Kybernetika

Xue-Fang Wang; Zhenhua Deng; Song Ma; Xian Du

Event-triggered design for multi-agent optimal consensus of Euler-Lagrangian systems

Kybernetika, Vol. 53 (2017), No. 1, 179–194

Persistent URL: `http://dml.cz/dmlcz/146715`

KYBERNETIKA — VOLUME *53* (2017), NUMBER 1, PAGES 179–194

EVENT-TRIGGERED DESIGN FOR MULTI-AGENT OPTIMAL CONSENSUS OF EULER–LAGRANGIAN SYSTEMS

Xue-Fang Wang, Zhenhua Deng, Song Ma and Xian Du

In this paper, a distributed optimal consensus problem is investigated to achieve the optimization of the sum of local cost function for a group of agents in the Euler–Lagrangian (EL) system form. We consider that the local cost function of each agent is only known by itself and cannot be shared with others, which brings challenges in this distributed optimization problem. A novel gradient-based distributed continuous-time algorithm with the parameters of EL system is proposed, which takes the distributed event-triggered control mechanism into account. A sufficient condition is given to show that the performance of the global convergence to the optimal point can be guaranteed under the proposed method. Moreover, the Zeno behavior of triggering time can be excluded. Finally, to show the effectiveness of the presented algorithm, an example is given along with simulation results.

Keywords: optimal consensus, multi-agent system, Euler–Lagrangian system, event-triggered control

Classification: 34K35, 34H05, 49K35, 65K10, 90C25

1. INTRODUCTION

Distributed optimization has drawn much attention since it has wide application background. One of the standard optimization problems is optimal consensus, which means that all the agents solve a optimization problem in a consensus way. This problem has been studied and applied to many research areas such as smart grids [3] and sensor networks [4]. In this distributed optimization problem, each agent only knows its own local cost function, while the objective for the whole network is to optimize a global cost function in the form of the sum of all local cost functions cooperatively. So far, there have been many meaningful results in continuous-time cases. For example, [5, 6] developed gradient-based or subgradient-based algorithms to solve the constrained distributed optimization problem, while [7, 8] studied the distributed optimization problem with external disturbances. Moreover, [16] designed an algorithm for distributed optimization problem with discrete-time communication. Additionally, [2] provided various connectivity conditions to solve the distributed continuous-time convex intersection computation.

DOI: 10.14736/kyb-2017-1-0179

In general, it should be noted that physical systems are indispensable part for achieving distributed optimization task, such as the cooperative search of radio sources [20], the optimization design of multiple mechanical agents [21] and the distributed optimal power flow [3]. Besides, Euler–Lagrangian (EL) systems have been widely studied, which are very powerful in the modeling and design for many physical systems such as mechanical/robotic systems [18, 19]. Up to now, EL systems have been widely applied to accomplish many complicated tasks by their cooperation like the cooperative search problem [20], the distributed tracking problem [17], and the set aggregation problem [23]. Because many practical tasks can be viewed as a special case of optimal consensus, it is very meaningful to study the optimization problem together with agent dynamics in the form of EL systems. For example, [9] designed a distributed optimization algorithm for multiple EL systems with global convergence, while [21] developed distributed algorithms for such system with semi-global stability.

On the other hand, the cooperation between agents is the basis of completing a distributed optimization task, which results in frequent communication among agents. However, in some practical situations, these agents may have limited energy or capacity. There are some strategies to deal with these problems including random sleep scheme [10] and event-triggered scheme [13]. In fact, the event-triggered strategy provides suitable rules to reduce the number of the communication rate and/or the actuator updates effectively. Following the ideas, event-triggered schemes have been used in multi-agent systems to reduce the cost of the communication between agents [11, 26]. However, few studies were done to pay attention to the reduction of communication for event-triggered control of optimal consensus problem, though some results can be found in [12, 14, 15, 16], where different distributed event-triggered optimization algorithms were presented for different situations.

The objective of this paper is to design a distributed event-triggered optimization algorithm to solve optimal consensus problem of EL agents for the reduction of communication. The contributions of this paper are summarized as follows. Different from many distributed optimization results with the agents in the single integrator form (such as [5, 6, 12, 14, 16]), a distributed event-triggered algorithm is designed for EL agents. Also, the set aggregation problem discussed in [23] is a special case of the optimization problem in this paper. In fact, the proposed algorithm of this paper can guarantee global convergence to the exact optimal solution by extending the result given in [9], which did not adopt any event-triggered strategy. Furthermore, the result of this paper is better than that of [16], in which the algorithm only achieved the exponential convergence to a neighborhood of the optimization point. Additionally, the proposed event-triggered strategy can not only reduce the cost of communication but also avoid the Zeno behavior of triggering time.

This paper is organized as follows. Section 2 formulates the problem and gives the preliminaries. Section 3 proposes the distributed optimization design process and shows the convergence results, and Section 4 gives an example to illustrate the effectiveness of the proposed algorithms. Finally, Section 5 gathers the conclusions.

Notations: $\mathbb{R}$ and $\mathbb{N}$ stand for the set of real and natural numbers, respectively. $\mathbb{R}^n$ is n-dimension Euclidean space. $\otimes$ and $\|\cdot\|$ denote the Kronecker product and the standard Euclidean norm, respectively. A^T is the transpose of matrix A. x_i is the ith element of vector x, and $col(x_1, \ldots, x_n) = [x_1^T, \ldots, x_n^T]^T$. I_n is $n \times n$ identity matrix. $\mathbf{1}_n$ and $\mathbf{0}_n$ are the column vectors of n ones and zeros, respectively. $\lambda_{\max}(A)$ is the maximal eigenvalue of matrix A.

2. PRELIMINARIES AND FORMULATION

In this section, some basic concepts are introduced for convex analysis [24] and graph theory [25], and then the considered problem is formulated.

2.1. Convex analysis

The following knowledge about convex analysis can be found in [24]. A function $f(\cdot) : \mathbb{R}^m \to \mathbb{R}$ is said to be convex if

$$f(a\zeta_1 + (1-a)\zeta_2) \leq af(\zeta_1) + (1-a)f(\zeta_2),\ \forall\, \zeta_1, \zeta_2 \in \mathbb{R}^m,\ a \in [0,1].$$

A differentiable function f is convex over $\mathbb{R}^m$ if

$$f(\zeta_1) - f(\zeta_2) \geq \nabla f(\zeta_2)^T(\zeta_1 - \zeta_2),\ \forall\, \zeta_1, \zeta_2 \in \mathbb{R}^m, \tag{1}$$

and f is strictly convex over $\mathbb{R}^m$ if the above inequality is strict whenever $\zeta_1 \neq \zeta_2$, and f is ω-strongly convex ($\omega > 0$) over $\mathbb{R}^m$ if

$$(\nabla f(\zeta_1) - \nabla f(\zeta_2))^T(\zeta_1 - \zeta_2) \geq \omega\|\zeta_1 - \zeta_2\|^2,\ \forall\, \zeta_1, \zeta_2 \in \mathbb{R}^m. \tag{2}$$

A function $f : \mathbb{R}^m \to \mathbb{R}^m$ is Lipschitz with constant $\theta > 0$, or simply θ-Lipschitz, if

$$\|f(\zeta_1) - f(\zeta_2)\| \leq \theta\|\zeta_1 - \zeta_2\|,\ \forall\, \zeta_1, \zeta_2 \in \mathbb{R}^m.$$

2.2. Graph theory

An undirected graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$ consists of a finite vertex set $\mathcal{V} = \{1, 2, \ldots, n\}$ and an edge set $\mathcal{E}$. An edge $(i,j) \in \mathcal{E}$ denotes that vertices i, j can obtain each other's information, that is, i and j are neighbors. $N_i = \{j : (j,i) \in \mathcal{E}\}$ denotes the neighbors of vertex i. A path of length ι from vertex i_1 to vertex $i_{\iota+1}$ is a sequence of $\iota+1$ distinct vertices $i_1, \ldots, i_{\iota+1}$ such that $(i_q, i_{q+1}) \in \mathcal{E}$ for $q = 1, \cdots, \iota$, in which i_0 and i_ι are called the end nodes of the path. If there is a path between any two vertices of a graph $\mathcal{G}$, then the graph is connected.

