Czechoslovak Mathematical Journal

Junfeng Liu

On invariant subspaces for polynomially bounded operators

*Czechoslovak Mathematical Journal*, Vol. 67 (2017), No. 1, 1–9

Persistent URL: http://dml.cz/dmlcz/146034

# ON INVARIANT SUBSPACES FOR POLYNOMIALLY BOUNDED OPERATORS

Junfeng Liu, Taipa

Received October 3, 2014. First published February 24, 2017.

*Abstract.* We discuss the invariant subspace problem of polynomially bounded operators on a Banach space and obtain an invariant subspace theorem for polynomially bounded operators. At the same time, we state two open problems, which are relative propositions of this invariant subspace theorem. By means of the two relative propositions (if they are true), together with the result of this paper and the result of C. Ambrozie and V. Müller (2004) one can obtain an important conclusion that every polynomially bounded operator on a Banach space whose spectrum contains the unit circle has a nontrivial invariant closed subspace. This conclusion can generalize remarkably the famous result that every contraction on a Hilbert space whose spectrum contains the unit circle has a nontrivial invariant closed subspace (1988 and 1997).

*Keywords*: polynomially bounded operator; invariant subspace

*MSC 2010*: 47A15

## 1. Introduction and preliminaries

In 1988, Brown, Chevreau and Pearcy in [3] proved that every contraction on a Hilbert space whose spectrum contains the unit circle has a nontrivial invariant closed subspace.

By the von Neumann inequality, every contraction on a Hilbert space is a polynomially bounded operator. Conversely, Pisier in [8] showed in 1997 that there are polynomially bounded operators on a Hilbert space that are not similar to a contraction. Thus one tries to generalize the result of Brown, Chevreau and Pearcy, see [3], to a polynomially bounded operator on a Banach space. To be more specific, a natural conjecture is as follows:

The research was partially supported by the Macao Science and Technology Development Fund (No. 083/2014/A2).

DOI: 10.21136/CMJ.2017.0459-14 

**Conjecture 1.** *Every polynomially bounded operator on a Banach space whose spectrum contains the unit circle has a nontrivial invariant closed subspace.*

In 2004, Ambrozie and Müller in [1] showed that every polynomially bounded operator of class $C_{0.}$ on a Banach space whose spectrum contains the unit circle has a nontrivial invariant closed subspace.

An operator $T$ on a Banach space $X$ is said to be polynomially bounded if there is a constant $k$ such that

$$\|p(T)\| \leqslant k\|p\|, \quad p \in P, \tag{1.1}$$

where $P$ denotes the normed space of all polynomials with the norm

$$\|p\| = \sup\{|p(z)|\colon\ z \in \mathbb{C},\ |z| \leqslant 1\}.$$

An operator $T$ on a Banach space $X$ is said to be a polynomially bounded operator of class $C_{0.}$ if $T$ is polynomially bounded and $\lim\limits_{n\to\infty} T^n x = 0$ for all $x \in X$. An operator $T$ on a Banach space $X$ is said to be a polynomially bounded operator of class $C_{.0}$ if $T$ is polynomially bounded and $\lim\limits_{n\to\infty} T^{*n}x^* = 0$ for all $x^* \in X^*$.

It is well known that there are many polynomially bounded operators of class $C_{.0}$ that are not polynomially bounded operators of class $C_{0.}$ (for example, the unilateral right shift operator).

In this paper, based on [1] we prove that every polynomially bounded operator of class $C_{.0}$ on a Banach space whose spectrum contains the unit circle has a nontrivial invariant closed subspace.

We first recall some basic notions and facts from [1] and others. For the notation and terminology not explained in the text we refer to [1], [9] and so on.

