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Ratio Type Statistics for Detection of Changes in Mean and the Bootstrap Method

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The paper presents procedures for detection of changes in mean. In particular test procedures based on ratio type test statistics that are functionals of partial sums of residuals are studied. We explore the possibility of applying the bootstrap method for obtaining critical values of the proposed test statistics and derive the limit behavior of the block bootstrap statistic for the L_2 procedure.

1. Introduction

Ratio type statistics studied in this paper are derived from non-ratio statistics based on partial sums of residuals. They do not need to be standardized by any variance estimate, which makes them a suitable alternative for non-ratio statistics, most of all in situations, when it is difficult to find a variance estimate with satisfactory properties. Such difficulty can occur in situations with dependent random errors (see e.g. [1]).

We describe basic properties and asymptotic behavior of statistics for change detection in location model with at most one abrupt change in the mean, while assuming to have data obtained in ordered time points and study the null hypothesis of no change against the alternative of a change occurring at some unknown time point. We extend the ideas presented by Horváth et al. in [5] and Hušková in [6]. In order to obtain critical values for the studied test statistic, we focus on the circular moving block bootstrap method. The method was introduced by Politis and Romano in [10] and applied in a similar situation by Kirch in [8].

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2. Model description and the test statistics

Let us consider observations $Y_1, \ldots, Y_n$ that were obtained at n time-ordered points. We study the location model with at most one abrupt change in mean:

$$Y_k = \mu + d\mathbf{I}\{k > z\} + e_k, \qquad k = 1, \ldots, n, \tag{1}$$

where μ, $d = d_n$ and $z = z_n$ are unknown parameters and $\mathbf{I}\{A\}$ denotes the indicator of set A, z is called the change-point. By $e_1, \ldots, e_n$, we denote the random error terms. We are going to test the null hypothesis that no change occurred

$$H_0 : z = n \tag{2}$$

against the alternative that change occurred at some unknown time-point z

$$H_1 : z < n,\ d \neq 0. \tag{3}$$

Following the ideas described in [5], [6] and [7], a test statistic based on M-residuals is considered:

$$T_n(\psi) = \max_{n\gamma \le k \le n - n\gamma} \frac{\max\limits_{1 \le i \le k} \left| \sum\limits_{1 \le j \le i} \psi(Y_j - \hat{\mu}_{1k}(\psi)) \right|}{\max\limits_{k \le i \le n} \left| \sum\limits_{i+1 \le j \le n} \psi(Y_j - \hat{\mu}_{2k}(\psi)) \right|}, \tag{4}$$

where $0 < \gamma < 1/2$ is a given constant, ψ is a score function, $\hat{\mu}_{1k}(\psi)$ is an M-estimate of parameter μ based on observations $Y_1, \ldots, Y_k$ and $\hat{\mu}_{2k}(\psi)$ is an M-estimate of μ based on observations $Y_{k+1}, \ldots, Y_n$.

For the choice of $\psi_{L_2}(x) = x$, we get one of statistics studied in [5]. By considering different score functions, we may construct similar statistics, but more robust against outliers and more suitable for heavy-tailed distributions.

3. Limit distribution under null hypothesis

At first we formulate assumptions concerning the score function ψ and the distribution of random errors $e_1, \ldots, e_n$.

Assumption 1. The random error terms $\{e_i, i \in \mathbb{N}\}$ form a strictly stationary α-mixing sequence with distribution function F, that is symmetric around zero and for some $\delta > 0$, $\Delta > 0$ there exists a constant $C_1(\delta, \Delta) > 0$ such that

$$\sum_{h=0}^{\infty} (h+1)^{\delta/2} \alpha(h)^{\Delta/(2+\delta+\Delta)} \le C_1(\delta, \Delta). \tag{5}$$

where $\alpha(k)$, $k = 0, 1, \ldots$ are the α-mixing coefficients.
Assumption 2. The score function ψ is a non-decreasing and antisymmetric function.

