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FINITE-TIME ADAPTIVE OUTER SYNCHRONIZATION BETWEEN TWO COMPLEX DYNAMICAL NETWORKS WITH NONIDENTICAL TOPOLOGICAL STRUCTURES

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In this paper, we investigate the finite-time adaptive outer synchronization between two complex dynamical networks with nonidentical topological structures. We propose new adaptive controllers, with which we can synchronize two complex dynamical networks within finite time. Sufficient conditions for the finite-time adaptive outer synchronization are derived based on the finite-time stability theory. Finally, numerical examples are examined to demonstrate the effectiveness and feasibility of the theoretical results.

Keywords: complex networks, outer synchronization, finite-time, adaptive feedback controllers

Classification: 34D06, 05C82

1. INTRODUCTION

Recently, complex networks have been investigated across many fields of the real world, such as Internet, traffic networks, biological networks [18], population evolution [27], communication networks, electrical power grids, neural networks [17, 30], neighborhood relationship [26], World Wide Web [9], and so on. Normally, a dynamical complex network is made of a large set of interconnected nodes in which a node is a fundamental unit with specific contents, and the edges connecting the nodes represent the interactions among the individual unit. Because the ways of connection are different, and whether there are weights or not between these nodes, we can get many different types of complex networks, such as directed weighted network, directed unweighted network, etc.

However, the complex networks' structure performs the network dynamical behaviors and results in considerable important research problems. Particularly, one interesting and significant investigation is the synchronization of complex dynamical networks, which not only can explain many real natural phenomena, but also has many potential applications in secure communication, information processing, biological system, control processing, chemical reactions, etc. [1, 4, 5, 14, 32, 34, 37].

In the past few years, various cases of synchronization in complex networks have been extensively studied. In Ref. [15], Lü and Chen introduced a time-varying complex dynamical network model and studied its controlled synchronization criteria. In Ref. [12],

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the authors investigated the synchronization in general complex dynamical networks with coupling delays. The cluster synchronization under the influence of advection was investigated in [7]. The synchronization investigated in the above references is called “inner synchronization”, which is concerned with the synchronization among the nodes within one network. Compared with the inner synchronization, the synchronization between two or more complex networks regardless of the inner network is called “outer synchronization”, which exists in our daily lives. In Ref. [13], the authors firstly explored the “outer synchronization”. Later on, various patterns of outer synchronization have been studied [19, 20, 22, 24, 25, 31]. In Ref. [31], the authors investigated the outer synchronization between drive-response networks with nonidentical nodes and unknown parameters. The impulsive synchronization of a nonlinear coupled complex network with a delay node was explored in [19]. In Ref. [20], the authors discussed the generalized outer synchronization between two uncertain dynamical networks. In Ref. [22], the generalized outer synchronization between two complex dynamical networks with time delay and noise perturbation was investigated. Lag synchronization and mixed outer synchronization were investigated in Refs. [24, 25].

It is worth noting that most of previous works on network synchronization focused on the asymptotical synchronization, which meant that synchronization of complex dynamical networks can not occur in a finite time. However, in the actual information networks, complex ecological networks, and many others, it is often necessary to achieve synchronization within a limited period of time, which is called the finite-time synchronization. Due to its significant applications in many areas, the finite-time synchronization of complex dynamical networks have received considerable attention among many researchers in recent years [16, 21, 23, 33]. In Ref. [16], the finite-time synchronization between two complex networks with delayed coupling was analyzed by using the impulsive and periodically intermittent control. In Refs. [21, 23], the finite-time stochastic outer synchronization between two different complex dynamical networks was investigated. In Ref. [33], the finite-time synchronization of complex networks with complex-variable chaotic systems was explored. As we all known, a focused problem in the study of finite-time synchronization is how to design a physically available and simple controller to guarantee the realization of the synchronization. Most of finite-time controllers usually contain a linear coupling part [35, 36]. However, it is very difficult to find the suitable coupling constant. Fortunately, the adaptive feedback control method can solve the problem perfectly [8], and this technique has been profoundly reflected in this article.

