Czechoslovak Mathematical Journal

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Czechoslovak Mathematical Journal, Vol. 64 (2014), No. 4, 1035–1044

Persistent URL: http://dml.cz/dmlcz/144159

TWO OPERATIONS ON A GRAPH PRESERVING THE (NON)EXISTENCE OF 2-FACTORS IN ITS LINE GRAPH

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(Received September 28, 2013)

Abstract. Let $G = (V(G), E(G))$ be a graph. Gould and Hynds (1999) showed a well-known characterization of G by its line graph $L(G)$ that has a 2-factor. In this paper, by defining two operations, we present a characterization for a graph G to have a 2-factor in its line graph $L(G)$. A graph G is called N^2-locally connected if for every vertex $x \in V(G)$, $G[\{y \in V(G);\ 1 \leqslant \mathrm{dist}_G(x,y) \leqslant 2\}]$ is connected. By applying the new characterization, we prove that every claw-free graph in which every edge lies on a cycle of length at most five and in which every vertex of degree two that lies on a triangle has two N^2-locally connected adjacent neighbors, has a 2-factor. This result generalizes the previous results in papers: Li, Liu (1995) and Tian, Xiong, Niu (2012), and is the best possible.

Keywords: 2-factor; claw-free graph; line graph; N^2-locally connected

MSC 2010: 05C35, 05C38, 05C45

1. Introduction

All graphs considered are simple finite undirected graphs and we refer to [1] for terminology and notation not defined here.

We will use $e(G)$ to denote the number of edges of G. We denote the minimum degree of G by $\delta(G)$, and the set of all vertices of degree k in G by $V_k(G)$. We denote $V_{\geqslant k}(G) = \bigcup_{i \geqslant k} V_i(G)$, and denote by $G[E]$ the subgraph of G induced by the edge set E of $E(G)$. The *distance* in G of two vertices $x, y \in V(G)$ is denoted by $\mathrm{dist}_G(x,y)$.

The research has been supported by the Natural Science Funds of China (Nos. 11471037, 11171129 and 11001197) and by Specialized Research Fund for the Doctoral Program of Higher Education (No. 20131101110048).

The *line graph* of H, denoted by $L(H)$, is the graph with $E(H)$ as vertex set, in which two vertices are adjacent if and only if the corresponding edges have a vertex in common. A graph is called *claw-free* if it has no induced subgraph isomorphic to $K_{1,3}$. A 2-*factor* of a graph G is a spanning subgraph of G in which every vertex has the same degree 2.

An *even* graph is a graph in which every vertex has positive even degree. A connected even subgraph is called a *circuit*. For $m \geqslant 2$, a star $K_{1,m}$ is a complete bipartite graph with independent sets $A = \{c\}$ and B with $|B| = m$; the vertex c is called the center and the vertices in B are called the leaves of $K_{1,m}$.

Let $\mathscr{S}$ be a set of edge-disjoint circuits and stars with at least three edges in a graph H. We call $\mathscr{S}$ a *system that dominates* H or simply a *dominating system* if every edge of H is either contained in one of the circuits or stars of $\mathscr{S}$ or is adjacent to one of the circuits. Gould and Hynds gave the following characterization of a graph H with $L(H)$ that has a 2-factor.

Theorem 1 (Gould and Hynds [4]). *Let H be a graph. Then $L(H)$ has a 2-factor if and only if there is a system that dominates H.*

Gould and Hynds in [4] also proved that the number of components in a 2-factor of $L(H)$ is equal to the number of elements in a system that dominates H.

It follows from either [2] or [3] that every claw-free graph G with $\delta(G) \geqslant 4$ has a 2-factor. Yoshimoto [9] showed that a claw-free graph G with $\delta(G) \geqslant 3$ has also a 2-factor if, additionally, G is 2-connected. Recently, by using Theorem 1, Tian, Xiong and Niu obtained the following result.

Theorem 2 (Tian, Xiong and Niu [8]). *Let G be a claw-free graph with $\delta(G) \geqslant 3$. If every edge of G lies on a cycle of length at most 5, then G has a 2-factor.*

In the following, we will give another characterization of a graph H for $L(H)$ to have a 2-factor. We first define two operations as follows.

To *split* a vertex v in a graph G with $N_G(v) = \{u', u''\}$ is to add two new vertices v' and v'', such that v' is adjacent to u' and v'' is adjacent to u'', see Figure 1.