Assumption 3.

$$\int |\psi(x)|^{2+\delta+\Delta} dF(x) < \infty \tag{6}$$

and

$$\int |\psi(x+t_2) - \psi(x+t_1)|^{2+\delta+\Delta} dF(x) \le C_2(\delta, \Delta)|t_2 - t_1|^{\eta},$$

$$|t_j| \le C_3(\delta, \Delta), \ j = 1, 2 \tag{7}$$

for some constants $1 \le \eta \le 2 + \delta + \Delta$, $\delta > 0$, $\Delta > 0$ as in (5) and constants $C_2(\delta, \Delta)$, $C_3(\delta, \Delta) > 0$ both depending only on δ and Δ.
Assumption 4. Let us denote $\lambda(t) = -\int \psi(e-t)dF(e)$, for $t \in \mathbb{R}$. We assume that $\lambda(0) = 0$ and that there exists a first derivative $\lambda'(\cdot)$ that is Lipschitz in the neighborhood of 0 and satisfies $\lambda'(0) > 0$.
Assumption 5. Let

$$0 < \sigma^2(\psi) = E\psi^2(e_1) + 2\sum_{i=1}^{\infty} E\psi(e_1)\psi(e_{i+1}) < \infty. \tag{8}$$

Remark 3.1 Assumption 1 is satisfied for example for ARMA processes with continuously distributed stationary innovations and bounded variance (see [4], Section 2.4).

Remark 3.2 The conditions regarding ψ reduce to moment restrictions for $\psi_{L_2}(x) = x$ (L_2-method). For $\psi_{L_1}(x) = \mathrm{sgn}(x)$ (L_1-method), the conditions reduce to F being a symmetric distribution and having continuous density f in a neighborhood of 0 with $f(0) > 0$. Similarly, we may consider the derivative of the Huber loss function

$$\psi_H(x) = x\mathbf{I}\{|x| \le K\} + K\mathrm{sgn}(x)\mathbf{I}\{|x| > K\} \tag{9}$$

for some $K > 0$. In that case, we need to assume F being a symmetric distribution with continuous density f in a neighborhood of K and $-K$ satisfying $f(K) > 0$.

Theorem 1 *Let us assume that the above stated Assumptions 1-5 hold. Then, under null hypothesis* (2)

$$T_n(\psi) \xrightarrow{\mathscr{D}} \sup_{\gamma \le t \le 1-\gamma} \frac{\sup_{0 \le u \le t} |W(u) - u/tW(t)|}{\sup_{t \le u \le 1} \left|\tilde{W}(u) - (1-u)/(1-t)\tilde{W}(t)\right|}, \tag{10}$$

as $n \to \infty$, where $\{W(u), 0 \le u \le 1\}$ is a standard Wiener process and $\tilde{W}(u) = W(1) - W(u)$.

Proof. The proof goes along the lines of proof of Theorem 1.1 in [5], using several results derived in [7]. Therefore we give only an outline of the proof.
Without loss on generality, we assume that μ=0. Let

$$Z_n(t) = \frac{1}{\sqrt{n}} \sum_{1 \le j \le nt} \psi(e_j), \qquad \tilde{Z}_n(t) = \frac{1}{\sqrt{n}} \sum_{nt < j \le n} \psi(e_j).$$

Then, by applying Theorem 1.5.1 from [4], we get

$$(Z_n(t), \tilde{Z}_n(t)) \xrightarrow{\mathscr{D}} \sigma(\psi)(W(t), \tilde{W}(t)),$$

where $\tilde{W}(t) = W(1) - W(t)$.
By the same way as in [7], it follows from Lemma 3 and Lemma 4 (pages 15–16 in [7]) that

$$\sup_{1\le i\le nt} \left\{ n^{\kappa} \sqrt{\frac{[nt]}{i([nt]-i)}} \left| \sum_{1\le j\le i} \psi(Y_j - \hat{\mu}_{1,[nt]}(\psi)) \right.\right.$$
$$\left.\left. - \left(\sum_{1\le j\le i} \psi(e_j) - \frac{i}{[nt]} \sum_{1\le j\le nt} \psi(e_j) \right) \right| \right\} \xrightarrow{P} 0, \quad \text{for some } \kappa > 0,$$

where $[a]$ denotes the integer part of $a \in \mathbb{R}$, which implies

$$\frac{1}{\sqrt{n}} \sup_{1<i\le nt} \left| \sum_{1\le j\le i} \psi(Y_j - \hat{\mu}_{1,[nt]}(\psi)) \right| = \sup_{1\le i\le nt} \left| Z_n\left(\frac{i}{n}\right) - \frac{i}{[nt]} Z_n(t) \right| + o_P(1).$$