In this paper, inspired by the above analyses, we investigate the finite-time adaptive outer synchronization between two complex dynamical networks with nonidentical topological structures. The main contribution of this paper is to propose adaptive feedback controllers which are constructed combining the finite-time control and adaptive control methods. Using the finite-time adaptive controllers, we can synchronize two different complex dynamical networks in finite time and do not need to give the exact value of the coupling strength. Based on the finite-time stability theory, analytical sufficient conditions for the finite-time outer synchronization are derived. Finally, two numerical examples are examined to illustrate the usefulness and effectiveness of the theoretical results. Especially, the simulation results for the networks with small-world topologies show that the small-word networks with large average degree has a faster convergence rate.

The rest of this paper is outlined as follows. Problem statement and preliminaries are given in Section 2. In Section 3, base on the finite-time stability theory, sufficient conditions for the finite-time outer synchronization are derived. In Section 4, two numerical simulations are given to show the usefulness and effectiveness of the theoretical results. Finally, some conclusions are given in Section 5.

2. PROBLEM STATEMENT AND PRELIMINARIES

Consider a dynamical network consisting of N linear coupled nodes, which can be described as follows:

$$\dot{x}_i(t) = f(x_i(t)) + \sum_{j=1}^{N} c_{ij}\Gamma x_j,\ i = 1, 2, \ldots, N, \tag{1}$$

where $x_i(t) = (x_{i1}, \ldots, x_{in})^T \in R^n$ is the state vector of the ith node, $f : R \times R^n \to R^n$ is a smooth nonlinear vector function. Γ is a constant matrix linking the coupled variables and $C = (c_{ij})_{N\times N}$ is the coupling configuration matrix of the complex network. The entries c_{ij} are defined as follows: if there exists a link from node j to node i $(i \neq j)$ then we set $c_{ij} > 0$, otherwise we set $c_{ij} = 0 (i \neq j)$, and $c_{ii} = -\sum_{j=1, j\neq i}^{N} c_{ij},\ i = 1, 2, \ldots, N$.

To investigate the finite-time outer synchronization between two complex networks with nonidentical topological structures, we take the above network (1) as the driving network, and the following model with an adaptive control scheme as the response network, which is given by

$$\dot{y}_i(t) = f(y_i(t)) + \sum_{j=1}^{N} d_{ij}\Gamma y_j + u_i(t),\ i = 1, 2, \ldots, N, \tag{2}$$

where $y_i(t) = (y_{i1}, \ldots, y_{in})^T \in R^n$ is the state vector of the ith node, D is the coupling configuration matrix of network (2), $e_i(t) = y_i(t) - x_i(t) (i = 1, 2, \ldots, N)$ are the synchronization errors between the driving network (1) and the response network (2). Adaptive controllers u_i are designed as follows:

$$u_i = \begin{cases} -\varepsilon_i e_i - k[\mathrm{sign}(e_i) + \frac{|\varepsilon_i - \xi| e_i}{\|e\|^2}] + \sum_{j=1}^{N}(c_{ij} - d_{ij})\Gamma x_j(t), & \text{if } \|e_i\| \neq 0; \\ 0, & \text{if } \|e_i\| = 0, \end{cases} \tag{3}$$

where the constants $k, \xi > 0$, the adaptive law of ε_i is $\dot{\varepsilon}_i(t) = e_i^T(t)\, e_i(t)$.

Remark 2.1. The first term of controllers $u_i(t)$ in Eq. (3) is based on the adaptive control method. The adaptive control method has been extensively used to investigated inner synchronization problems of complex networks. For example, the adaptive synchronization of an uncertain complex dynamical network was investigated in Ref. [38]. The pinning adaptive synchronization of a general complex dynamical network was discussed in Ref. [39]. However, the first term of controllers $u_i(t)$ can only ensure the infinite-time synchronization of complex networks. The second term of $u_i(t)$ is based on the finite-time control technology which can synchronize networks in finite time. The previous finite-time controllers usually contain a linear coupling part [35, 36]. And it is very difficult to find the suitable coupling constant. Here, we solve the difficulty by combining the adaptive control and finite-time control methods. The last term of $u_i(t)$ is due to the structure difference between the networks (1) and (2), which is not necessary when networks (1) and (2) have the same structures.

Remark 2.2. Throughout this paper, the configuration matrices C and D of the driving network (1) and the response network (2) are not necessary to be symmetric or irreducible. Namely, driving network (1) and response network (2) can be directed or not, and they may also have isolated nodes and clusters.

For getting our main results in the next section, we state here an assumption on nonlinear function $f(x)$, a definition of finite-time outer synchronization and a necessary lemma.