$A = [a_{ij}] \in \mathbb{R}^{n\times n}$ is the weighted adjacency matrix of $\mathcal{G}$ with a_{ij} as the weighting of edge (i,j), where $a_{ii} = 0$, $a_{ij} > 0$ if $(j,i) \in \mathcal{E}$, otherwise, $a_{ij} = 0$. $D = diag\{d_1, \ldots, d_n\} \in \mathbb{R}^{n\times n}$ is the degree matrix of $\mathcal{G}$, where $d_i = \sum_{j=1}^n a_{ij}$ for $i = 1, \ldots, n$, and $L = D - A$ is the Laplacian matrix of $\mathcal{G}$. The eigenvalues of $\mathcal{L}$ are denoted by $\lambda_1, \ldots \lambda_n$ with $\lambda_i \leq \lambda_j$ for $i \leq j$. Besides, $\lambda_1 = 0$ is an eigenvalue with $\mathbf{1}_n$ as its corresponding eigenvector of L, and $\lambda_2 > 0$ if and only if the graph $\mathcal{G}$ is connected. More details of graph theory can be found in [25].

2.3. Problem formulation

Consider a network of n agents with an interaction topology described by a graph $\mathcal{G}$. Agent i is equipped with a local cost function $f_i : \mathbb{R}^m \to \mathbb{R}$, only known by itself, and its dynamics is described by the following EL equation:

$$M_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i + G_i(q_i) = u_i, \quad i = 1, 2, \dots, n, \tag{3}$$

where q_i, $\dot{q}_i \in \mathbb{R}^m$ denote the generalized position and velocity vectors, respectively; $M_i(q_i) \in \mathbb{R}^{m\times m}$ is the positive definite inertia matrix; $C_i(q_i, \dot{q}_i)\dot{q}_i \in \mathbb{R}^m$ is the Coriolis and centripetal forces vector; $G_i(q_i) \in \mathbb{R}^m$ is the gravity vector; and $u_i \in \mathbb{R}^m$ is the control force. It is known that an EL system satisfies the following properties [22]:

(i) $\dot{M}_i(q_i) - 2C_i(q_i, \dot{q}_i)$ is skew symmetric.

(ii) For any $x, y \in \mathbb{R}^m$, $M_i(q_i)x + C_i(q_i, \dot{q}_i)y + G_i(q_i) = \Omega_i(q_i, \dot{q}_i, x, y)\varphi_i$, where $\Omega_i(q_i, \dot{q}_i, x, y) \in \mathbb{R}^{m\times p}$ is a known regression matrix and $\varphi_i \in \mathbb{R}^p$ is a constant vector consisting of the uncertain parameters of system (3).

As for the EL multi-agent system, agent $i \in \mathcal{V}$ is presented with a local cost function $f_i : \mathbb{R}^m \to \mathbb{R}$, only known by agent i. The objective of the EL systems is to cooperatively solve the following distributed optimal consensus problem:

$$\min_{p\in\mathbb{R}^m} f(p), \quad f(p) = \sum_{i=1}^{n} f_i(p), \tag{4}$$

where $f(p)$ is called the global cost function of the network. To be strict, we give the following definition.

Definition 2.1. The distributed optimization problem is solved for the system (3), if, for any initial condition $q_i(0)$, $\dot{q}_i(0) \in \mathbb{R}^m$ with $i \in \mathcal{V}$, all the Euler–Lagrangian agents converge to the global optimal solution of problem (4), i. e.,

$$\lim_{t\to\infty} q_i(t) = p^*, \ \lim_{t\to\infty} \dot{q}_i(t) = \mathbf{0}_m, \ \ i \in \mathcal{V}, \tag{5}$$

where $p^* = arg \min\limits_{p\in\mathbb{R}^m} f(p)$.

The following assumption is about the graph and the cost functions, which was used in [7, 9, 12, 16, 21].

Assumption 2.2. (a) The undirected graph $\mathcal{G}$ is connected; (b) The local cost function f_i is w_i-strongly convex and differentiable, and its gradient is θ_i-Lipschitz on $\mathbb{R}^m$.

From Assumption 2.2, it is obvious that $f(p)$ is strongly convex, which implies that problem (4) has a unique optimal solution $p^* := \arg\min_{p\in\mathbb{R}^m} f(p)$. Here we consider the case of $-\infty < \|p^*\| < +\infty$, i. e., the optimal solution is bounded.

The task in this paper is to design a control protocol u_i with concerning the reduction of communication cost such that the multi-agent system (3) still solves the optimization problem $p^* = \arg \min\limits_{p\in\mathbb{R}^m} f(p)$ by driving q_i to p^*. To this end, an event-triggered scheme will be given, which is a nonsmooth scheme. Therefore, we need the following definition in the sequel.

Definition 2.3. The upper Dini derivative of continuous function $g : \mathbb{R} \to \mathbb{R}$ is denoted by g'_+, defined as

$$g'_+ \triangleq \limsup_{h\to 0^+} \frac{g(t+h)-g(h)}{h}.$$

3. MAIN RESULT

In this section, we provide a event-based optimal consensus protocol for EL multi-agent systems with the optimization problem (4).

For each agent i, we assume that the system parameters are available, and then the following gradient-based event-triggered optimization controller is developed:

$$\begin{aligned} u_i &= C_i(q_i,\dot q_i)\dot q_i + G_i(q_i) - kM_i(q_i)\dot q_i - M_i(q_i)\sum_{j\in N_i} a_{ij}(q_i(t^i_k) - q_j(t^j_k)) \\ &\quad - M_i(q_i)\nabla f_i(q_i) - M_i(q_i)v_i, \\ \dot v_i &= \sum_{j\in N_i} a_{ij}(q_i(t^i_k) - q_j(t^j_k) + \dot q_i(t^i_l) - \dot q_j(t^j_l)),\ i\in\mathcal{V},\ t\in[t^i_k,t^i_{k+1})\cup[t^i_l,t^i_{l+1}), \end{aligned} \tag{6}$$

where $k \geq \theta(1+\frac{\theta}{4\omega}) + 2 + \frac{1}{4}\lambda_{\max}(L)$, $\theta = \max\{\theta_1,\dots,\theta_n\}$, and $\omega = \min\{\omega_1,\dots,\omega_n\}$. t^i_k and t^j_k are the last event times of agent i and j corresponding to q, respectively. t^j_l and t^j_l are the last event times of agent i and j corresponding to $\dot q$, respectively.

