we consider the following stochastic model

$$\begin{aligned} \dot{x} &= -x \exp\left(-\frac{1}{y}\right) + l(a-x) + \varepsilon_1 \xi_1(t), \\ \dot{y} &= x \exp\left(-\frac{1}{y}\right) + m(b-y) + \varepsilon_2 \xi_2(t). \end{aligned} \tag{15}$$

Here, $\xi_{1,2}(t)$ are the standard Gaussian uncorrelated processes, and $\varepsilon_{1,2}$ are the noise intensities. In what follows, we put $\varepsilon_1 = \varepsilon_2 = \varepsilon$.

Consider how random forcing deforms deterministic dynamics of the system modelling the equilibrium operation mode of the thermochemical process under consideration.

Bistability zone

First, consider the stochastic dynamics of system (15) in the bistability zone $a_1 < a < a_2$ where the stable equilibrium M and limit cycle coexist. Their basins of attraction are separated by the unstable cycle. Let the random trajectory starts from the equilibrium M. For weak noise, random trajectories reside near the point M with small-amplitude stochastic oscillations (see, i. e. time series for $\varepsilon = 0.00003$ in Figure 2a, blue color).

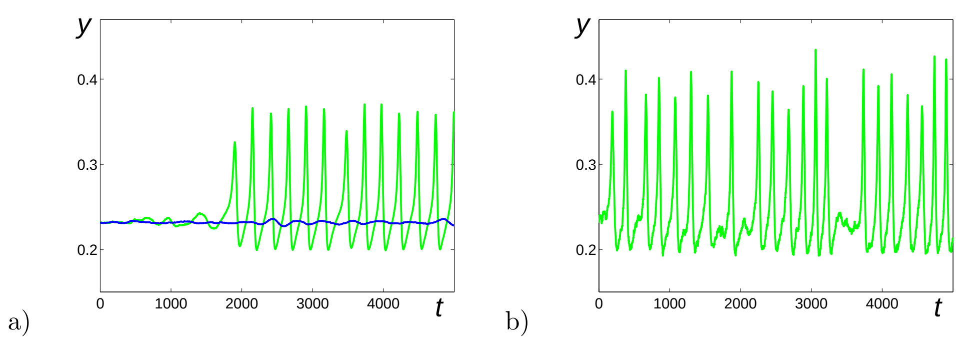

Fig. 2. Time series $y(t)$ of stochastic system (15) for $a = 1.582$:
a) with $\varepsilon = 0.00003$ (blue), $\varepsilon = 0.0001$ (green); b) with $\varepsilon = 0.001$.

When noise increases, the amplitude of stochastic oscillations increases too, and the trajectory can cross the separatrix (unstable cycle) and fall into the basin of attraction of the limit cycle and continue oscillations near this cycle (see time series for $\varepsilon = 0.0001$ in Figure 2a, green color). As one can see, the system exhibits a transition from small- to large-amplitude stochastic oscillations. Here, transitions from the basin of attraction of the cycle to equilibrium are also possible. As a result, mixed-mode stochastic oscillations are observed (see Figure 2b for $\varepsilon = 0.001$). Note that the frequency of such transitions grows with increasing noise.