Let $\mathbb{D} = \{z\colon\ z \in \mathbb{C},\ |z| < 1\}$ be the open unit disc in the complex plane $\mathbb{C}$. Let us denote by $A(\mathbb{D})$ the disc algebra consisting of all functions continuous on $\overline{\mathbb{D}}$ and analytic on $\mathbb{D}$ with the norm $\|f\| = \sup\{|f(z)|\colon\ z \in \mathbb{D}\}$. Let $\mathbb{T} = \{z\colon\ z \in \mathbb{C},\ |z| = 1\}$ be the unit circle in the complex plane $\mathbb{C}$. Let $C(\mathbb{T})$ denote the Banach space of all continuous functions on $\mathbb{T}$ with the norm $\|f\|_{\mathbb{T}} = \sup\{|f(z)|\colon\ z \in \mathbb{C},\ |z| = 1\}$. By the maximum modulus principle, we have

$$\|f\| = \sup\{|f(z)|\colon\ z \in \mathbb{D}\} = \sup\{|f(z)|\colon\ z \in \mathbb{T}\} = \|f\|_{\mathbb{T}}$$

for each $f \in A(\mathbb{D})$, and hence $P$ and $A(\mathbb{D})$ can be regarded as subspaces of $C(\mathbb{T})$. By the Hahn-Banach theorem, every $\varphi \in P^*$ can be extended without changing the norm to a functional on $C(\mathbb{T})$, which is still denoted by the same symbol $\varphi$. By the

Riesz theorem, there is a complex-valued regular Borel measure $\mu$ on $\mathbb{T}$ such that $\|\mu\| = \|\varphi\|$, and

$$\varphi(f) = \int_{\mathbb{T}} f \, \mathrm{d}\mu, \quad f \in A(\mathbb{D}). \tag{1.2}$$

Let $L^1(\mathbb{T})$ be the Banach space of all complex integrable functions on $\mathbb{T}$ with norm $\|f\|_1 = (2\pi)^{-1} \int_{-\pi}^{\pi} |f(\mathrm{e}^{\mathrm{i}t})| \, \mathrm{d}t$. For every $h \in L^1(\mathbb{T})$, define a functional $M_h$ on $A(\mathbb{D})$ by

$$M_h(f) = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(\mathrm{e}^{\mathrm{i}t}) h(\mathrm{e}^{\mathrm{i}t}) \, \mathrm{d}t, \quad f \in A(\mathbb{D}).$$

Then $M_h$ is a bounded linear functional on $A(\mathbb{D})$, and $\|M_h\| \leqslant \|h\|_1$. By [1], page 337, if $h = 1$ then $M_1(p) = p(0)$ for every $p \in P$, and so it is easy to see that for every $f \in A(\mathbb{D})$, we have

$$M_1(f) = f(0). \tag{1.3}$$

From now on, by $T$ we denote a polynomially bounded operator, and $k$ is as in (1.1). It is well known that for every $f \in A(\mathbb{D})$, there is a sequence of polynomials $\{p_n(z)\}$ such that $\|p_n - f\| \to 0$, $n \to \infty$. Thus we have

$$\|p_n(T)x - p_m(T)x\| \leqslant k\|p_n - p_m\|\|x\| \to 0, \quad n \to \infty, \ m \to \infty$$

for each $x \in X$, which implies $\{p_n(T)x\}$ is a convergent sequence in $X$. Define an operator $f(T)\colon X \to X$ by

$$f(T)x = \lim_{n \to \infty} p_n(T)x, \quad x \in X. \tag{1.4}$$

It is easy to see that the definition of $f(T)$ does not depend on the particular choice of $\{p_n\}$, and $f(T)$ is a linear operator on $X$. Moreover, we have

$$\|f(T)x\| = \lim_{n \to \infty} \|p_n(T)x\| \leqslant \lim_{n \to \infty} k\|p_n\| \, \|x\| = k\|f\| \, \|x\|$$

for every $x \in X$, and so

$$\|f(T)\| \leqslant k\|f\|, \quad f \in A(\mathbb{D}). \tag{1.5}$$

For every $x \in X$, $x^* \in X^*$, define a functional $x \otimes x^*$ on $P$ by

$$(x \otimes x^*)(p) = \langle p(T)x, x^* \rangle, \quad p \in P.$$

Then $x \otimes x^*$ is a bounded linear functional on $P$, and $\|x \otimes x^*\| \leqslant k\|x\|\|x^*\|$.