Combining (3) and (6), we obtain the following closed-loop system

$$\begin{aligned} \ddot q_i &= -k\dot q_i - \sum_{j\in N_i} a_{ij}(q_i(t^i_k) - q_j(t^j_k)) - \nabla f_i(q_i) - v_i, \\ \dot v_i &= \sum_{j\in N_i} a_{ij}(q_i(t^i_k) - q_j(t^j_k) + \dot q_i(t^i_l) - \dot q_j(t^j_l)). \end{aligned} \tag{7}$$

Without loss of generality, we assume that the first event is generated at time t_0. Then the triggering time sequence $\{t^i_k\}$ of position and sequence $\{t^i_l\}$ of velocity for agent i are defined iteratively by

$$\begin{aligned} t^i_{k+1} &= inf\{t : t > t^i_k, f^i_p(t) > 0\}, \\ t^i_{l+1} &= inf\{t : t > t^i_l, f^i_v(t) > 0\}, \end{aligned} \tag{8}$$

where

$$\begin{aligned} f^i_p(t) &= \|e^i_p(t)\| - \alpha\beta_1\|\sum_{j\in N_i} a_{ij}(q_i(t^i_k) - q_j(t^j_k))\| - \beta_2 e^{-\gamma t}, \\ f^i_v(t) &= \|e^i_v(t)\| - \alpha\beta_3\|\dot q_i(t^i_l)\| - \beta_4 e^{-\gamma t} \end{aligned} \tag{9}$$

are said to be the trigger functions for positive real numbers β_1, β_2, β_3, β_4, α (to be determined later) and γ satisfying $\alpha\beta_1 < \frac{1}{d+a\sqrt{6n}}$, $\alpha\beta_3 < 1$ and $0 < \gamma < \frac{\beta}{2\lambda_{\max}(\Phi)}$ with constant β to be determined later, $d = \max_{i\in\mathcal{V}}\{d_i\}$, $a = \max_{i,j\in\mathcal{V}}\{a_{ij}\}$,

$$\begin{aligned}\Phi &= \begin{bmatrix} \Phi_1 & \frac{1}{2}I_n & \frac{\alpha}{2}\begin{pmatrix} \mathbf{0}_{n-1}^T \\ I_{n-1}\end{pmatrix} \\ * & \frac{k}{2}I_n & \mathbf{0}_{n\times(n-1)} \\ * & * & \frac{1}{2}(U^TLU)^{-1} + \frac{\alpha}{2}I_{n-1}\end{bmatrix},\\ \Phi_1 &= \frac{1}{2}I_n + \frac{\alpha}{2}\begin{pmatrix} 0 & \mathbf{0}_{n-1}^T \\ * & I_{n-1}\end{pmatrix},\end{aligned}$$

$e_p^i(t) = q_i(t_k^i) - q_i(t)$ and $e_v^i(t) = \dot{q}_i(t_l^i) - \dot{q}_i(t)$. Therefore, $e_p^i(t)$ and $e_v^i(t)$ are reset to 0 at $t = t_k^i$ and $t = t_l^i$, respectively.

It is time to give the main result to show that the distributed optimal consensus of EL system (3) can be exponentially achieved under controller (6) with event-triggered condition (8).

Theorem 3.1. Consider the heterogeneous EL multi-agent system (3), and suppose that Assumption 1 holds. Then, for any initial conditions with $\sum_{i=1}^n v_i(0) = \mathbf{0}_m$, the distributed optimal consensus of system (3) can be achieved exponentially under the controller (6) with event-triggered condition (8). Furthermore, the Zeno behavior of the event-triggered strategies can be excluded.

Proof. Let

$$\begin{aligned}\eta = col(\eta_1,\dots,\eta_n) = q - q^*, \quad \xi = col(\xi_1,\dots,\xi_n) = [\dot{q}_1^T,\dots,\dot{q}_n^T]^T,\\ \vartheta = col(\vartheta_1,\dots,\vartheta_n) = [v_1^T,\dots,v_n^T]^T - \varphi^*, \quad h = \nabla_q\tilde{f}(q) - \nabla_q\tilde{f}(q^*),\end{aligned}$$

where $q = [q_1^T,\dots,q_n^T]^T$, $q^* = \mathbf{1}_n \otimes p^*$, $\varphi^* = -\nabla_q\tilde{f}(q^*)$, and $\tilde{f}(q) := \sum_{i=1}^n f_i(q_i)$. Actually, one can find that $e_p^i(t) = \big(q_i(t_k^i) - q^*\big) - (q_i(t) - q^*) = \eta_i(t_k^i) - \eta_i(t)$.

Then, for system (7), we have

$$\begin{aligned}\dot{\eta}_i &= \xi_i,\\ \dot{\xi}_i &= -k\xi_i - \sum_{j\in N_i} a_{ij}(\eta_i(t_k^i) - \eta_j(t_k^j)) - \nabla f_i(q_i) - v_i,\\ \dot{\vartheta}_i &= \sum_{j\in N_i} a_{ij}[(\eta_i(t_k^i) - \eta_j(t_k^j)) + (\xi_i(t_l^i) - \xi_j(t_l^j))].\end{aligned} \tag{10}$$

Moreover, it yields the compact form of system (10)

$$\begin{aligned}\dot{\eta} &= \xi,\\ \dot{\xi} &= -k\xi - L(\eta + e_p) - \vartheta - h,\\ \dot{\vartheta} &= L(\eta + e_p + \xi + e_v).\end{aligned} \tag{11}$$

Since L is the Laplacian matrix associated with $\mathcal{G}$, we have $L\mathbf{1}_n = \mathbf{0}_n$, and then $\sum_{i=1}^n \dot{v}_i = \mathbf{0}_m$, which implies that

$$\sum_{i=1}^n v_i(t) = \sum_{i=1}^n v_i(0) = \mathbf{0}_m, \ \forall t \geq 0. \tag{12}$$

Since $\mathcal{G}$ is connected, zero is the simple eigenvalue of L and all the other eigenvalues are positive. With singular value decomposition, there is an orthogonal matrix $P = (r, U) \in \mathbb{R}^{n\times n}$ such that $P^T LP = diag\{0, \lambda_2, \ldots, \lambda_n\}$, where $r = \frac{\mathbf{1}_n}{\sqrt{n}}$ and $U \in \mathbb{R}^{n\times(n-1)}$. For convenience, we change some variables as follows:

$$\begin{aligned}\hat{\eta} &= (P^T \otimes I_m)\eta,\ \hat{\xi} = (P^T \otimes I_m)\xi,\\ \hat{\vartheta} &= (P^T \otimes I_m)\vartheta,\ \hat{e}_p = (P^T \otimes I_m)e_p,\\ \hat{e}_v &= (P^T \otimes I_m)e_v, \end{aligned} \tag{13}$$

then, system (11) can be rewritten as follows together with (12)

$$\begin{aligned}\dot{\hat{\xi}} &= -k\hat{\xi} - (P^T LP \otimes I_m)(\hat{\eta} + \hat{e}_p) - (P^T \otimes I_m)(\vartheta + h),\\ \dot{\hat{\eta}} &= \hat{\xi},\\ \dot{\hat{\vartheta}} &= (P^T LP \otimes I_m)(\hat{\eta} + \hat{e}_p + \hat{\xi} + \hat{e}_v). \end{aligned} \tag{14}$$

Furthermore, system (14) can be written as follows.

$$\begin{aligned}\dot{\hat{\xi}}_1 &= -k\hat{\xi}_1 - (r^T \otimes I_m)h,\\ \dot{\hat{\xi}}_{2:n} &= -\hat{\xi}_{2:n} - (U^T LU \otimes I_m)(\hat{\eta}_{2:n} + \hat{e}_{p_{2:n}}) - \hat{\vartheta}_{2:n} - (U^T \otimes I_m)h,\\ \dot{\hat{\eta}}_1 &= \hat{\xi}_1,\ \dot{\hat{\eta}}_{2:n} = \hat{\xi}_{2:n},\\ \dot{\hat{\vartheta}}_1 &= \mathbf{0}_m,\ \dot{\hat{\vartheta}}_{2:n} = (U^T LU \otimes I_m)(\hat{\eta}_{2:n} + \hat{e}_{p_{2:n}} + \hat{\xi}_{2:n} + \hat{e}_{v_{2:n}}), \end{aligned} \tag{15}$$

where $\hat{\xi}_1$, $\hat{\eta}_1$, $\hat{\vartheta}_1 \in \mathbb{R}^m$ and $\hat{\xi}_{2:n}$, $\hat{\eta}_{2:n}$, $\hat{\vartheta}_{2:n} \in \mathbb{R}^{(n-1)m}$.