Assumption 2.3. For nonlinear function $f(x)$, there exists a constant $l > 0$ such that

$$[x(t) - y(t)]^T [f(x(t)) - f(y(t))] \leq [x(t) - y(t)]^T l [x(t) - y(t)], \forall x, y \in R^n. \tag{4}$$

Definition 2.4. The driving network (1) and response network (2) are said to achieve outer synchronization in finite time. If, for arbitrary solutions of networks (1) and (2) denoted by $x_i(t) = (x_{i1}, \ldots, x_{in})^T$ and $y_i(t) = (y_{i1}, \ldots, y_{in})^T$ with different initial states $x_i(0), y_i(0)$, there exists a finite time function $T_0 > 0$, such that

$$\lim_{t \to T_0} ||x_i(t, x_i(0)) - y_i(t, y_i(0))|| = 0,$$

and $T_0 = \inf\{T : ||x_i(t, x_i(0)) - y_i(t, y_i(0))|| \equiv 0, \ \forall t \geq T, \ i = 1, 2, \ldots, n\}$ is called settling time.

Lemma 2.5. (Vincent and Guo [28]) Assume that a continuous, positive-definite function $V(t)$ satisfies the following differential inequality:

$$\dot{V}(t) \leq -\rho V^{\eta}(t), \quad \forall t \geq t_0, \quad V(t_0) \geq 0,$$

where $\rho > 0, 0 < \eta < 1$ are all constants. Then, for any given t_0, $V(t)$ satisfies the following inequality:

$$V^{1-\eta}(t) \leq V^{1-\eta}(t_0) - \lambda(1 - \eta)(t - t_0), t_0 \leq t \leq t_1,$$

and

$$V(t) = 0, \quad \forall t \geq t_1,$$

with t_1 given by

$$t_1 = t_0 + \frac{V^{1-\eta}(t_0)}{\rho(1 - \eta)}.$$

Lemma 2.6. (M. P. Aghababa and H. P. Aghababa [2, 3]) For $x_1, x_2, \ldots, x_n \in R$, the following inequality holds:

$$|x_1| + |x_2| + \cdots + |x_n| \geq \sqrt{x_1^2 + x_2^2 + \cdots + x_n^2}.$$

3. MAIN RESULTS

In this section, we will focus on the sufficient conditions for the finite-time adaptive outer synchronization between networks (1) and (2), and the main results are summarized in the following theorem.

Theorem 3.1. Suppose that Assumption 2.3 holds and there exists a sufficiently large positive constant ξ such that $\xi > l + \lambda_{\max}(\mathcal{D}^s)$, where $\mathcal{D} = D \otimes \Gamma, \mathcal{D}^s = \frac{\mathcal{D}+\mathcal{D}^T}{2}$. Then, under the set of controllers (3), networks (1) and (2) can reach finite-time outer synchronization.

P r o o f. From networks (1) and (2), the error system can be described by

$$\dot{e}_i(t) = f(y_i) - f(x_i) + \sum_{j=1}^{N} d_{ij}\Gamma e_j(t) - \varepsilon_i e_i - k\Big[\mathrm{sign}(e_i) + \frac{|\varepsilon_i - \xi| e_i}{\|e\|^2}\Big], \; i = 1, 2, \ldots, N. \quad (5)$$

Thus, according to the Assumption 2.3 the above error system (5) for any initial data $e_i(0) = y_i(0) - x_i(0)$ possesses a unique global solution $e_i(t, e_i(0))$ on $t \geq 0$, and $e_i(t, 0) \equiv 0$ is a trivial solution of the error dynamics (5). Apparently, outer synchronization between networks (1) and (2) could be realized in finite time if this trivial solution is finite-time stable.