## 2. Main results

**Lemma 1** (parallel with Lemma 7.1 in [1] for the class $C_{0.}$). *Let $T$ be a polynomially bounded operator of class $C_{.0}$ on a Banach space $X$. If $x \in X$, $x^* \in X^*$, then there is $h \in L^1(\mathbb{T})$ such that*

$$(x \otimes x^* - M_1)(p) = \frac{1}{2\pi}\int_{-\pi}^{\pi} p(\mathrm{e}^{\mathrm{i}t})h(\mathrm{e}^{\mathrm{i}t})\,\mathrm{d}t, \quad p \in P,$$

*and $\|h\|_1 = \|x \otimes x^* - M_1\|$.*

Proof. Since $x\otimes x^* - M_1$ is a bounded linear functional on $P$, it follows from (1.2) that there is a Borel measure $\mu$ on $\mathbb{T}$ such that $\|\mu\| = \|x \otimes x^* - M_1\|$, and

$$(x \otimes x^* - M_1)(f) = \int_{\mathbb{T}} f\,\mathrm{d}\mu, \quad f \in A(\mathbb{D}). \tag{2.1}$$

Since $x \otimes x^* \in P^*$, it follows that $x \otimes x^*$ can be extended without changing the norm to a functional on $C(\mathbb{T})$, which is still denoted by the same symbol $x \otimes x^*$. On the other hand, it follows from (1.4) that for every $f \in A(\mathbb{D})$ there exists $p_n \in P$ such that $\|p_n - f\| \to 0$, $n \to \infty$, $\lim\limits_{n\to\infty} p_n(T)x = f(T)x$. Therefore we have

$$x \otimes x^*(f) = \lim_{m\to\infty} x \otimes x^*(p_m) = \lim_{m\to\infty}\langle p_m(T)x, x^*\rangle = \langle f(T)x, x^*\rangle. \tag{2.2}$$

Let $\{f_n\}$ be a Montel sequence in $A(\mathbb{D})$, that is, $f_n \in A(\mathbb{D})$, $\sup_n \|f_n\| < \infty$, and $\lim\limits_{n\to\infty} f_n(z) = 0$ for all $z \in \mathbb{D}$. We now show that for every $\varepsilon \in (0, 2k)$ there exists $n_0$ such that $|\langle f_n(T)x,\, x^*\rangle| < \varepsilon$ for all $n \geqslant n_0$. Assume without loss of generality that $\|f_n\| \leqslant 1$, $\|x\| \leqslant 1$, $\|x^*\| \leqslant 1$.

Since $T$ is a polynomially bounded operator of class $C_{.0}$ on $X$ it follows that $\lim\limits_{n\to\infty} T^{*n}u^* = 0$ for all $u^* \in X^*$, so that there exists a positive integer $m$ such that $\|T^{*m}x^*\| < \varepsilon/(4k)$.

Let $f_n(z) = \sum\limits_{j=0}^{\infty} c_{n,j}z^j$ be the Taylor expansion of $f_n$. By the Cauchy formula and the Lebesgue domination theorem we have $c_{n,j} \to 0$, $n \to \infty$, for every $j$. Hence there exists $n_0$ such that for every $n \geqslant n_0$ we have $|c_{n,j|} < \varepsilon/(2mk)$, $j = 0, 1, 2, \ldots, m$. For every $n \geqslant n_0$, write $p_n(z) = \sum\limits_{j=0}^{m-1} c_{n,j}z^j$, then there exists $g_n \in A(D)$ such that $f_n(z) = p_n(z) + z^n g_n(z)$. Consequently $\|p_n\| \leqslant \sum\limits_{j=0}^{m-1} |c_{n,j}| < \varepsilon/(2k)$ and $\|g_n\| =$