Take the following candidate Lyapunov function

$$V = V_1 + \alpha V_2 = q_0^T(\Phi \otimes I_m)q_0,\ q_0 = [\hat{\xi}^T, \hat{\eta}^T, \hat{\vartheta}_{2:n}^T]^T, \tag{16}$$

where

$$\begin{aligned}V_1 &= \frac{1}{2}\|\hat{\xi}_1\|^2 + \hat{\xi}_1^T\hat{\eta}_1 + \frac{k}{2}\|\hat{\eta}_1\|^2 + \frac{1}{2}\|\hat{\xi}_{2:n}\|^2 + \hat{\xi}_{2:n}^T\hat{\eta}_{2:n} + \frac{k}{2}\|\hat{\eta}_{2:n}\|^2\\ &\quad + \frac{1}{2}\hat{\vartheta}_{2:n}((U^T LU)^{-1} \otimes I_m)\hat{\vartheta}_{2:n},\\ V_2 &= \frac{1}{2}(\hat{\xi}_{2:n} + \hat{\vartheta}_{2:n})^2. \end{aligned} \tag{17}$$

Then the derivative of V_1 along the trajectories of system (15) can be written as

$$\begin{aligned}\dot{V}_1 &= -(k-1)\|\hat{\xi}_1\|^2 - (k-1)\|\hat{\xi}_{2:n}\|^2 - (\xi + \eta)^T h - \hat{\xi}_{2:n}^T(U^T LU \otimes I_m)\hat{\eta}_{2:n}\\ &\quad -\hat{\xi}_{2:n}^T(U^T LU \otimes I_m)\hat{e}_{p_{2:n}} - \hat{\eta}_{2:n}^T(U^T LU \otimes I_m)\hat{\eta}_{2:n} - \hat{e}_{p_{2:n}}^T(U^T LU \otimes I_m)\hat{\eta}_{2:n}\\ &\quad +\hat{\vartheta}_{2:n}\hat{e}_{p_{2:n}} + \hat{\vartheta}_{2:n}\hat{e}_{v_{2:n}}\\ &= -(\hat{\eta}_{2:n}, \hat{\xi}_{2:n})^T(M \otimes I_m)(\hat{\eta}_{2:n}^T, \hat{\xi}_{2:n}^T)^T - (\alpha_0 + 1)\|\hat{\xi}_{2:n}\|^2 - (k-1)\|\hat{\xi}_1\|^2 - (\xi+\eta)^T h\\ &\quad -\hat{\xi}_{2:n}^T(U^T LU \otimes I_m)\hat{e}_{p_{2:n}} - \hat{e}_{p_{2:n}}^T(U^T LU \otimes I_m)\hat{\eta}_{2:n} + \hat{\vartheta}_{2:n}\hat{e}_{p_{2:n}} + \hat{\vartheta}_{2:n}\hat{e}_{v_{2:n}}, \end{aligned} \tag{18}$$

where $\alpha_0 = \theta(1+\frac{\theta}{4\omega})$ and $M = \begin{bmatrix} U^TLU & \frac{1}{2}U^TLU \\ * & (k-\alpha_0-2)I_{n-1} \end{bmatrix}$. According to Schur Complement Lemma, it is clear that M is nonnegative definite. Since ∇f_i is θ_i-Lipschitz,

$$\begin{aligned} -\xi^T h &= -\sum_{i=1}^{n}\xi_i^T(\nabla f_i(\eta_i+p^*)-\nabla f_i(p^*)) \le \theta\sum_{i=1}^{n}|\xi_i||\eta_i| \\ &\le \theta(\frac{\theta}{4\omega}+1)\|\hat{\xi}\|^2+\frac{\theta\omega}{\theta+4\omega}\|\hat{\eta}\|^2. \end{aligned} \tag{19}$$

Moreover, since f_i is ω_i-strongly convex, it follows that

$$\eta^T h = -\sum_{i=1}^{n}\eta_i^T(\nabla f_i(\eta_i+p^*)-\nabla f_i(p^*)) \ge \omega\sum_{i=1}^{n}\|\eta_i\|^2 = \omega\|\eta\|^2. \tag{20}$$

The trigger condition (9) enforces that

$$\begin{aligned} \|e_p^i\| &\le \alpha\beta_1\|\sum_{j\in N_i}a_{ij}(\eta_i(t_k^i)-\eta_j(t_k^j))\|+\beta_2 e^{-\gamma t} \\ &\le \alpha\beta_1\left(\sum_{j\in N_i}a_{ij}\|e_p^i\|+\sum_{j\in N_i}a_{ij}\|\eta_i\|+\sum_{j\in N_i}a_{ij}\|\eta_j\|+\sum_{j\in N_i}a_{ij}\|e_p^j\|\right)+\beta_2 e^{-\gamma t}. \end{aligned}$$

Then

$$\|e_p\|^2 \le \frac{6\alpha^2\beta_1^2(d^2+a^2n)}{(1-\alpha\beta_1 d)^2-6\alpha^2\beta_1^2a^2n}\|\eta\|^2+\frac{2n\beta_2^2}{(1-\alpha\beta_1 d)^2-6\alpha^2\beta_1^2a^2n}e^{-2\gamma t}.$$

In the same way, we obtain

$$\|e_v\|^2 \le \frac{2\alpha^2\beta_3^2}{(1-\alpha\beta_3)^2}\|\xi\|^2+\frac{2n\beta_4^2}{(1-\alpha\beta_3)^2}e^{-2\gamma t}.$$

Furthermore, in view of (13), it concludes that

$$\|\hat{e}_p\|^2 \le \frac{6\alpha^2\beta_1^2(d^2+a^2n)}{(1-\alpha\beta_1 d)^2-6\alpha^2\beta_1^2a^2n}\|\hat{\eta}\|^2+\frac{2n\beta_2^2}{(1-\alpha\beta_1 d)^2-6\alpha^2\beta_1^2a^2n}e^{-2\gamma t}, \tag{21}$$

$$\|\hat{e}_v\|^2 \le \frac{2\alpha^2\beta_3^2}{(1-\alpha\beta_3)^2}\|\hat{\xi}\|^2+\frac{2n\beta_4^2}{(1-\alpha\beta_3)^2}e^{-2\gamma t}. \tag{22}$$