Let $e(t) = (e_1^T(t), e_2^T(t), \ldots, e_N^T(t))^T$, and take the following Lyapunov function:

$$V = \frac{1}{2}\sum_{i=1}^{N} e_i^T(t)\, e_i(t) + \frac{1}{2}\sum_{i=1}^{N}(\varepsilon_i - \xi)^2.$$

Then, we get the derivative of V along the error system (5) gives

$$\begin{aligned} \dot{V} \;=\; & \sum_{i=1}^{N} e_i^T(t)\Big\{f(y_i) - f(x_i) + \sum_{j=1}^{N} d_{ij}\Gamma e_j(t) - \varepsilon_i e_i - k\Big[\mathrm{sign}(e_i) + \frac{|\varepsilon_i - \xi| e_i}{\|e\|^2}\Big]\Big\} \\ & + \sum_{i=1}^{N}(\varepsilon_i - \xi)\, e_i^T(t)\, e_i(t). \end{aligned} \quad (6)$$

From the Assumption 2.3, we obtain

$$\begin{aligned} \dot{V} \;\leq\; & l\sum_{i=1}^{N} e_i^T(t)\, e_i(t) + \sum_{i=1}^{N} e_i^T(t)\sum_{j=1}^{N} d_{ij}\Gamma e_j(t) - \sum_{i=1}^{N} e_i^T(t)\varepsilon_i e_i(t) - k\Big[\sum_{i=1}^{N} e_i^T(t)\mathrm{sign}(e_i) \\ & + \sum_{i=1}^{N} e_i^T(t)\frac{|\varepsilon_i - \xi| e_i}{\|e\|^2}\Big] + \sum_{i=1}^{N}(\varepsilon_i - \xi)\, e_i^T(t)\, e_i(t). \end{aligned}$$

Since $\sum\limits_{i=1}^{N} e_i^T(t)\frac{e_i}{\|e\|^2} = 1$ is always satisfied, one has

$$\dot{V} \;\leq\; (l - \xi)\sum_{i=1}^{N} e_i^T(t)\, e_i(t) + \sum_{i=1}^{N} e_i^T(t)\sum_{j=1}^{N} d_{ij}\Gamma e_j(t) - k\Big(\sum_{i=1}^{N}|e_i| + \sum_{i=1}^{N}|\varepsilon_i - \xi|\Big).$$

Noting that

$$\begin{aligned}\sum_{i=1}^{N} e_i^T(t)\sum_{j=1}^{N} d_{ij}\Gamma e_j(t) &= e^T(t)D\otimes\Gamma e(t)\\ &\leq \lambda_{\max}(\mathcal{D}^s)\, e^T(t)\, e(t),\end{aligned} \tag{7}$$

we obtain

$$\dot{V} \leq (l-\xi+\lambda_{\max}(\mathcal{D}^s))\sum_{i=1}^{N} e_i^T(t)\, e_i(t) - k\Big(\sum_{i=1}^{N}|e_i| + \sum_{i=1}^{N}|\varepsilon_i-\xi|\Big).$$

If

$$\xi > l + \lambda_{\max}(\mathcal{D}^s), \tag{8}$$

then we get

$$\dot{V} \leq -k\Big(\sum_{i=1}^{N}|e_i| + \sum_{i=1}^{N}|\varepsilon_i-\xi|\Big).$$

By Lemma 2.6, we have

$$\begin{aligned}\dot{V} &\leq -k\Big[\sum_{i=1}^{N} e_i^T(t)\, e_i(t) + \sum_{i=1}^{N}(\varepsilon_i-\xi)^2\Big]^{\frac{1}{2}}\\ &= -k(2V)^{\frac{1}{2}}.\end{aligned}$$

According to the Lemma 2.5, the trivial solution of the error system (5) is finite-time stable. Therefore, for any arbitrary initial conditions, networks (1) and (2) can realize finite-time outer synchronization, and it's easy to obtain an estimation of the settling time

$$t_1 = t_0 + \frac{\sqrt{2}}{k}V^{\frac{1}{2}}(t_0).$$

The proof is completed. □

If the networks (1) and (2) have the same structures, the last term of the controllers in (3) is no longer required. Thus, we have the following corollary.

Corollary 3.2. Suppose that Assumption 2.3 holds. If the networks (1) and (2) have the same structures, i. e. C=D, then the two networks can realize finite-time outer synchronization under the following adaptive control scheme:

$$u_i = \begin{cases} -\varepsilon_i e_i - k[\mathrm{sign}(e_i) + \frac{|\varepsilon_i-\xi|e_i}{\|e\|^2}], & \text{if } \|e_i\| \neq 0;\\ 0, & \text{if } \|e_i\| = 0,\end{cases}$$

where the constants $k, \xi > 0$, the adaptive law of ε_i is $\dot{\varepsilon}_i(t) = e_i^T(t)\, e_i(t)$.