$\|f_n - p_n\|$. Thus by (1.5) we have

$$\begin{aligned}|\langle f_n(T)x, x^*\rangle| &= |\langle x, [f_n(T)]^*x^*\rangle| \leqslant \|[f_n(T)]^*x^*\|\\ &\leqslant \|[p_n(T)]^*x^*\| + \|[g_n(T)]^*\|\|T^{*m}x^*\|\\ &\leqslant \|[p_n(T)]^*\| + \|g_n(T)\|\|T^{*m}x^*\| = \|[p_n(T)]\| + \|g_n(T)\|\|T^{*m}x^*\|\\ &\leqslant k\|p_n\| + k\|f_n - p_n\|\,\frac{\varepsilon}{4k}\\ &\leqslant k\|p_n\| + k(\|f_n\| + \|p_n\|)\frac{\varepsilon}{4k} < \varepsilon,\end{aligned}$$

from which and (2.2) we obtain

$$(x \otimes x^*)(f_n) = \langle f_n(T)x, x^*\rangle \to 0, \quad n \to \infty. \tag{2.3}$$

On the other hand, it follows from (1.3) and the definition of the Montel sequences that

$$M_1(f_n) = f_n(0) \to 0, \quad n \to \infty.$$

Thus by (2.1) and (2.3) we have

$$\int_{\mathbb{T}} f_n \,\mathrm{d}\mu = (x \otimes x^* - M_1)(f_n) \to 0, \quad n \to \infty.$$

Therefore $\mu$ is a Henkin measure. By [9], page 189, Remark 9.2.2 (c), $\mu$ is absolutely continuous with respect to the Lebesgue measure on $\mathbb{T}$. It follows from (2.1) and the Radon-Nikodym theorem that there exists $h \in L^1(\mathbb{T})$ such that for all $p \in P$ we have

$$(x \otimes x^* - M_1)(p) = \int_{\mathbb{T}} p \,\mathrm{d}\mu = \frac{1}{2\pi}\int_{-\pi}^{\pi} p(\mathrm{e}^{\mathrm{i}t})h(\mathrm{e}^{\mathrm{i}t})\,\mathrm{d}t,$$

and

$$\|x \otimes x^* - M_1\| = \|\mu\| = |\mu|(\mathbb{T}) = \frac{1}{2\pi}\int_{-\pi}^{\pi} |h(\mathrm{e}^{\mathrm{i}t})|\,\mathrm{d}t = \|h\|_1.$$

□

**Lemma 2.** *Let $T$ be a polynomially bounded operator of class $C_{.0}$ on a Banach space $X$. Suppose that $\sigma(T) \supset \mathbb{T}$ and that $T$ has no nontrivial invariant closed subspace. Let $h \in L^1(\mathbb{T})$ be nonnegative. If $w \in X$, $w^* \in X^*$, $\delta > 0$, then there exist vectors $u \in X$, $u^* \in X^*$ such that*

(1) $\|u\| \leqslant 2\sqrt{2}\,kb\|h\|_1^{1/2}$, $\|u^*\| \leqslant k\|h\|_1^{1/2}$;

(2) $\|w \otimes u^*\| < \delta$;

(3) $\|u \otimes (u^* + w^*) - M_h\| < c_3\|h\|_1$, *where $b > 0$, $c_3 \in (0, 1)$ are constants in Theorem 7.2 of* [1].