Therefore, combining (19), (20), (21) and (22), we conclude that

$$\begin{aligned} \dot{V}_1 \le& -\|\hat{\xi}_{2:n}\|^2-(k-1-\alpha_0)\|\hat{\xi}_1\|^2+\frac{\alpha}{2}\|\hat{\xi}_{2:n}\|^2-\frac{4\omega^2}{\theta+4\omega}\|\hat{\eta}\|^2+\frac{\alpha}{2}\|\hat{\eta}_{2:n}\|^2+\frac{\alpha}{12}\|\hat{\vartheta}_{2:n}\|^2 \\ &+\frac{6\alpha\lambda_{\max}^2(L)\beta_1^2(d^2+a^2(n-1))}{(1-\alpha\beta_1 d)^2-6\alpha^2\beta_1^2a^2(n-1)}\|\hat{\eta}_{2:n}\|^2+\frac{2(n-1)\lambda_{\max}^2(L)\beta_2^2}{\alpha(1-\alpha\beta_1 d)^2-6\alpha^3\beta_1^2a^2(n-1)}e^{-2\gamma t} \\ &+\frac{18\alpha\beta_1^2(d^2+a^2(n-1))}{(1-\alpha\beta_1 d)^2-6\alpha^2\beta_1^2a^2(n-1)}\|\hat{\eta}_{2:n}\|^2+\frac{6(n-1)\beta_2^2}{\alpha(1-\alpha\beta_1 d)^2-6\alpha^3\beta_1^2a^2(n-1)}e^{-2\gamma t} \\ &+\frac{\alpha}{12}\|\hat{\vartheta}_{2:n}\|^2+\frac{6\alpha\beta_3^2}{(1-\alpha\beta_3)^2}\|\hat{\xi}_{2:n}\|^2+\frac{6\beta_4^2(n-1)}{\alpha(1-\alpha\beta_3)^2}e^{-2\gamma t}. \end{aligned} \tag{23}$$

Likewise, we obtain

$$
\begin{aligned}
\dot{V}_2 &= -\|\hat{\vartheta}_{2:n}\|^2 - k\|\hat{\xi}_{2:n}\|^2 - (k+1)\hat{\vartheta}_{2:n}^T\hat{\xi}_{2:n} - \hat{\xi}_{2:n}^T(U^T \otimes I_m)h - \hat{\vartheta}_{2:n}^T(U^T \otimes I_m)h \\
&\quad +\hat{\xi}_{2:n}^T(U^T LU \otimes I_m)\hat{\xi}_{2:n} + \hat{\vartheta}_{2:n}^T(U^T LU \otimes I_m)\hat{\xi}_{2:n} + \hat{\xi}_{2:n}^T(U^T LU \otimes I_m)\hat{e}_{v_{2:n}} \\
&\quad +\hat{\vartheta}_{2:n}^T(U^T LU \otimes I_m)\hat{e}_{v_{2:n}} \\
&\leq -\frac{1}{4}\|\hat{\vartheta}_{2:n}\|^2 + 2\theta^2\|\hat{\eta}\|^2 + k_0\|\hat{\xi}_{2:n}\|^2 + \frac{1}{2}\|\hat{\xi}_{2:n}\|^2 + \frac{13\alpha^2\lambda_{\max}^2(L)\beta_3^2}{(1-\alpha\beta_3)^2}\|\hat{\xi}_{2:n}\|^2 \\
&\quad +\frac{1}{24}\|\hat{\vartheta}_{2:n}\|^2 + \frac{13\lambda_{\max}^2(L)\beta_4^2(n-1)}{(1-\alpha\beta_3)^2}e^{-2\gamma t}, \qquad (24)
\end{aligned}
$$

where $k_0 = k^2 + k + \frac{5}{4} + \lambda_{\max}^2(L) + \lambda_{\max}(L)$. Then, in view of (23) and (24), we give

$$
\begin{aligned}
\dot{V} &= \dot{V}_1 + \alpha\dot{V}_2 \\
&\leq -\frac{\alpha}{24}\|\hat{\vartheta}_{2:n}\|^2 - R_1\|\hat{\xi}_1\|^2 - R_2\|\hat{\eta}\|^2 - R_3\|\hat{\xi}_{2:n}\|^2 + R_4 e^{-2\gamma t}, \qquad (25)
\end{aligned}
$$

where

$$
\begin{aligned}
R_1 &= k - 1 - \alpha_0, \\
R_2 &= \frac{4\omega^2}{\theta + 4\omega} - \frac{6\alpha\lambda_{\max}^2(L)\beta_1^2(d^2 + a^2(n-1))}{(1-\alpha\beta_1 d)^2 - 6\alpha^2\beta_1^2 a^2(n-1)} - \frac{\alpha}{2} \\
&\quad -\frac{18\alpha\beta_1^2(d^2 + a^2(n-1))}{(1-\alpha\beta_1 d)^2 - 6\alpha^2\beta_1^2 a^2(n-1)} - 2\alpha\theta^2, \\
R_3 &= 1 - \alpha - \frac{6\alpha\beta_3^2}{(1-\alpha\beta_3)^2} - k_0\alpha - \frac{13\alpha^3\lambda_{\max}^2(L)\beta_3^2}{(1-\alpha\beta_3)^2}, \\
R_4 &= \frac{2(n-1)\lambda_{\max}^2(L)\beta_2^2}{\alpha(1-\alpha\beta_1 d)^2 - 6\alpha^3\beta_1^2 a^2(n-1)} + \frac{6(n-1)\beta_2^2}{\alpha(1-\alpha\beta_1 d)^2 - 6\alpha^3\beta_1^2 a^2(n-1)} \\
&\quad +\frac{6\beta_4^2(n-1)}{\alpha(1-\alpha\beta_3)^2} + \frac{13\lambda_{\max}^2(L)\beta_4^2(n-1)}{(1-\alpha\beta_3)^2}. \qquad (26)
\end{aligned}
$$

By taking $\alpha\left(1 + \frac{6\beta_3^2}{(1-\alpha\beta_3)^2} + k_0 + \frac{13\alpha^2\lambda_{\max}^2(L)\beta_3^2}{(1-\alpha\beta_3)^2}\right) < 1$ and

$$
\alpha\left(\frac{6\lambda_{\max}^2(L)\beta_1^2(d^2+a^2(n-1))}{(1-\alpha\beta_1 d)^2-6\alpha^2\beta_1^2 a^2(n-1)} + \frac{18\beta_1^2(d^2+a^2(n-1))}{(1-\alpha\beta_1 d)^2-6\alpha^2\beta_1^2 a^2(n-1)}\right) + \alpha(\frac{1}{2}+2\theta^2) < \frac{4\omega^2}{\theta+4\omega},
$$

according to (16), we have

$$
\dot{V}(t) \leq -\beta\|q_0(t)\|^2 + R_4 e^{-2rt}, \qquad (27)
$$

where $\beta \triangleq \min\{\frac{\alpha}{24}, R_1, R_2, R_3\}$.

Then it results from (16) and (27) that

$$
\|q_0(t)\|^2 \leq \frac{\lambda_{\max}(\Phi)}{\lambda_{\min}(\Phi)}e^{-\frac{\beta}{\lambda_{\max}(\Phi)}t}\|q_0(0)\|^2 + \frac{R_4\lambda_{\max}(\Phi)}{\lambda_{\min}(\Phi)(\beta - 2\gamma\lambda_{\max}(\Phi))}(e^{-2\gamma t} - e^{-\frac{\beta}{\lambda_{\max}(\Phi)}t}).
$$

Further, by applying the condition $0 < \gamma < \frac{\beta}{2\lambda_{\max}(\Phi)}$ for $t \geq 0$, we can give the following result

$$\|q_0(t)\|^2 \quad \leq \quad \left(\frac{\lambda_{\max}(\Phi)}{\lambda_{\min}(\Phi)} \|q_0(0)\|^2 + \frac{R_4 \lambda_{\max}(\Phi)}{\lambda_{\min}(\Phi)(\beta - 2\gamma\lambda_{\max}(\Phi))} \right) e^{-2\gamma t}, \tag{28}$$

which implies that the optimal consensus can be achieved exponentially.