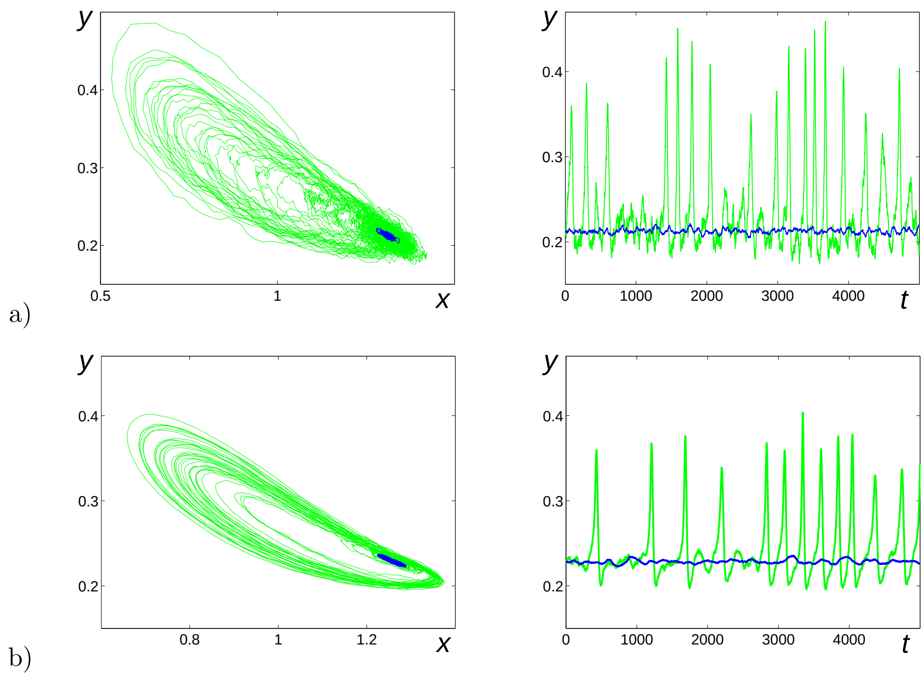

Fig. 3. Phase trajectories and time series $y(t)$ of stochastic system for a) $a = 1.55$ with $\varepsilon = 0.0005$ (blue), $\varepsilon = 0.003$ (green); b) $a = 1.58$ with $\varepsilon = 0.0001$ (blue), $\varepsilon = 0.0005$ (green).

Thus, in the bistability zone, noise generates a new more complicated regime of mixed-mode oscillations that can not be observed in the deterministic case. It is worth noting that such noise-induced complexity is not connected with the bistability only. In what follows we will consider the monostability zone where the stable equilibrium is a single attractor. Despite the apparent simplicity of this case, noise can here essentially complicate the system behaviour.

Excitability in the monostability zone

Consider now the stochastic dynamics of system (15) in the monostability zone $a < a_1$ where the deterministic system (14) has the single attractor in a form of the stable equilibrium M. Peculiarities of the stochastic dynamics are shown in Figure 3 for two values of the parameter a. Here, the random trajectories start from the equilibrium M.

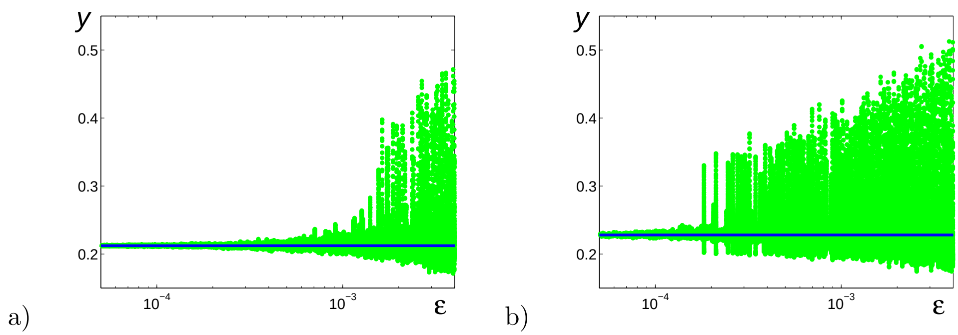

Fig. 4. Random states of system for a) $a = 1.55$, b) $a = 1.58$.

First fix the value $a = 1.55$. Forced by a weak noise, these trajectories are localised nearby the M, and exhibit small-amplitude stochastic oscillations (see phase trajectories and time series for $\varepsilon = 0.0005$ in Figure 3a, blue color). As noise increases, the amplitude of stochastic oscillations increases too, and the trajectory can falls into the zone of the phase plane where large amplitude scrolls are observed (see phase trajectories and time series for $\varepsilon = 0.003$ in Figure 3a, green color). As one can see, even in the monostability zone $a < a_1$, the stochastic system exhibits the intermittency of small- and large-amplitude stochastic oscillations. The similar scenario of the transition from unimodal small-amplitude stochastic oscillations ($\varepsilon = 0.0001$) to the the intermittency regime ($\varepsilon = 0.0005$) is shown in Figure 3b for $a = 1.58$.