P r o o f. Assume without loss of generality that $\|h\|_1 \neq 0$. By Theorem 7.2 in [1] applied to the function $\|h\|_1^{-1}\|h$ and the functional $\|h\|_1^{-1/2}w^*$, there are vectors $v \in X$ and $v^* \in X^*$ such that $\|v\| \leqslant 2\sqrt{2}\,kb$, $\|v^*\| \leqslant 1$, and

$$\|v \otimes (T^{*n}v^* + \|h\|_1^{-1/2}\omega^*) - M_{\|h\|_1^{-1}h}\| < c_3. \tag{2.4}$$

Set $u = \|h\|_1^{1/2}v$, $u^* = \|h\|_1^{1/2}T^{*n}v^*$. So we have $\|u\| \leqslant 2\sqrt{2}\,kb\|h\|_1^{1/2}$, $\|u^*\| \leqslant k\|h\|_1^{1/2}$, and the estimate

$$\|w \otimes u^*\| = \|h\|_1^{1/2}\|w \otimes T^{*n}v^*\| \leqslant k\|w\|\|T^{*n}v^*\|\|h\|_1^{1/2} < \delta$$

holds if $n$ is large enough. Moreover, by (2.4) we have

$$\|u \otimes (u^* + w^*) - M_h\| = \|h\|_1\,\|v \otimes (T^{*n}v^* + \|h\|_1^{-1/2}w^*) - M_{\|h\|_1^{-1}h}\| < c_3\|h\|_1.$$

$\square$

Fix an integer $N$ such that $c_3 + \pi N^{-1} < 1$, and a positive constant $c$ such that $1 - N^{-1}(1 - c_3 - \pi N^{-1}) < c < 1$.

**Lemma 3.** *Let $T$ be a polynomially bounded operator of class $C_{.0}$ on a Banach space $X$. Suppose that $\sigma(T) \supset \mathbb{T}$ and that $T$ has no nontrivial invariant closed subspace. If $h \in L^1(\mathbb{T})$, $x \in X$, $x^* \in X^*$, then there exist vectors $y \in X$, $y^* \in X^*$ such that*
(1) $\|y - x\| \leqslant 2\sqrt{2}\,kb\|h\|_1^{1/2}$;
(2) $\|y^* - x^*\| \leqslant k\|h\|_1^{1/2}$;
(3) $\|y \otimes y^* - x \otimes x^* - M_h\| < c\|h\|_1$,
*where $b > 0$ and $c \in (0,1)$ are the constants above.*

P r o o f. From Lemma 2, we derive Lemma 3 as in Theorem 7.4 of [1]. $\square$

**Theorem 1.** *Let $T$ be a polynomially bounded operator of class $C_{.0}$ on a Banach space $X$. If $\sigma(T) \supset \mathbb{T}$, then $T$ has a nontrivial invariant closed subspace.*

P r o o f. Using Lemma 1 and Lemma 3, one can prove Theorem 1 as in the proof of Theorem B of [1]. For the convenience of the reader we state the main ideas. Let $b > 0$ and $c \in (0,1)$ be constants in Lemma 3. Assume that $T$ has no nontrivial invariant closed subspace. Take $x_0 = 0$, $x_0^* = 0$. Then $\|x_0 \otimes x_0^* - M_1\| = 1 = c^0$. By induction, assume that we have chosen vectors $x_n \in X$, $x_n^* \in X^*$ such that $\|x_n \otimes x_n^* - M_1\| < c^n$. By Lemma 1, there is a vector $h_n \in L^1(\mathbb{T})$ such that $\|h_n\|_1 = \|x_n \otimes x_n^* - M_1\| < c^n$, and

$$(x_n \otimes x_n^* - M_1)(p) = \frac{1}{2\pi}\int_{-\pi}^{\pi} p(\mathrm{e}^{\mathrm{i}t})h_n(\mathrm{e}^{\mathrm{i}t})\,\mathrm{d}t, \quad p \in P. \tag{2.5}$$

By Lemma 3, there are vectors $x_{n+1} \in X$, $x^*_{n+1} \in X^*$ such that

$$\|x_{n+1} - x_n\| \leqslant 2\sqrt{2}\, kb\|h_n\|_1^{1/2} \leqslant 2\sqrt{2}\, kbc^{n/2}, \tag{2.6}$$

$$\|x^*_{n+1} - x^*_n\| \leqslant k\|h_n\|_1^{1/2} \leqslant\ kc^{n/2}, \tag{2.7}$$

and

$$\|x_{n+1} \otimes x^*_{n+1} - x_n \otimes x^*_n - M_{-h_n}\| \leqslant c\|h_n\|_1 \leqslant c^{n+1}.$$