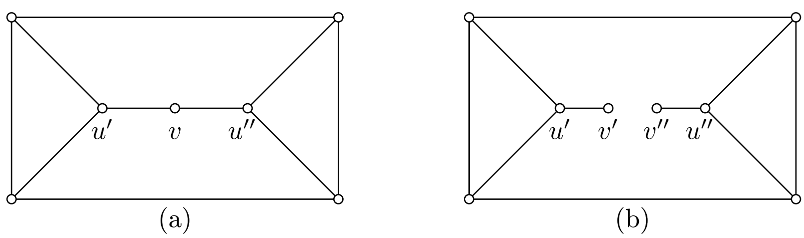

Figure 1. (a) A graph G with its vertex v of degree 2; (b) splitting the vertex v in G.

Denote $D'(T) = \{v \in V_3(T)\colon\ N(v) \cap V_1(T) \neq \emptyset\}$.

Operation 1. Let T be a tree and $v \in V_2(T)$. Then split the vertex v in T.

Operation 2. Let T be a tree and $v \in D'(T)$. Then delete the vertex v from T.

We call H' a *reduction* of a graph H if it is obtained from H by repeatedly performing Operations 1 and 2, until this is impossible. Note that a graph may have different reductions.

We denote by $[Y, Z]$ the set of all the edges with one end in Y and the other end in Z, and denote by $N(X)$ the set of vertices outside X that have a neighbor in X. Define

$$F_H(X) = H\big[[X, N(X) \cap V_{\geqslant 3}(H)] \cup E(H - (V(X) \cup (N(X) \cap V_1(H))))\big],$$

which denotes the edge-induced subgraph of H by the edges in $[X, N(X) \cap V_{\geqslant 3}(H)]$, and by those edges obtained from H by deleting the vertices both in X and in $N(X) \cap V_1(H)$.

Lemma 3. *Let H be a graph and X an even subgraph of H with $|E(X)|$ maximized. Then $F_H(X)$ is a forest.*

P r o o f. Suppose that $F_H(X)$ has a cycle C. Then $X \cup C$ is an even subgraph of H which has more edges than X; this contradicts the maximality of X. $\square$

The forest $F_H(X)$ is illustrated in Figure 2. Let $F_H^*(X)$ be the forest obtained from $F_H(X)$ by identifying each vertex of $V(X) \cap V(F_H(X))$ and the center of one of $|V(X) \cap V(F_H(X))|$ additional $K_{1,3}$'s, respectively.

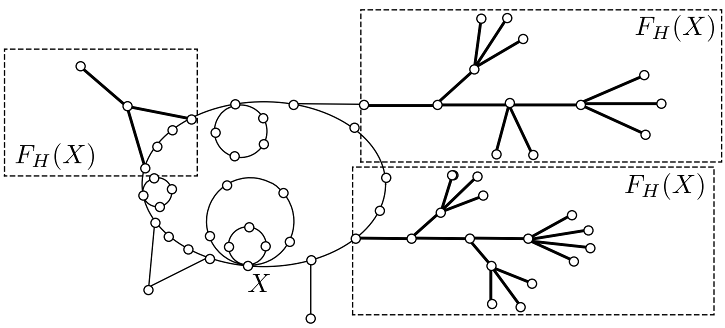

Figure 2. An even subgraph X and the forest $F_H(X)$ in H. The edges of $F_H(X)$ in three rectangular boxes are labeled by the thick lines.

Now we present our characterization.

Theorem 4. *Let H be a graph. Then the line graph $L(H)$ has a 2-factor if and only if H has a maximal even subgraph C such that $F_H^*(C)$ has no reduction which has a component that is an edge.*

Applying Theorem 4, we obtain Theorem 5 below, which generalizes Theorem 2.

We first give some definitions. For $x \in V(G)$ and an integer $k \geqslant 1$, let $N_G^k(x) = \{y \in V(G);\ 1 \leqslant \operatorname{dist}_G(x,y) \leqslant k\}$. A vertex v of G is *locally connected* if $G[N_G^1(v)]$ is connected; otherwise, it is *locally disconnected*. A graph G is *N^2-locally connected* if, for every vertex $x \in V(G)$, $G[N_G^2(x)]$ is a connected graph.

Theorem 5. *Every claw-free graph in which every edge lies on a cycle of length at most five and in which every locally connected vertex of degree two has two N^2-locally connected adjacent neighbors, has a 2-factor.*

The following result, which was proved by Li and Liu long time ago, is obtained straightforwardly from Theorem 5.