In the following, we will explain why the Zeno behavior can be avoided.

According to paper [26], we first show that Zeno behavior is free for the communication of the position. Compute the upper Dini derivative of $\|e_p^i(t)\|$ over time interval $[t_k^i, t_{k+1}^i)$, then we can derive that

$$D^+\|e_p^i(t)\| \leq \|\dot{e}_p^i(t)\| = \|\dot{q}_i(t)\| = \|\xi_i(t)\| = \|(P^{-T} \otimes I_m)\hat{\xi}_i(t)\| \leq \|\hat{\xi}_i(t)\|.$$

By applying (28), we have

$$D^+\|e_p^i(t)\| \leq \sqrt{N} e^{-\gamma t}, \tag{29}$$

where $N = \frac{\lambda_{\max}(\Phi)}{\lambda_{\min}(\Phi)} \|q_0(0)\|^2 + \frac{R_4 \lambda_{\max}(\Phi)}{\lambda_{\min}(\Phi)(\beta - 2\gamma\lambda_{\max}(\Phi))}$.

Combining (29) with $e_p^i(t_k^i) = 0$, it follows that

$$\|e_p^i(t)\| \leq \frac{\sqrt{N}}{\gamma}(e^{-\gamma t_k^i} - e^{-\gamma t}),\ t \in [t_k^i, t_{k+1}^i).$$

The next communication will not be executed until the first trigger function of (9) crosses zero, i. e.,

$$\alpha\beta_1 \| \sum_{j \in N_i} a_{ij}(q_i(t_k^i) - q_j(t_k^j))\| + \beta_2 e^{-\gamma t_{k+1}^i} = \|e_p^i(t_{k+1}^i)\| \leq \frac{\sqrt{N}}{\gamma}(e^{-\gamma t_k^i} - e^{-\gamma t_{k+1}^i}).$$

Take $T_k^i = t_{k+1}^i - t_k^i$, which yields

$$\beta_2 e^{-\gamma T_k^i} \leq \frac{\sqrt{N}}{\gamma}(1 - e^{-\gamma T_k^i}). \tag{30}$$

By (30), we obtain that $\{T_k^i > 0 : \frac{\sqrt{N}}{\gamma}(1 - e^{-\gamma T_k^i}) - \beta_2 e^{-\gamma T_k^i} \geq 0\}$ is nonempty and $T = \inf_k\{T_k^i\} > 0$ for any i, which implies that the Zeno behavior is excluded for any agent i.

Next, we present that velocity communication is free of Zeno behavior.

With the same approach, we can derive the upper right-hand Dini derivative of $\|e_v^i(t)\|$ described as follows:

$$\begin{aligned}D^+\|e_v^i(t)\| \leq &\|\dot{e}_v^i(t)\| = \|\dot{\xi}_i(t)\| = \|(P^{-T}\otimes I_m)\dot{\hat{\xi}}_i(t)\| \leq \|\dot{\hat{\xi}}_i(t)\|\\ \leq &k\|\hat{\xi}_i(t)\| + \lambda_{\max}(L)\|\hat{\eta}_i(t_k^i)\| + \|\vartheta_i\| + \|h_i\|\\ \leq &k\sqrt{N}e^{-\gamma t} + \lambda_{\max}(L)\|\hat{\eta}_i(t_k^i)\| + \sqrt{N}e^{-\gamma t} + \theta\|\hat{\eta}_i(t)\|\\ \leq &(k+1+\theta)\sqrt{N}e^{-\gamma t} + \lambda_{\max}(L)\|\hat{\eta}_i(t_k^i)\|,\ t\in[t_l^i, t_{l+1}^i). \end{aligned}\tag{31}$$

It follows from (31) and $e_v^i(t_l^i) = 0$ that

$$\|e_v^i(t)\| \quad \leq \quad \frac{(k+1+\theta)\sqrt{N}}{\gamma}(e^{-\gamma t_l^i} - e^{-\gamma t}) + \lambda_{\max}(L)\|\hat{\eta}_i(t_k^i)\|(t - t_l^i).$$

In the same sense, the next event will not be triggered until the second trigger function of (9) crosses zero, i. e.,

$$\begin{aligned}\alpha\beta_3\|\xi_i(t_l^i)\| + \beta_4 e^{-\gamma t_{l+1}^i} \quad &= \quad \|e_v^i(t_{l+1}^i)\|\\ &\leq \quad \frac{(k+1+\theta)\sqrt{N}}{\gamma}(e^{-\gamma t_l^i} - e^{-\gamma t_{l+1}^i}) + \lambda_{\max}(L)\|\hat{\eta}_i(t_k^i)\|T_l^i,\end{aligned}$$

where $T_l^i = t_{l+1}^i - t_l^i$.

Therefore,

$$\beta_4 e^{-\gamma T_l^i} \leq \frac{(k+1+\theta)\sqrt{N}}{\gamma}(1 - e^{-\gamma T_l^i}) + \lambda_{\max}(L)\|\hat{\eta}_i(t_k^i)\|T_l^i. \tag{32}$$

By (32), we see that $\{T_l^i > 0 : \frac{(k+1+\theta)\sqrt{N}}{\gamma}(1-e^{-\gamma T_l^i})+\lambda_{\max}(L)\|\hat{\eta}_i(t_k^i)\|T_l^i-\beta_4 e^{-\gamma T_l^i} \geq 0\}$ is nonempty and $T = \inf_l\{T_l^i\} > 0$ for any agent i, which implies no Zeno behavior. Thus, the conclusion follows. □

Remark 3.2. In distributed optimization field, the dynamics of agents usually are with the form of single integrator as given in [5, 6, 12, 14, 15, 16], while the considered multi-agent system in this paper consists of multiple EL systems, which is more difficult than single integrator. Thus the algorithm design of this paper is more complex than that of single integrator. Moreover, compared with the results of [21] with exponential convergence in the sense of semi-globally, the algorithm of this paper, which basically extends the results given in [9] with global convergence, can also keep the global convergence to the exact optimal solution along with the reduction of communication among agents. Furthermore, the result of this paper is better than that of [16] since the algorithm of [16] only achieved the exponential convergence to a neighborhood of the optimal point. Additionally, the proposed event-triggered strategy can not only reduce the cost of communication but also avoid the Zeno behavior of triggering time.