Thus by (2.5) and the definition of $M_{h_n}$ we obtain

$$\begin{aligned}\|x_{n+1} &\otimes x^*_{n+1} - M_1\| \\ &= \|(x_{n+1} \otimes x^*_{n+1} - x_n \otimes x^*_n) + (x_n \otimes x^*_n - M_1)\| \\ &= \|x_{n+1} \otimes x^*_{n+1} - x_n \otimes x^*_n + M_{h_n}\| < c^{n+1} \to 0, \quad n \to \infty.\end{aligned} \tag{2.8}$$

Moreover, by (2.6) and (2.7) it follows that $\{x_n\}$ and $\{x^*_n\}$ are Cauchy sequences in $X$ and $X^*$, respectively. Suppose that $x_n \to x$, $x^*_n \to x^*$, $n \to 0$, then we have

$$\begin{aligned}&\|x_n \otimes x^*_n - x \otimes x^*\| \\ &\quad = \sup\{\|x_n \otimes x^*_n(p) - x \otimes x^*(p)\|;\ \|p\| \leqslant 1\} \\ &\quad \leqslant \sup\{|\langle p(T)x_n, x^*_n\rangle - \langle p(T)x_n, x^*\rangle| + |\langle p(T)x_n, x^*\rangle - \langle p(T)x, x^*\rangle|;\ \|p\| \leqslant 1\} \\ &\quad \leqslant \sup\{k\|p\|(\|x_n\|\|x^*_n - x^*\| + \|x_n - x\|\|x^*\|);\ \|p\| \leqslant 1\} \to 0, \quad n \to \infty.\end{aligned}$$

Thus by (2.8) we can obtain $x \otimes x^* = M_1$. This shows that $\langle x, x^*\rangle = 1$, $\langle T^n x, x^*\rangle = 0$, $n = 1, 2, \ldots$ Therefore we have $x \neq 0$, $x^* \neq 0$, and $T^n x \in \ker x^*$, $n = 1, 2, \ldots$ If $Tx = 0$, then $\ker T$ is a nontrivial invariant closed subspace for $T$. If $Tx \neq 0$, then $M = \overline{\operatorname{span}}\{Tx, T^2x, \ldots, T^n x, \ldots\}$ ($\subset \ker x^*$) is a nontrivial invariant closed subspace for $T$. $\square$

## 3. Some remarks

**Remark 1.** If $T$ is a polynomially bounded operator (of class $C_{\cdot 0}$) on a Banach space such that $\sigma(T) \supset \mathbb{T}$, then it follows from Theorem $A$ of [1] that $T^*$ has a nontrivial invariant closed subspace (see also [1], page 344), but it is well known that it is impossible for the conclusion of Theorem 1 to follow from Theorem A of [1] (see [1], [2], [4], [5], [6], [7] and so on).

In fact, it is well known there are much differences between the properties of invariant subspaces of an operator $A$ and its adjoint operator $A^*$. For example, if $M$ is a nontrivial invariant closed subspace of an operator $A$, then $M^\perp$ is a nontrivial invariant closed subspace of $A^*$; conversely, if $N$ is a nontrivial invariant closed subspace of $A^*$, then ${}^\perp N$ is an invariant closed subspace of $A$, but ${}^\perp N$ may be trivial (if $N$ is $w^*$-dense).

In particular, Jiang in [5] gave an example on a Banach space in which the operator $A^*$ has a nontrivial invariant closed subspace, but A has no nontrivial invariant closed subspace.