Such transitions from the unimodal regime to the bimodal one are shown in detail in Figure 4 where random states versus noise intensity are plotted for $a = 1.55$ (a) and $a = 1.58$ (b). As one can see, stochastic phenomena, shown here, are observed for the very week noise. Moreover, the closer a to the bifurcation value a_2 the smaller critical value of the noise intensity corresponding to the onset of the generation of large-amplitude stochastic oscillations. The underlying reason of these effects lies in the high sensitivity of the equilibrium M to the random disturbances in the model under consideration.

The stochastic sensitivity of the equilibrium is characterised by the matrix W (see equation (8) with $K = 0$ in Section 2). For the equilibrium $M(\bar{x}, \bar{y})$ of system (15), this matrix is a solution of the equation

$$FW + WF^{\top} + S = 0, \tag{16}$$

where

$$W = \begin{bmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \end{bmatrix}, \quad F = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix}, \quad S = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$f_{11} = -\exp\left(-\frac{1}{\bar{y}}\right) - l, \quad f_{12} = -\bar{x}\exp\left(-\frac{1}{\bar{y}}\right)\frac{1}{\bar{y}^2},$$

$$f_{21} = \exp\left(-\frac{1}{\bar{y}}\right), \quad f_{22} = \bar{x}\exp\left(-\frac{1}{\bar{y}}\right)\frac{1}{\bar{y}^2} - m.$$

For $a = 1.55$, we have $w_{11} = 4.62 \cdot 10^2$, $w_{12} = w_{21} = -1.02 \cdot 10^2$, $w_2 = 3.4 \cdot 10^1$. For $a = 1.58$, we have $w_{11} = 1.1 \cdot 10^4$, $w_{12} = w_{21} = -2.26 \cdot 10^3$, $w_2 = 5.21 \cdot 10^2$. Plots of the functions $w_{11}(a)$ and $w_{22}(a)$ are shown in Figure 5.

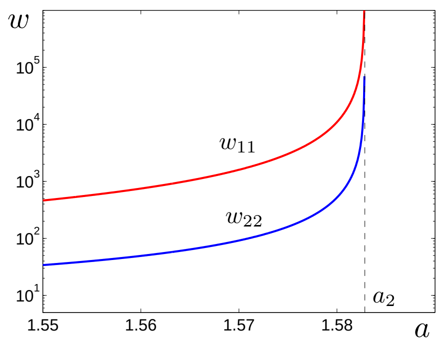

Fig. 5. Stochastic sensitivity of the equilibrium M.

As one can see, the stochastic sensitivity of the equilibrium unlimitedly grows as the parameter a tends to a_2.

In the next Section, we consider how in the system with regulator one can reduce a stochastic sensitivity of the equilibrium and stabilize the normal operating mode with acceptable small-amplitude oscillations around the equilibrium.

4. STABILIZATION OF THE STOCHASTIC FLOW REACTOR

Consider now the system (15) with additional control inputs:

$$\begin{aligned} \dot{x} &= -x\exp\left(-\frac{1}{y}\right) + l(a - x) + u_1 + \varepsilon_1\xi_1(t), \\ \dot{y} &= x\exp\left(-\frac{1}{y}\right) + m(b - y) + u_2 + \varepsilon_2\xi_2(t). \end{aligned} \tag{17}$$

In the case of complete information, when coordinates x, y of the system state are known for any time exactly, to stabilize an operating mode of the stochastically forced reactor, we will use the regulator

$$u_1 = k_{11}(x - \bar{x}) + k_{12}(y - \bar{y}), \qquad u_2 = k_{21}(x - \bar{x}) + k_{22}(y - \bar{y}). \tag{18}$$

In this case, from the formula (11), taking into account $B = C = S = I$, $Q = 0$, we have

$$K = -F - 0.5W^{-1}.$$

This formula gives the matrix $K = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix}$ of the regulator (18) which provides the assigned stochastic sensitivity matrix W.