Similarly, we get

$$\frac{1}{\sqrt{n}} \sup_{nt<i\le n} \left| \sum_{i\le j\le n} \psi(Y_j - \hat{\mu}_{2,[nt]}(\psi)) \right| = \sup_{nt<i\le n} \left| \tilde{Z}_n\left(\frac{i}{n}\right) - \frac{n-1}{n-[nt]} \tilde{Z}_n(t) \right| + o_P(1).$$

The rest of the proof is the same as in the proof of Theorem 1.1 in [5]. □

Remark 3.3 The null hypothesis is rejected for large values of $T_n(\psi)$. Explicit form of the limit distribution (10) under the null hypothesis is not known. Therefore, in order to obtain critical values, we have to use either simulation from the limit distribution or resampling methods.

Remark 3.4 For $\psi_{L_2} : \psi_{L_2}(x) = x, x \in \mathbb{R}$, the above stated Assumptions 2 and 4 are satisfied. We can also drop the requirement of symmetry of F in Assumption 1 and replace it by $Ee_1 = 0$. Assumptions 3 and 5 reduce to:
Assumption 3'.

$$E|e_1|^{2+\beta} < \infty. \tag{11}$$

for some constant $\beta > 0$.
Assumption 5'.

$$0 < \sigma^2(\psi_{L_2}) = Ee_1^2 + 2\sum_{i=1}^{\infty} Ee_1 e_{i+1} < \infty. \tag{12}$$

Remark 3.5 The limit results for $T_n(\psi_{L_2})$ were derived in [5] and [6] under less restrictive assumptions regarding the random errors. According to Theorem 1.2 in [5], the test based on $T_n(\psi_{L_2})$ is consistent, if $z = [n\zeta]$ for some $\zeta : \gamma < \zeta < 1 - \gamma$ and $\sqrt{n} d_n \to \infty$.

For other score functions ψ, results regarding limit behavior under fixed as well as under local alternative for the related non-ratio statistic are presented in [7]. The result for the ratio statistic under fixed alternative may be derived by a modification of the proof therein.

4. Block bootstrap with replacement

In the following section, we are going to study only the case of $\psi_{L_2}(x) = x$. Extension to the case of a general score function ψ from the previous sections is straightforward, but the proofs are much more complex.

There are several different approaches that may be used when resampling dependent observations. Classical resampling methods are not suitable, since they do not take into account the underlying dependency structure. Here we focus our attention to the so called circular moving block bootstrap method, which was introduced by Politis and Romano in [10]. Overlapping blocks of consequent observations are formed from the original observations. The first few consequent observations from the original sequence are appended after the last observation, so that for a sequence of length n, we always have n possible blocks of subsequent observations to choose from. With this method, there is equal probability for each observation to be included in the bootstrap sample. For more details on the method, we also refer to [8].

Let L denote the number of blocks and let K be the block length. In order to keep the notation as simple as possible, we restrict ourselves to situation, where $n = KL$, i.e. if the set of n observations can be divided in exactly L blocks of length K. It can be proven (cf. [8]) that the limit results remain the same after omitting the last K_1 observations, if $n = KL + K_1$, $1 \leq K_1 \leq K - 1$. We will assume that K and n are both functions of L such that $n = KL$ and we let $L \to \infty$.

First, let us define the following subsets of $\mathbb{N} \times \mathbb{N}$ for integer numbers l, k, L, K and real number $0 < \gamma \leq 1/2$:

$$\Pi_{l,k,L,K} = \{(p,q): \; p,q \in \mathbb{N}, \\ 1 \leq p \leq l, \; 1 \leq q \leq K, \; (p-1)K + q \leq (l-1)K + k\}, \quad (13)$$

$$\tilde{\Pi}_{l,k,L,K} = \{(p,q): \; p,q \in \mathbb{N}, \\ l \leq p \leq L, \; 1 \leq q \leq K, \; (p-1)K + q \geq (l-1)K + k + 1\}, \quad (14)$$

$$\Omega_{L,K}(\gamma) = \{(p,q): \; p,q \in \mathbb{N}, \\ 1 \leq p \leq L, \; 1 \leq q \leq K, \; KL\gamma \leq (p-1)K + q \leq KL(1-\gamma)\}. \quad (15)$$