Corollary 6 (Li and Liu [5]). *Every N^2-locally connected claw-free graph with $\delta(G) \geqslant 2$ has a 2-factor.*

2. Notation and preliminary results

Before we present the proofs of Theorems 4 and 5, we first introduce some additional terminology and notation.

Let $G = (V(G), E(G))$ be a graph with vertex set $V(G)$ and edge set $E(G)$. The *neighborhood* and the *degree of vertex u* in G are denoted by $N(u) = \{x \in V(G);\ xu \in E(G)\}$ and $d_G(u)$ (or $d(u)$ when no confusion is possible), respectively. An edge of G is a *pendant edge* if some of its vertices is of degree 1. The *edge degree* of an edge $e = uv$ of G is defined as $\xi_G(e) = d(u) + d(v) - 2$ and the *minimum edge degree* $\delta_e(G)$ is the minimum value of the edge degrees of all edges in G.

2.1. The closure of a claw-free graph. Let x be a vertex of a claw-free graph G. If the subgraph induced by $N(x)$ is connected, we add edges joining all pairs of nonadjacent vertices in $N(x)$. This operation is called *local completion* of G at x. The *closure* $\operatorname{cl}(G)$ of G is the graph obtained by recursively repeating the local completion operation, as long as this is possible. Ryjáček [6] showed that the closure of G is uniquely determined and G is hamiltonian if and only if $\operatorname{cl}(G)$ is hamiltonian. The latter result was extended to 2-factors as follows.

Theorem 7 (Ryjáček, Saito and Schelp [7]). *If G is a claw-free graph, then G has a 2-factor if and only if* $\mathrm{cl}(G)$ *has a 2-factor.*

Ryjáček [6] also established the following relationship between claw-free graphs and triangle-free graphs.

Theorem 8 (Ryjáček [6]). *If G is a claw-free graph, then there is a triangle-free graph H such that $L(H) = \mathrm{cl}(G)$.*

2.2. Some auxiliary results for the proof of Theorem 5. Observing that every new edge of the closure $\mathrm{cl}(G)$ lies on a triangle, we have the following result.

Lemma 9. *If every edge of a claw-free graph G lies on a cycle of length at most five, then every edge of* $\mathrm{cl}(G)$ *also lies on a cycle of length at most five.*

By the definitions of a locally disconnected and N^2-locally connected vertex, we obtain the following result.

Lemma 10. *Let G be a claw-free graph. Then a locally disconnected vertex v is N^2-locally connected in G if and only if v lies on an induced cycle of length* 4 *or* 5 *in G.*

Lemma 11. *Let G be a graph and $u \in V(G)$. If u is N^2-locally connected in G, then u is N^2-locally connected in* $\mathrm{cl}(G)$.

P r o o f. Suppose that u is locally connected in $\mathrm{cl}(G)$. Then u is N^2-locally connected in $\mathrm{cl}(G)$. Now suppose that u is locally disconnected in $\mathrm{cl}(G)$. Then u is locally disconnected in G. Since u is N^2-locally connected in G, by Lemma 10, u lies on an induced cycle of length 4 or 5 in G. Notice that u is locally disconnected in $\mathrm{cl}(G)$ and u lies on an induced cycle of length 4 or 5 in $\mathrm{cl}(G)$. By Lemma 10, u is N^2-locally connected in $\mathrm{cl}(G)$. □

Lemma 12. *Let G be a claw-free graph in which every edge of G lies on a cycle of length at most five. If every locally connected vertex of degree two in G has two N^2-locally connected adjacent neighbors, then every locally connected vertex of degree two in* $\mathrm{cl}(G)$ *has also two N^2-locally connected adjacent neighbors.*

P r o o f. Suppose that x is a locally connected vertex in $\mathrm{cl}(G)$ with degree 2. Let $N(x) = \{z_1, z_2\}$. Since $d_{\mathrm{cl}(G)}(x) = 2$ and by the hypothesis that every edge of G lies on a cycle, $d_G(x) = 2$.

Suppose first that x is locally disconnected in G (i.e., $z_1 z_2 \notin E(G)$), let $G = G_1, G_2, \ldots, G_k = \mathrm{cl}(G)$ be the sequence of graphs that yields $\mathrm{cl}(G)$ (i.e., G_{i+1} is obtained from G_i by a local completion at some vertex x_i), and let G_{i_0} be the first graph in which $z_1z_2 \in E(G_{i_0})$. Then $x_{i_0}z_1z_2$ is a triangle in G_{i_0}, but then z_1 is locally connected in G_{i_0}, hence $xx_{i_0} \in E(\mathrm{cl}(G))$, implying $d_{\mathrm{cl}(G)}(x) \geqslant 3$, a contradiction.