Here, we restrict ourselves by the diagonal stochastic sensitivity matrices $W = \begin{bmatrix} w & 0 \\ 0 & w \end{bmatrix}$ with any assigned $w > 0$. In this case,

$$k_{11} = -f_{11} - \frac{1}{2w}, \quad k_{12} = -f_{12}, \quad k_{21} = -f_{21}, \quad k_{22} = -f_{22} - \frac{1}{2w}. \tag{19}$$

For the equilibrium M, the regulator (18) with parameters (19) provides the stability and constant stochastic sensitivity for any value of the parameter a. Results of the control based on the synthesis of the stochastic sensitivity $w = 10$ are shown in Figure 6. Note

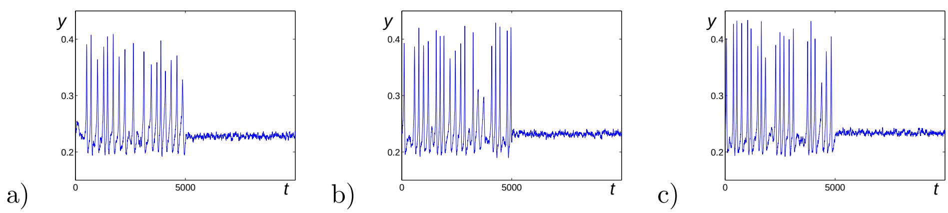

Fig. 6. Time series of the stochastic system with $\varepsilon = 0.001$: a) $a = 1.58$; b) $a = 1.582$; c) $a = 1.583$. The control is switched on at $t = 5000$, the regulator provides $w = 10$.

that a decrease of the control parameter w allows us to decrease a dispersion of these small-amplitude oscillations (compare time series in Figure 7 for $w = 0.1$ and $w = 10$).

Consider now the case of the incomplete information when the only coordinate x is observable. Here, we use the following regulator:

$$u_1 = k_1(x - \bar{x}), \qquad u_2 = k_2(x - \bar{x}). \tag{20}$$

In this case,

$$K = \begin{bmatrix} k_1 \\ k_2 \end{bmatrix}, \qquad C = [\,1 \;\; 0\,], \qquad F + BKC = \begin{bmatrix} f_{11} + k_1 & f_{12} \\ f_{21} + k_2 & f_{22} \end{bmatrix}.$$

The set $\mathbf{K}$ of the regulator (20) parameters which provide the exponential stability of the equilibrium M is defined by the system of linear inequalities:

$$\begin{aligned} \mathrm{tr}(F + BKC) &= f_{11} + f_{22} + k_1 < 0, \\ \det(F + BKC) &= f_{11}f_{22} - f_{12}f_{21} + f_{22}k_1 - f_{12}k_2 > 0. \end{aligned} \tag{21}$$

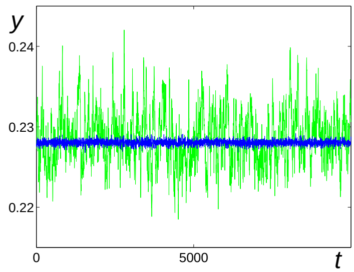

Fig. 7. Time series of the controlled stochastic system for $a = 1.58$, $\varepsilon = 0.001$ with the regulator providing $w = 0.1$ (blue) and $w = 10$ (green).