For a set of i.i.d. random variables $\mathbf{U} = (U_1, \dots, U_n)$, uniformly distributed on the set $\{0, \dots, n-1\}$, we define the following block bootstrap statistics:

$$S^{\mathbf{U}}_{L,K}(p,q,l,k) = \sum_{i=1}^{p-1}\sum_{j=1}^{K}\left(Y_{U_i+j} - m^{\mathbf{U}}_{L,K}(l,k)\right) + \sum_{j=1}^{q}\left(Y_{U_p+j} - m^{\mathbf{U}}_{L,K}(l,k)\right), \tag{16}$$

where

$$m^{\mathbf{U}}_{L,K}(l,k) = \frac{1}{(l-1)K+k}\left(\sum_{r=1}^{l-1}\sum_{s=1}^{K} Y_{U_r+s} + \sum_{s=1}^{k} Y_{U_l+s}\right)$$

for $p, l = 1, \dots, L$, $q, k = 1, \dots, K$, $p \le l$, $(p-1)K + q \le (l-1)K + k$. Similarly, we define

$$\tilde{S}^{\mathbf{U}}_{L,K}(p,q,l,k) = \sum_{j=k+1}^{K}\left(Y_{U_l+j} - \tilde{m}^{\mathbf{U}}_{L,K}(l,k)\right)\mathbf{I}\{p \ge l+1\}$$
$$+ \sum_{i=l+1}^{p-1}\sum_{j=1}^{K}\left(Y_{U_i+j} - \tilde{m}^{\mathbf{U}}_{L,K}(l,k)\right)\mathbf{I}\{p \ge l+2\} + \sum_{j=1}^{q}\left(Y_{U_p+j} - \tilde{m}^{\mathbf{U}}_{L,K}(l,k)\right), \tag{17}$$

where $\mathbf{I}\{A\}$ denotes the indicator of set A and

$$\tilde{m}^{\mathbf{U}}_{L,K}(l,k) = \frac{1}{(L-l+1)K-k}\left(\sum_{s=k+1}^{K} Y_{U_l+s} + \sum_{s=l+1}^{L}\sum_{r=1}^{K} Y_{U_r+s}\right)$$

for $p, l = 1, \dots, L$, $q, k = 1, \dots, K$ such that $p \ge l$, $(p-1)K + q \ge (l-1)K + k + 1$.

Now define the block bootstrap version of $T_n(\psi_{L_2})$ in (4):

$$T^*_{L,K}(\psi_{L_2}) = \max_{(l,k)\in\Omega_{L,K}(\gamma)} \frac{\max_{(p,q)\in\Pi_{l,k,L,K}}\left|S^{\mathbf{U}}_{L,K}(p,q,l,k)\right|}{\max_{(p,q)\in\tilde{\Pi}_{l,k,L,K}}\left|\tilde{S}^{\mathbf{U}}_{L,K}(p,q,l,k)\right|}. \tag{18}$$

We are going to prove that $T^*_{L,K}(\psi_{L_2})$ provides asymptotically correct critical values for the test based on $T_n(\psi_{L_2})$, when observations follow either the null hypothesis or alternative one.

Theorem 2 *Let $E|e_1|^\nu < \infty$ for some $\nu > 4$. Let Assumption 1 be satisfied for $\delta_1, \Delta_1 > 0$ and for $\delta_2, \Delta_2 > 0$ such that $2 + 2\kappa < \delta_1 < \nu - 2$, $\Delta_1 = \nu - 2 - \delta_1$ and $0 < \delta_2 < (\delta_1 - 2 + 2\kappa)/(2+\kappa)$, $\Delta_2 = (\delta_1 - 2 + 2\kappa)/(2+\kappa) - \delta_2$ for some $0 < \kappa < (\nu-4)/2$. Moreover, let Assumption 5' be satisfied, and let*

$$K \le L^{\delta_2/2-\varepsilon} \tag{19}$$

for some $\varepsilon > 0$. For K bounded, also assume that $\operatorname{var}(\sum_{k=1}^{K} e_k) \ge c$ *for some $c > 0$, as $L \to \infty$.*
Under alternative, let $z = [n\zeta]$ for some $\zeta : \gamma < \zeta < 1 - \gamma$.