4. EXAMPLE

In this section, an example is given to illustrate the effectiveness of the proposed control algorithm. Consider a network composed of five EL agents with all edge weights 1, which is depicted by Figure 1, and their local cost functions are presented respectively as follows

$$
\begin{aligned}
f_1(q) =& (q_x-2)^2+(q_y-2)^2,\\
f_2(q) =& \frac{q_x^2}{\sqrt{q_x^2+1}}+\frac{q_y^2}{\sqrt{q_y^2+1}}+\|q\|^2,\\
f_3(q) =& \frac{q_x^2}{\ln\left(q_x^2+2\right)}+\frac{q_y^2}{\ln\left(q_y^2+2\right)}\\
&+(q_x-5)^2+(q_y-5)^2,\\
f_4(q) =& \ln\left(e^{-0.05q_x}+e^{0.05q_x}\right)\\
&+\ln\left(e^{-0.05q_y}+e^{0.05q_y}\right)+\|q\|^2,\\
f_5(q) =& \frac{q_x^2}{\sqrt{q_x^2+1}}+\frac{q_y^2}{\sqrt{q_y^2+1}}+q_x+q_y+\|q\|^2,
\end{aligned}
$$

where $q=[q_x\ q_y]^T$ is a vector.

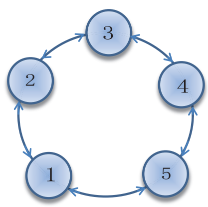

Fig. 1. The interaction topology of system.

Similar to [21, 22, 23], the system dynamics are

$$
\begin{bmatrix} m_{11,i} & m_{12,i} \\ m_{21,i} & m_{22,i} \end{bmatrix}\begin{bmatrix} \ddot{q}_{ix} \\ \ddot{q}_{iy} \end{bmatrix}+\begin{bmatrix} c_{11,i} & c_{12,i} \\ c_{21,i} & c_{22,i} \end{bmatrix}\begin{bmatrix} \dot{q}_{ix} \\ \dot{q}_{iy} \end{bmatrix}=\begin{bmatrix} \tau_{ix} \\ \tau_{iy} \end{bmatrix}
$$

where $m_{11,i}=\theta_{1i}+2\theta_{2i}\cos q_{iy}$, $m_{12,i}=m_{21,i}=\theta_{3i}+\theta_{2i}\cos q_{iy}$, $m_{22,i}=\theta_{3i}$, $c_{11,i}=-\theta_{2i}\sin q_{iy}\dot{q}_{iy}$, $c_{12,i}=-\theta_{2i}\sin q_{iy}\left(\dot{q}_{ix}+\dot{q}_{iy}\right)$, $c_{21,i}=\theta_{2i}\sin q_{iy}\dot{q}_{ix}$, $c_{22,i}=0$, $\theta_{1i}=1.301$, $\theta_{2i}=0.256$, $\theta_{3i}=0.096$, and $G_i(q_i)=[0\ 0]^T$, $i=1,\dots 5$.

The initial conditions are set as follows: $q_1(0)=[4\ 3]^T$, $q_2(0)=[3\ -5]^T$, $q_3(0)=[0\ -2.5]^T$, $q_4(0)=[-1\ -2]^T$, $q_5(0)=[-2\ 4]^T$, and $\dot{q}_i(0)=[0\ 0]^T$ with $i=1,\dots,5$. The parameters of controller are: $\alpha=0.0002$, $\beta_1=2$, $\beta_2=10$, $\beta_3=5$, $\beta_4=20$, $\gamma=0.2$, $\theta=2$, $\omega=1.96$ and $k=5$. The simulation results are shown in Figures 2−4. From Figures 2 and 3, it is clear that, under (6), the state of each agent converges to the global optimal point q^*, and meanwhile, the velocity of each agent tends to zero. Besides, Figure 4 shows that, the Zeno behavior of triggering time can be avoided.

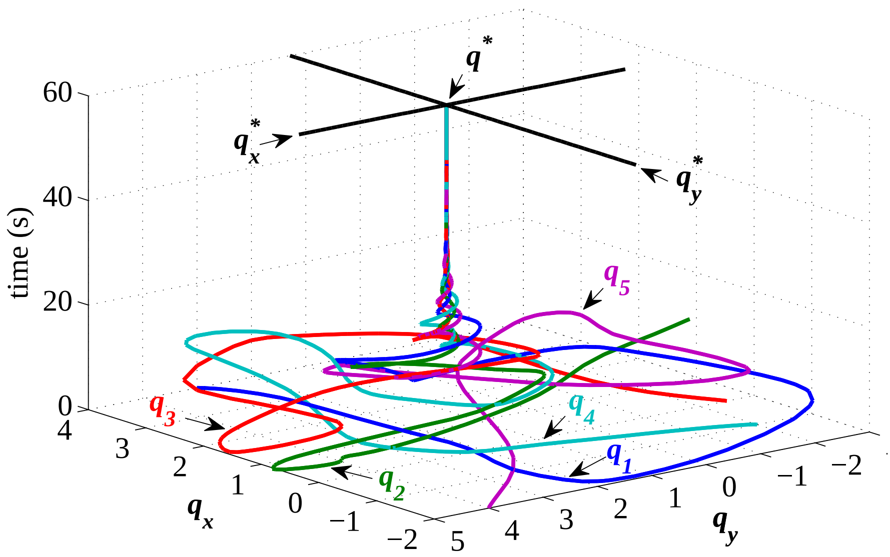

Fig. 2. The position performance under control algorithm (6).

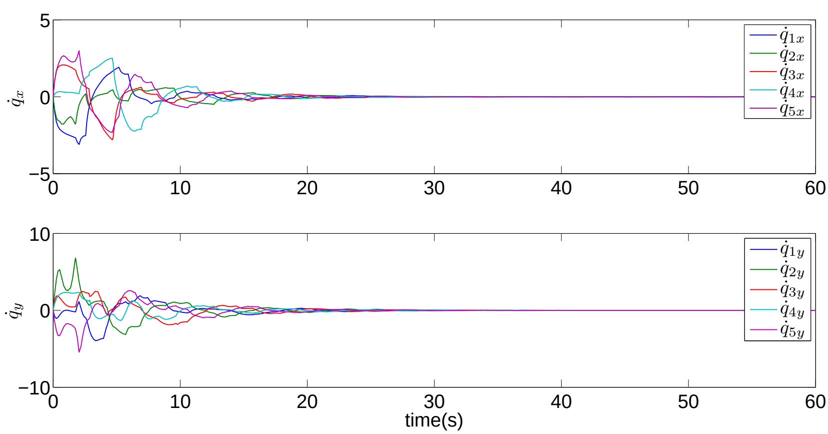

Fig. 3. The velocity performance under control algorithm (6).

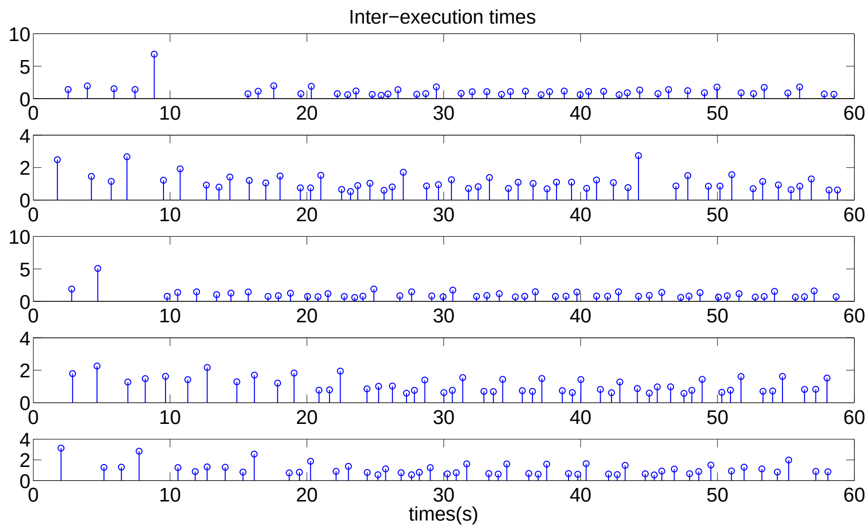

Fig. 4. The inter-execution times of position.