Hence x is locally connected in G. Then, since $d_G(x) = 2$, z_1 and z_2 are N^2-locally connected in G. Thus by Lemma 11, z_1 and z_2 are N^2-locally connected in $\mathrm{cl}(G)$. □

3. Some lemmas

In order to prove Theorem 4, we first present a useful result which was proved in [8].

Lemma 13 (Tian, Xiong and Niu [8]). *Let T be a tree with $\delta_e(T) \geqslant 3$. If $V_2(T) = \emptyset$, then T has a dominating system.*

We also give the following lemmas, which are needed in the proof of Theorem 4.

Lemma 14. *Let T be a tree and $v \in V_2(T)$. Let T_1 and T_2 be two trees obtained from T by performing Operation 1 on the vertex v. Then $L(T)$ has a 2-factor if and only if both $L(T_1)$ and $L(T_2)$ have a 2-factor.*

P r o o f. By Theorem 1, $L(T)$ has a 2-factor if and only if T has a dominating system $\mathscr{S}$ such that $\mathscr{S} = \bigcup\limits_{i=1} S_i$, where S_i is the i-th star in $\mathscr{S}$ which has at least three edges. Since the vertex of degree two cannot be the center of a star in $\mathscr{S}$, T has a dominating system if and only if both T_1 and T_2 have a dominating system. Hence the lemma holds by Theorem 1. □

Lemma 15. *Let T be a tree other than $K_{1,3}$. Then for any $v \in D'(T)$, $L(T)$ has a 2-factor if and only if $L(T - v)$ has a 2-factor.*

P r o o f. Since $v \in D'(T)$, v must be chosen as the center of one of the stars in a dominating system. Thus T has a dominating system if and only if $T - v$ has a dominating system. Therefore the lemma holds by Theorem 1. □

Lemma 16. *Let T be a tree. Then $L(T)$ has a 2-factor if and only if T has a reduction T' such that $\xi_{T'}(e) \geqslant 3$ for each edge $e \in E(T')$.*

P r o o f. Sufficiency. Let T' be a reduction of T such that $\xi_{T'}(e) \geqslant 3$ for each edge $e \in E(T')$. Then we have $\delta_e(T') \geqslant 3$ by the assumption, and $V_2(T') = \emptyset$ since T' is a reduction of T. By Lemma 13 and Theorem 1, $L(T')$ has a 2-factor. Thus $L(T)$ has a 2-factor by Lemmas 14 and 15.

Conversely, suppose that $L(T)$ has a 2-factor. Then T has a dominating system by Theorem 1, and so T' has a dominating system by Lemmas 14 and 15. Let $e = uv$ be an edge of T'. Without loss of generality, assume that $d_{T'}(u) \leqslant d_{T'}(v)$. If $d_{T'}(u) \geqslant 4$, then $\delta_e(T') \geqslant 6$ and we are done.

It remains to consider the case when $d_{T'}(u) \leqslant 3$. We distinguish the following two cases.

Case 1. $d_{T'}(u) = 1$. Then $d_{T'}(v) \geqslant 1$. If $d_{T'}(v) = 1$, then e is an isolated edge in T'. This is impossible since T' has a dominating system. If $d_{T'}(v) = 2$ or $d_{T'}(v) = 3$, then we can perform Operation 1 or Operation 2 on v in T', a contradiction. If $d_{T'}(v) \geqslant 4$, then $\xi_{T'}(e) \geqslant 3$.

Case 2. $2 \leqslant d_{T'}(u) \leqslant 3$. Then $d_{T'}(v) \geqslant 2$. Since T' is a reduction of T, $d_{T'}(v) \neq 2$. So $d_{T'}(v) \geqslant 3$. Thus $\xi_{T'}(e) \geqslant 3$. □

Lemma 17. *Let T be a tree. Then $L(T)$ has a 2-factor if and only if T has no reduction T' such that T' has a component that is an edge.*

Proof. Suppose first that $L(T)$ has a 2-factor. Then T has a dominating system by Theorem 1. Thus by Lemmas 14 and 15, T' has a dominating system, where T' is a reduction of T. So T' has no component that is an edge.