Then we have for all $y \in \mathbb{R}$, as $L \to \infty$

$$P\left(T^*_{L,K}(\psi_{L_2}) \le y | Y_1, \dots, Y_n\right) \to$$
$$P\left(\sup_{\gamma \le t \le 1-\gamma} \frac{\sup_{0 \le u \le t} |W(u) - u/tW(t)|}{\sup_{t \le u \le 1} \left|\tilde{W}(u) - (1-u)/(1-t)\tilde{W}(t)\right|} \le y \right) \text{ a.s.},$$

as $L \to \infty$, where $\{W(u), 0 \le u \le 1\}$ is a standard Wiener process and $\tilde{W}(u) = W(1) - W(u)$.

Proof. The proof goes along the lines of proof of Theorem 3.6.2 in [8] and uses several results derived there. Therefore we only give an outline of the proof. In contrast to [8], we dropped the assumption of random errors being a linear process. The crucial part of the proof is verification of the assumptions of Theorem 3.6.1 in [8], i.e. that

$$\frac{1}{n} \sum_{i=0}^{n-1} \left(\max_{k=0,\dots,K-1} \left| \frac{1}{\sqrt{K}} \sum_{j=k+1}^{K} (a_n(i+j) - \bar{a}_n) \right|^{\rho} \right) \le D_1 \tag{20}$$

for some $2 < \rho \le 4$ and satisfying

$$\tau_n^2(\mathbf{a}) = \frac{1}{n} \sum_{i=0}^{n-1} \left(\frac{1}{\sqrt{K}} \sum_{j=1}^{K} (a_n(i+j) - \bar{a}_n) \right)^2 \ge D_2 \tag{21}$$

for some constants $D_1, D_2 > 0$ and for appropriately chosen scores

$$\mathbf{a} = (a_n(1), \dots, a_n(n))$$

such that $\bar{a}_n = \frac{1}{n} \sum_{l=0}^{n} a_n(l)$. The assertion of the theorem then follows as a direct consequence of the proof of Theorem 3.6.1 in [8].

In order to show validity of (20) and (21), we notice that if $\{e_i, i \in \mathbb{N}\}$ is an α-mixing sequence, then $\{1/K \sum_{j=1}^{K} e_{Kl+k+j-1}, l \in \mathbb{N}\}$ is also α-mixing for each $k = 1, \dots, K$, but with smaller or equal mixing coefficients (see [2], Theorem 5.2).

Using Theorem B.6 in [8] for the sequences of partial sums, in combination with (19), we get

$$\frac{1}{n} \sum_{l=0}^{n-1} \left(\frac{1}{\sqrt{K}} \sum_{k=1}^{K} e_{l+k} \right)^2 \to \operatorname{var}\left(\frac{1}{\sqrt{K}} \sum_{k=1}^{K} e_k \right) \ge D_1, \quad \text{a.s.,} \tag{22}$$

$$\frac{1}{n} \sum_{l=0}^{n-1} \max_{k=0,\dots,K-1} \left| \frac{1}{\sqrt{K}} \sum_{j=k+1}^{K} e_{l+j} \right|^{2+\kappa} \to E \max_{k=0,\dots,K-1} \left| \frac{1}{\sqrt{K}} \sum_{j=k+1}^{K} e_{1+j} \right|^{2+\kappa} \le D_2, \quad \text{a.s.} \tag{23}$$

for some constants $D_1, D_2 > 0$.

Now, we consider three different situations and for each of them we choose the appropriate scores $\mathbf{a} = (a_n(1), \dots, a_n(n))$. We derive the results condtionally on given $Y_1, \dots, Y_n$.

(1) $Kd_n^2 = O(1)$. This case also includes the null hypothesis (with $d_n = 0$). We choose $a_n(i) = Y_i$.
(2) $1/(Kd_n^2) = O(1)$. In this case, we let $a_n(i) = Y_i/(\sqrt{K}d_n)$.
(3) Both $Kd_n^2 \leq 1$ and $Kd_n^2 > 1$ is true for infinitely many $n \in \mathbb{N}$. In this case, we use a combination of both score choices.