5. CONCLUSIONS

In this paper, one of the distributed optimization problems, optimal consensus was studied for a group of multiple Euler–Lagrangian systems. A novel event-based distributed continuous-time controller was designed. With the proposed event-triggered condition to reduce the communication cost and also avoid the Zeno phenomena, the optimal consensus of the considered system was shown to be achieved globally under the given controller. An example with simulations was given to demonstrate the effectiveness of the algorithms.

ACKNOWLEDGEMENT

This work was supported by the National Key Research and Development Program of China (2016YFB0901902), National Natural Science Foundation of China (61325014, 61333001), and Beijing Natural Science Foundation (4152057).

(Received August 24, 2016)

REFERENCES

[1] A. Nedic and A. Ozdaglar: Distributed subgradient methods for multi-agent optimization. IEEE Trans. Automat. Control *54* (2009), 48–61. DOI:10.1109/tac.2008.2009515

[2] G. Shi, K. H. Johansson, and Y. Hong: Reaching an optimal consensus: dynamical systems that compute intersections of convex sets. IEEE Trans. Automat. Control *58* (2013), 610–622. DOI:10.1109/tac.2012.2215261

[3] S. Bose, S. H. Low, T. Teeraratkul, and B. Hassibi: Equivalent relaxations of optimal power flow. IEEE Trans. Automat. Control *60* (2015), 729–742. DOI:10.1109/tac.2014.2357112

[4] Y. Zhang, Y. Lou, Y. Hong. and L. Xie: Distributed projection-based algorithms for source localization in wireless sensor networks. IEEE Trans. Wireless Commun. *14* (2015), 3131–3142. DOI:10.1109/twc.2015.2402672

[5] Q. Liu and J. Wang: A second-order multi-agent network for bound-constrained distributed optimization. IEEE Trans. Automat. Control *60* (2015), 3310–3315. DOI:10.1109/tac.2015.2416927

[6] P. Yi, Y. Hong, and F. Liu: Distributed gradient algorithm for constrained optimization with application to load sharing in power systems. Systems Control Lett. *83* (2015), 45–52. DOI:10.1016/j.sysconle.2015.06.006

[7] X. Wang, P. Yi, and Y. Hong: Dynamical optimization for multi-agent systems with external disturbance. Control Theory Technol. *12* (2014), 132–138. DOI:10.1007/s11768-014-0036-y

[8] X. Wang, Y. Hong, and H. Ji: Distributed optimization for a class of nonlinear multiagent systems With disturbance rejection. IEEE Trans. Cybernet. *46* (2016), 1655–1666. DOI:10.1109/tcyb.2015.2453167

[9] Y. Zhang, Z. Deng, and Y. Hong: Distributed optimal coordination for multiple heterogenous Euler–Lagrangian systems. Automatica *79* (2017), 207–213. DOI:10.1016/j.automatica.2017.01.004

[10] P. Yi and Y. Hong: Stochastic sub-gradient algoirthm for distributed optimization with random sleep scheme. Control Theory Technol. *13* (2015), 333–347. DOI:10.1007/s11768-015-5100-8

[11] J. Hu, G. Chen, and H. Li: Distributed event-triggered tracking control of leader-follower multi-agent systems with communication delays. Kybernetika *47* (2011), 630–643.

[12] Z. Deng and Y. Hong: Distributed event-triggered optimization for multi-agent systems with disturbance rejection. In: 12th IEEE Int. Conf. Control and Autom., Kathmandu 2016, pp 13–18. DOI:10.1109/icca.2016.7505245

[13] P. Tabuada: Event-triggered real-time scheduling of stabilizing control tasks. IEEE Trans. Automat. Control *52* (2007), 1680–1685. DOI:10.1109/tac.2007.904277

[14] W. S. Chen and W. Ren: Event-triggered zero-gradient-sum distributed consensus optimization over directed networks. Automatica *65* (2016), 90–97. DOI:10.1016/j.automatica.2015.11.015

[15] Z. Deng, X. Wang, and Y. Hong: Distributed optimization design with triggers for disturbed continuous-time multi-agent systems. IET Control Theory Appl. *11* (2017), 2, 282–290. DOI:10.1049/iet-cta.2016.0795

[16] S. S. Kia, J. Cortes, and S. Martinez: Distributed convex optimization via continuous-time coordination algorithms with discrete-time communication. Automatica *55* (2015), 254–264. DOI:10.1016/j.automatica.2015.03.001

[17] H. Cai and J. Huang: Leader-following consensus of multiple uncertain Euler–Lagrange systems under switching network topology. Int. J. Gene. Sys., *43* (2014), 294–304. DOI:10.1080/03081079.2014.883714

[18] S. J. Chung and J. J. E. Slotine: Cooperative robot control and concurrent synchronization of lagrangian systems. IEEE Trans. Robotics *25* (2009), 686–700. DOI:10.1109/tro.2009.2014125

[19] W. E. Dixon: Nonlinear Control of Engineering Systems: A Lyapunov-Based Approach. Birkhäuser, Boston 2003.

[20] C. Y. Kim, D. Z. Song, Y. L. Xu, J. G. Yi and X. Y. Wu: Cooperative search of multiple unknown transient radio sources using multiple paired mobile robots. IEEE Trans. Rob. *30* (2014), 1161–1173. DOI:10.1109/tro.2014.2333097

[21] Z. Deng and Y. Hong: Multi-agent optimization design for autonomous lagrangian systems. Unmanned Systems *4* (2016), 5–13. DOI:10.1142/s230138501640001x

[22] M. Spong, S. Hutchinson, and M. Vidyasagar: Robot Modeling and Control. John Wiley and Sons, Hoboken 2006. DOI:10.1108/ir.2006.33.5.403.1

[23] Z. Meng, T. Yang, G. Shi, D. V. Dimarogonas, Y. Hong, and K. H. Johansson: Set target aggregation of multiple mechanical systems. In: IEEE 53rd Ann. Conf. Decision and Control (CDC), Los Angeles 2014, pp. 6830–6835. DOI:10.1109/cdc.2014.7040462

[24] R. Rockafellar: Convex Analysis. Princeton University Press, Princeton 1970. DOI:10.1017/s0013091500010142

[25] C. D. Godsil and G. Royle: Algebraic Graph Theory. Springer, New York 2001. DOI:10.1007/978-1-4613-0163-9

[26] W. Zhu and Z. P. Jiang: Event-based leader-following consensus of multi-agent systems with input time delay. IEEE Trans. Automat. Control *60* (2015), 1362–1367. DOI:10.1109/tac.2014.2357131

Xue-Fang Wang, the School of Control Science and Engineering, Dalian University of Technology, Dalian 116024. P. R. China.
e-mail: xfwang2015@mail.dlut.edu.cn

Zhenhua Deng, Key Laboratory of System and Control, Chinese Academy of Sciences, Beijing, China. He is the corresponding author of this paper.
e-mail: deng_dzh@163.com

Song Ma, AVIC Shenyang Aircraft Design and Research Institute, Shenyang 110035. P. R. China.
e-mail: masonghit@sina.com

Xian Du, the School of Control Science and Engineering, Dalian University of Technology, Dalian 116024. P. R. China.
e-mail: mydx.1@hotmail.com