4. SIMULATION RESULTS

In this section, a three-dimensional chaotic system and a four-dimensional hyperchaotic system are performed to verify the feasibility and effectiveness of the above synchronization scheme. In the simulations, all equations are integrated with step 0.01 and the initial values are taken randomly from the interval $[-4, 4]$. For simplicity, we always assume that $\Gamma = I$.

Example 4.1. In the first example, we take the three-dimensional Genesio system [6] as the node dynamics of driving and response networks which can be described by:

$$\dot{x} = f(x) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \alpha & \beta & \lambda \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ x_1^2 \end{pmatrix} \triangleq Ax + g(x), \tag{9}$$

where $x = (x_1, x_2, x_3)^T \in R^3$ is the state vector, α, β, and λ are real constants. The Genesio system has a chaotic attractor when $\alpha = -6$, $\beta = -2.92$, and $\lambda = -1.2$. It is easy to compute that $\lambda_{\max}(A + A^T) = 5.6297$, and the Assumption 2.3 is satisfied.

Here, we adopt the configuration matrix for the driving network (1) as follows:

$$C = \begin{pmatrix} -3 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & -2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -4 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -3 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & -3 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & -3 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & -2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & -2 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & -3 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & -4 \end{pmatrix},$$

and the configuration matrix for the response network (2) as follows:

$$D = \begin{pmatrix} -2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & -2 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & -2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & -2 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -2 \end{pmatrix}.$$

It is easy to compute that $\lambda_{\max}(\mathcal{D}^s) = 0.2891$. Figures 1 (a) and (b) show the synchronization error trajectories $e_{ij}(t)$ $(i = 1, 2, \ldots, 10; j = 1, 2, 3)$ and the total synchronization error trajectory $\delta(t)$, where $\delta(t) = \|e(t)\|$. Figures 1 (c) and (d) show the time response of the adaptive controllers (3) and the corresponding feedback strengths ε_i

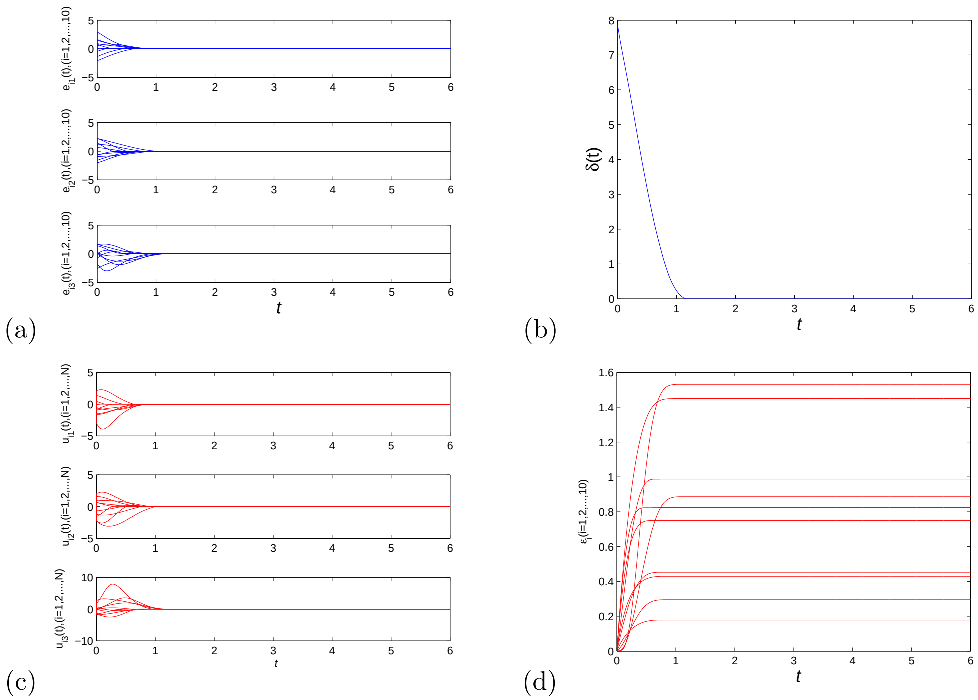


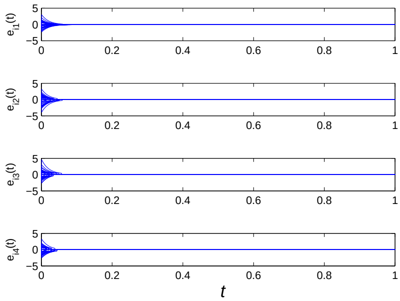
Fig. 1. Synchronization error trajectories $e_{ij}(t)(i = 1, 2, \ldots, 10; j = 1, 2, 3)$ (a) and the total synchronization error trajectory $\delta(t)$ (b) between the driving network (1) and response network (2) with 10 nodes under the adaptive controllers (3). Time response of the adaptive controllers (c) and feedback strengths ε_i (d) of adaptive controllers (3) for the networks (1) and (2).