Conversely, by Lemma 16, we only need to prove that $\xi_{T'}(e) \geqslant 3$ for each edge $e \in E(T')$. Let $e = uv$ be an edge of T'. Since T' has no component that is an edge, $\xi_{T'}(e) \neq 0$. We claim that $\xi_{T'}(e) \neq 1$: Otherwise, if $\xi_{T'}(e) = 1$, then $d_{T'}(u) = 2$ or $d_{T'}(v) = 2$, which contradicts the definition of reduction. We also claim that $\xi_{T'}(e) \neq 2$: Otherwise, $(d_{T'}(u), d_{T'}(v)) \in \{(2,2),(1,3),(3,1)\}$, which is impossible since T' is a reduction. Therefore, $\xi_{T'}(e) \geqslant 3$ for each edge $e \in E(T')$. □

The following lemma follows directly from Lemma 17 and Theorem 1.

Lemma 18. *Let T be a tree. Then T has a dominating system if and only if T has no reduction T' such that T' has a component that is an edge.*

4. Proof of Theorem 4

Suppose that C is a maximal even subgraph in H. For convenience, denote $F_H^*(C)$ and $F_H(C)$ by F_1 and F_2, respectively. Let $F_1^{(1)}$ be composed of all the components of F_1 such that $V(F_1^{(1)}) \cap N(C) \subseteq V_2(H)$, and let $F_1^{(2)}$ be composed of all the components of F_1 such that $V(F_1^{(2)}) \cap N(C) \subseteq V_{\geqslant 3}(H)$. Evidently, $H = F_1^{(1)} \cup (H - V(F_1^{(1)})) \cup [V(C), N(C) \cap V_2(H)]$ and $F_1 = F_1^{(1)} \cup F_1^{(2)}$.

Claim 1. $(H - V(F_1^{(1)})) \cup [V(C), N(C) \cap V_2(H)]$ has a dominating system if and only if $F_1^{(2)}$ has a dominating system.

P r o o f. To show sufficiency, suppose that $F_1^{(2)}$ has a dominating system $\mathscr{S}$. Let $\mathcal{T}$ be the set of all the stars in $\mathscr{S}$ with centers in $V(F_1^{(2)}) \cap C$. Then

$$(\mathscr{S} \setminus \mathcal{T}) \cup \{\text{all the circuits in } C\}$$

is a dominating system of $(H - V(F_1^{(1)})) \cup [V(C), N(C) \cap V_2(H)]$.

Conversely, suppose that $(H - V(F_1^{(1)})) \cup [V(C), N(C) \cap V_2(H)]$ has a dominating system $\mathscr{S}'$. Let $\mathcal{T}'$ be the set of all the stars in $\mathscr{S}'$ with centers in $V(F_1^{(2)}) \cap C$. Then

$$(\mathscr{S}' \setminus \{\text{all the circuits in } C\}) \cup \mathcal{T}'$$

is a dominating system of $F_1^{(2)}$. □

By the definition of $F_1^{(1)}$, $F_1^{(1)}$ has a dominating system in H if and only if it has a dominating system in F_1. Hence by Claim 1, we conclude that

(4.1) H has a dominating system if and only if F_1 has a dominating system.

To prove sufficiency, suppose that F_1 has no reduction which has a component that is an edge. By Lemma 18, F_1 has a dominating system. Thus by (4.1), H has a dominating system. So by Theorem 1, $L(H)$ has a 2-factor.

We prove necessity. Suppose, to the contrary, that H has a maximal even subgraph X such that X_1 has a reduction which has a component that is an edge, where $X_1 = F_H^*(X)$. Thus by Lemma 18, X_1 has no dominating system. Hence by (4.1), H has no dominating system. Therefore $L(H)$ has no 2-factor by Theorem 1, a contradiction. □

5. Proof of Theorem 5

In this section, we apply Theorem 4 to prove Theorem 5. The following lemma will be needed in our arguments.

Lemma 19 (Lemma 12, [8]). *Let H be a subgraph of a graph G. If C is a cycle of G such that $|E(C) \cap E(H)| \geqslant e(C) - 1$, then $V(C) \subseteq V(H)$.*

P r o o f of Theorem 5. Suppose that G satisfies the conditions of Theorem 5. Then by Lemmas 9 and 12, $\mathrm{cl}(G)$ also satisfies the conditions of Theorem 5. Thus by Theorem 8, we may assume that $\mathrm{cl}(G) = L(H)$, where H is triangle-free.