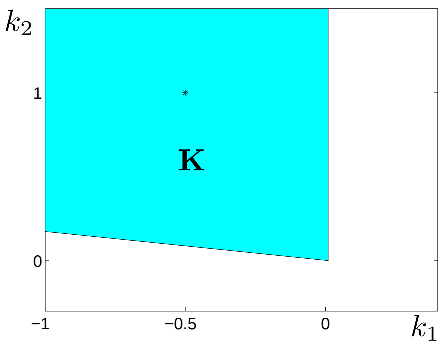

Fig. 8. The stability domain $\mathbf{K}$ for $a = 1.58$.

For $a = 1.58$, the stability domain $\mathbf{K}$ is shown in Figure 8.
For any $(k_1, k_2) \in \mathbf{K}$, elements $w_{ij}(k_1, k_2)$ of the stochastic sensitivity matrix $W(k_1, k_2)$ are found from the system (8). Further, the problem of the reducing the sensitivity can be solved by the standard descent procedures.

For example, for $k_1 = -0.5,\ k_2 = 1$ (see asterisk in Figure 8) we have

$$w_{11} = 1.317,\ w_{12} = -0.794,\ w_{22} = 5.735.$$

Remember, that for the system without control, we have

$$w_{11} = 1.1 \cdot 10^4,\ w_{12} = -2.26 \cdot 10^3,\ w_2 = 5.21 \cdot 10^2.$$

As one can see, the regulator (20) with $k_1 = -0.5,\ k_2 = 1$ essentially decreases the stochastic sensitivity. Results of the numerical simulation of the system (17) with regulator (20) are shown in Figure 9.

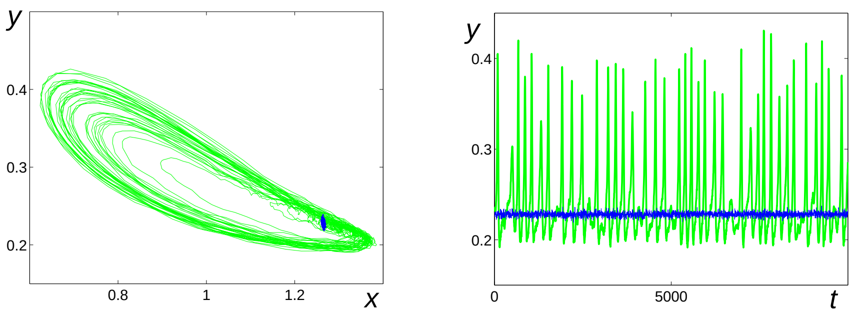

Fig. 9. Phase trajectories and time series $y(t)$ of stochastic system for $a = 1.58$, $\varepsilon = 0.001$ without control (green), and with regulator (20) with $k_1 = -0.5$, $k_2 = 1$ (blue).

It is worth noting that in the case of the incomplete information, the suggested control approach also allows us to stabilize the equilibrium regime of the stochastic flow reactor and suppress unwanted large-amplitude stochastic outbreaks.

CONCLUSION

Deterministic stability of the equilibrium is considered as a standard condition of the proper operation of the various technical processes. However, there is a class of so-called excitable systems in which such stability is insufficient, especially under the random disturbances. In excitable systems, even weak noise can destroy a stable operating mode, and cause unacceptable large-amplitude stochastic oscillations. In this paper, the mathematical details of the similar noise-induced excitability were considered on the example of the simple conceptual thermochemical model of flow reactor. To prevent the equilibrium regime of this reactor from unacceptable large-amplitude stochastic oscillations, a new control approach was suggested. A main point of this approach is to construct feedback regulators which reduce the stochastic sensitivity and keep the randomly forced system near the stable equilibrium. Mathematically, the problem of the synthesis of the assigned stochastic sensitivity for the equilibrium of the nonlinear randomly forced system by feedback regulator under incomplete information was reduced to the solution of the corresponding quadratic matrix equations for regulator's parameters. The analysis of solvability of such equations with the help of the pseudo-inversion technique was given. It was shown how this general mathematical approach can be constructively applied to the stabilization of the excitable regime in the stochastic flow reactor.

ACKNOWLEDGEMENT

This work was partially supported by RFBR (16-08-00388).

(Received May 7, 2017)

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