which reach certain constants. From Figure 1 (b), we can see that the outer synchronization is realized at $t = 1.2109$. Therefore, the numerical results fully support the theoretical analysis.

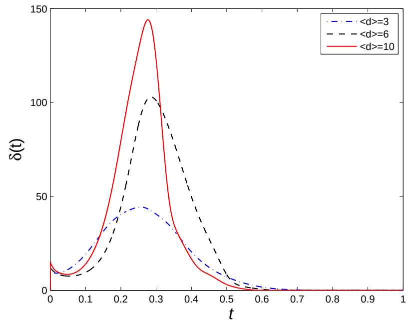
Example 4.2. To show the generality of the present method, we take the hyperchaotic Rössler system [10, 11] as the second example. The hyperchaotic Rössler system can be described by a four-dimensional differential equation as follows:

$$\dot{x} = f(x) = \begin{pmatrix} 0 & -1 & -1 & 0 \\ 1 & \vartheta & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -\nu & \varsigma \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ x_1 x_3 + \varrho \\ 0 \end{pmatrix}, \tag{10}$$

where $x = (x_1, x_2, x_3, x_4)^T \in R^4$ is the state vector, ϑ, ϱ, ν, and ς are real constants. When $\vartheta = 0.25$, $\varrho = 3$, $\nu = 0.5$, and $\varsigma = 0.05$, system (10) has a hyperchaotic attractor.



(a)



(b)

Fig. 2. (a) Synchronization error trajectories $e_{ij}(t)(i = 1, 2, \ldots, 100; j = 1, 2, 3, 4)$ between the small-word networks (1) and (2) with 100 nodes and the average degree $\langle d \rangle = 3$. (b) The total synchronization error trajectories between the small-word networks (1) and (2) with different average degrees $\langle d \rangle = 3, 6, 10$.

Some real networks often have complex topology, and the network topology may play a vital role in synchronization. In this example, we demonstrate the effectiveness of the theoretical results on small-world networks. We assume that networks (1) and (2) are small-world networks. To construct a small-world network we use the algorithm proposed by Watts and Strogatz [29]. The algorithm starts from a regular lattice with N nodes and with a certain probability p each link is rewired to another node randomly chosen from all possible nodes that avoid self-loops and link duplications. First, we generate a small-world network with $N = 100, p = 0.5$ and the average degree $\langle d \rangle = 3$. Figures 2 (a) and (b) show the synchronization error trajectories $e_{ij}(t)$ $(i = 1, 2, \ldots, 100; j = 1, 2, 3, 4)$ and the total synchronization error trajectories $\delta(t)$. From Figure 2 (b), we can easily make an important observation that the outer synchronization is realized in finite time. Therefore, the simulations correspond to the theoretical analysis successfully. Next, we consider the impact of the average degree on the convergence rate. Figure 2 (b) shows the total synchronization errors $\delta(t)$ with the different average degrees $\langle d \rangle = 3, 6, 10$ respectively. From Figure 2 (b), one can easily see that the small-word networks with large average degree has a faster convergence rate.

5. CONCLUSIONS

In this paper, we have investigated the finite-time adaptive outer synchronization between two complex dynamical networks with nonidentical topological structures. By applying the finite-time stability theory and proposing an adaptive control method for synchronization between two complex dynamical networks, sufficient conditions are obtained to ensure the finite-time outer synchronization. It is worth noting that the coupling configuration matrix is not necessary to be symmetric or irreducible. The controllers in our paper can also be used to investigate the inner or outer synchronization of time-varying complex networks. In addition, for large scale networks, time delay and noise perturbation are unavoidable and should be taken into account due to the finite information transmission and processing speeds among the network nodes. Therefore, it is important to study the finite-time adaptive synchronization of complex networks with time delay and noise perturbation. These are our next research topics.

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