Let Y be a maximal even subgraph of H such that any even subgraph Y' of H satisfies $e(Y') \leqslant e(Y)$. For convenience, denote $F_H^*(Y)$ and $F_H(Y)$ by F^1 and F^2, respectively.

Claim 2 (Claim 3, [8]). Let C be a cycle of H. Then $|E(C) \cap E(Y)| \geqslant e(C)/2$.

Claim 3 (Claim 4, [8]). For $v \in V_2(H)$, either $v \in V(Y)$, or $v \in V_0(H-Y)$.

Claim 4. If $x \in V_3(H)$ and $y \in N(x) \cap V_1(H)$, then either $x \in V(Y)$ or $e = xy$ is an edge of a claw which is a component of F^2.

Proof. We may assume that $x \notin V(Y)$. Since $d_H(x) = 3$, suppose that $N_H(x) \setminus \{y\} = \{w_1, w_2\}$. Let $e_1 = xw_1$ and $e_2 = xw_2$. Since ee_1e_2e is a triangle in $\mathrm{cl}(G)$, e is locally connected in $\mathrm{cl}(G)$. Moreover, since $d_{\mathrm{cl}(G)}(e) = 2$, e_1 and e_2 are N^2-locally connected in $\mathrm{cl}(G)$. Note that, since $\mathrm{cl}(G)$ is claw-free, $e_1, e_2 \in V(\mathrm{cl}(G))$ lie on a common induced cycle of length at most 5 in $\mathrm{cl}(G)$. Thus, since H is triangle-free, $e_1, e_2 \in E(H)$ lie on a common induced cycle C of length 4 or 5 in H.

First suppose that $e(C) = 4$. Then by Claim 2, $|E(C) \cap E(Y)| \geqslant 2$. If $|E(C) \cap E(Y)| \geqslant e(C) - 1 = 3$, then $x \in V(Y)$ by Lemma 19, a contradiction. Therefore, $|E(C) \cap E(Y)| = 2$. Since $x \notin V(Y)$, we have $E(C) \setminus E(Y) = \{e_1, e_2\}$. Thus $H[\{e, e_1, e_2\}]$ is a component of F^2. Noting that $H[\{e, e_1, e_2\}]$ is also a claw, we are done.

Next suppose that $e(C) = 5$. Then by Claim 2, $|E(C) \cap E(Y)| \geqslant 3$. If $|E(C) \cap E(Y)| \geqslant e(C) - 1 = 4$, then by Lemma 19, $x \in V(Y)$, a contradiction. Therefore, $|E(C) \cap E(Y)| = 3$. Since $x \notin V(Y)$, $E(C) \setminus E(Y) = \{e_1, e_2\}$. Thus $H[\{e, e_1, e_2\}]$ is a component of F^2. Noting that $H[\{e, e_1, e_2\}]$ is also a claw, we are done. □

If T is a component of F^1, then, by Claims 3 and 4, T is of one of the following two types: (i) T is a tree obtained from a claw by identifying two of its leaves with the centers of 2 additional $K_{1,3}$'s, (ii) T is a tree which has no vertex of degree 2 and has no vertex of degree 3 which is adjacent to a vertex of degree 1. In the former case, T has a unique reduction which is edgeless, and in the latter, T equals its reduction. Thus, F^1 has a unique reduction, each component of which satisfies (ii). By Claim 3, no component in case (ii) is an edge. Hence, the reduction of F^1 has no component that is an edge. Thus $L(H)$ has a 2-factor by Theorem 4. □

6. Sharpness of Theorem 5

We give an example to show that 5 cannot be weakened to an integer $l \geqslant 6$ in Theorem 5. The graph H_0 in Figure 3 is obtained from $K_{2,3}$ by subdividing the three edges that are incident with exactly one vertex of degree three in $K_{2,3}$ and attaching some pendant edges to every vertex of degree three. The line graph $L(H_0)$ of H_0 is a claw-free graph in which there exists an edge that lies on a cycle of length exactly six and in which there is no locally connected vertex of degree two. However, H_0 has no dominating system, hence $L(H_0)$ has no 2-factor.

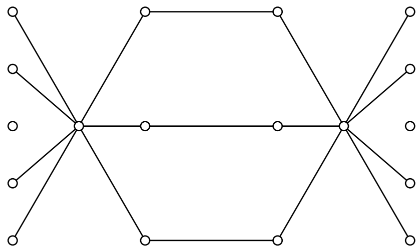

Figure 3. The graph H_0.

Acknowledgement. The authors would like to thank the referees for their valuable comments and suggestions.

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