**Remark 2.** Let $T$ be a polynomially bounded operator on a Banach space $X$. Set $M = \{x \in X \colon \lim_{n\to\infty} T^n x = 0\}$, $N = \{x^* \in X^* \colon \lim_{n\to\infty} T^{*n} x^* = 0\}$, then $M$ and ${}^\perp N$ are invariant closed subspaces for $T$. Moreover, if $\{0\} \neq M \neq X$, it is clear that $T$ has a nontrivial invariant closed subspace.

Ambrozie and Müller tried to show in [1] that every polynomially bounded operator $T$ on a Banach space whose spectrum contains the unit circle has a nontrivial invariant closed subspace (i.e. Conjecture 1). As pointed in [1], one can reduce the invariant subspace problem of (general) polynomially bounded operators (whose spectrum contains the unit circle) in a standard way. To be more specific, to prove Conjecture 1 it suffices to show the following propositions:

**Proposition 1.** *When $M = X$, $T$ has a nontrivial invariant closed subspace.*

**Proposition 2.** *When $N = X^*$, $T$ has a nontrivial invariant closed subspace.*

**Proposition 3.** *When $\{0\} \neq N \neq X^*$, $T$ has a nontrivial invariant closed subspace.*

**Proposition 4.** *When $M = \{0\}$ and $N = \{0\}$, $T$ has a nontrivial invariant closed subspace.*

In [1], Ambrozie and Müller proved Proposition 1 (it is the main result of [1]). In this paper, we proved Proposition 2. But Proposition 3 and Proposition 4 remain open so far.

By the way, in Proposition 3 it is clear that ${}^\perp N \neq \{0\}$. To prove Proposition 3 it therefore suffices to prove ${}^\perp N \neq X$. On Proposition 4, Ambrozie and Müller [1] proved that $T^*$ has a nontrivial invariant closed subspace when $X$ is a (general) Banach space (see also [4], Theorem 4.2.9), therefore $T$ has a nontrivial invariant closed subspace when $X$ is a reflexive Banach space. But we need to prove in Proposition 4 that $T$ has a nontrivial invariant closed subspace when $X$ is a (general) Banach space.

**Acknowledgement.** The authors wish to thank Professor M. Liu for helpful discussions.

*References*

[1] *C. Ambrozie, V. Müller*: Invariant subspaces for polynomially bounded operators. J. Funct. Anal. *213* (2004), 321–345. zbl MR doi
[2] *B. Beauzamy*: Introduction to Operator Theory and Invariant Subspaces. North-Holland Mathematical Library 42, North-Holland, Amsterdam, 1988. zbl MR
[3] *S. W. Brown, B. Chevreau, C. Pearcy*: On the structure of contraction operators. II. J. Funct. Anal. *76* (1988), 30–55. zbl MR doi
[4] *I. Chalendar, J. R. Partington*: Modern Approaches to the Invariant-Subspace Problem. Cambridge Tracts in Mathematics 188, Cambridge University Press, Cambridge, 2011. zbl MR
[5] *J. Jiang*: Bounded Operators without Invariant Subspaces on Certain Banach Spaces. Thesis (Ph.D.), The University of Texas at Austin, ProQuest LLC, Ann Arbor, 2001. MR
[6] *K. B. Laursen, M. M. Neumann*: An Introduction to Local Spectral Theory. London Mathematical Society Monographs. New Series 20, Clarendon Press, Oxford, 2000. zbl MR
[7] *V. Lomonosov*: An extension of Burnside's theorem to infinite-dimensional spaces. Isr. J. Math. *75* (1991), 329–339. zbl MR doi
[8] *G. Pisier*: A polynomially bounded operator on Hilbert space which is not similar to a contraction. J. Am. Math. Soc. *10* (1997), 351–369. zbl MR doi
[9] *W. Rudin*: Function Theory in the Unit Ball of $C^n$. Grundlehren der mathematischen Wissenschaften 241, Springer, Berlin, 1980. zbl MR doi

*Author's address*: Junfeng Liu, Faculty of Information Technology, Macau University of Science and Technology, Avenida Wai Long, Taipa, Macau 999078, P. R. China, e-mail: `jfliu997@163.com`.