Using the arguments given in the proof of Theorem 3.6.2 in [8], we prove that the expressions in (22) and (23) are asymptotically equivalent to the left hand sides of (20) and (21) for all of the three considered situations. Hence, the assumptions of Theorem 2 are satisfied both under null hypothesis and under alternative and we get that along almost all samples $Y_1, \ldots, Y_n$, it holds that

$$\left(\max_{(p,q)\in\Pi_{l,k,L,K}} \frac{\left|S^{\mathbf{U}}_{L,K}(p,q,l,k)\right|}{\tau_{L,K}\sqrt{LK}}, \max_{(p,q)\in\tilde{\Pi}_{l,k,L,K}} \frac{\left|\tilde{S}^{\mathbf{U}}_{L,K}(p,q,l,k)\right|}{\tau_{L,K}\sqrt{LK}} \right)$$
$$\xrightarrow{\mathscr{D}} \left(\sup_{0<u<t} |W(u) - u/tW(t)|, \sup_{t<u<1} \left|\tilde{W}(u) - (1-u)/(1-t)\tilde{W}(t)\right| \right),$$

conditionally on $Y_1, \ldots, Y_n$, where

$$\tau^2_{L,K} = \frac{1}{n} \sum_{i=0}^{n-1} \left(\frac{1}{\sqrt{K}} \sum_{j=1}^{K} \left(Y_{i+j} - \bar{Y}_n\right) \right)^2. \tag{24}$$

The assertion of Theorem 3 is now straightforward, since the considered bootstrap statistic is a continuous function of the above vector of statistics. □

5. Simulation

We were interested in the performance of the test based on test statistic $T_n(\psi)$ with $\psi_{L_2}(x) = x$ and $\psi_{L_1}(x) = \mathrm{sgn}(x)$. We focused on comparison of the accuracy of critical values obtained by circular moving block bootstrap method with the accuracy of critical values obtained by simulation from the limit distribution. Some simulation results concerning the test based on asymptotic critical values for the studied type of test statistic can be also found in [5] and [9].

On Figures 1 (L_2 method) and 2 (L_1 method), one may see the size-power plots for choices of $n = 100$, 200 and $\gamma = 0.1$, 0.2. The ideal situation under null hypothesis is depicted by the straight dotted line. Under alternative, the desired situation would be a steep function with values close to 1. For more details on size-power plots we may refer e.g. to [8]. The random errors were simulated as an AR(1) process with AR coefficients 0.3 (dark gray lines) and 0.5 (light gray lines), and as a set of i.i.d. random errors with standard normal distribution (black lines). Rejection rates based on simulated asymptotic critical values are depicted by the dashed line, rejection rates based on block bootstrap with block length $K = 5$ are depicted by the solid line. On

Table 1. Simulated rejection rates with $n = 200$, $\gamma = 0.2$ under H_1 with $d = 1$ and $z = 1/2$. Random errors were simulated either as $N(0,1)-$ or as t_5-distributed AR(1) sequences with several values of autoregression coefficient φ

	L_2 statistic		L_1 statistic	
	$N(0,1)$	t_5	$N(0,1)$	t_5
$\varphi = 0$	0.922	0.925	0.801	0.906
$\varphi = 0.3$	0.718	0.733	0.585	0.696
$\varphi = 0.5$	0.500	0.508	0.404	0.485

figures 3 and 4, the results of similar simulations for AR(1) sequences with student t-distribution with 5 degrees of freedom are shown.

In the simulation study, we generated 10,000 independent samples in order to compute asymptotic critical values. When bootstrapping, for each sample we used 1000 bootstrap samples to compute bootstrap critical values. In simulations of rejection rates, we used 1000 repetitions.

In all of 4 figures depicting situation under the null hypothesis, we may see that comparing to the critical values obtained by simulation from the asymptotic distribution, critical values obtained by bootstrapping are more accurate. When comparing the accuracy of critical values for different choices of score function ψ, the L_1 method seem to be better then the L_2 method. However, when using the L_1-method, power of the test slightly decreases, as we may see from Table 1. Similarly, the choice of $\gamma = 0.2$ seem to provide more accurate critical values than the choice of $\gamma = 0.1$, but the test power is larger in the latter case. Furthermore, with the choice of ψ_{L_2}, the simulated rejection rates under H_0 are higher than the corresponding theoretical α-levels for larger values of the autoregression coefficient, while for the L_1-method they remain much more stable. Comparing the case of $N(0,1)$ innovations with the case of t_5 innovations, rejection rates for the L_1 version of the test statistic tend to be slightly higher for t_5 distribution, while they remain more or less the same for the L_2 version. As we expected, the accuracy of the critical values tend to be better for larger n.

6. Summary

Ratio type statistics provide an alternative to non-ratio statistics in situations, in which variance estimation is problematic. We proved that the block bootstrap method provides asymptotically correct critical values for the studied ratio type statistic in the

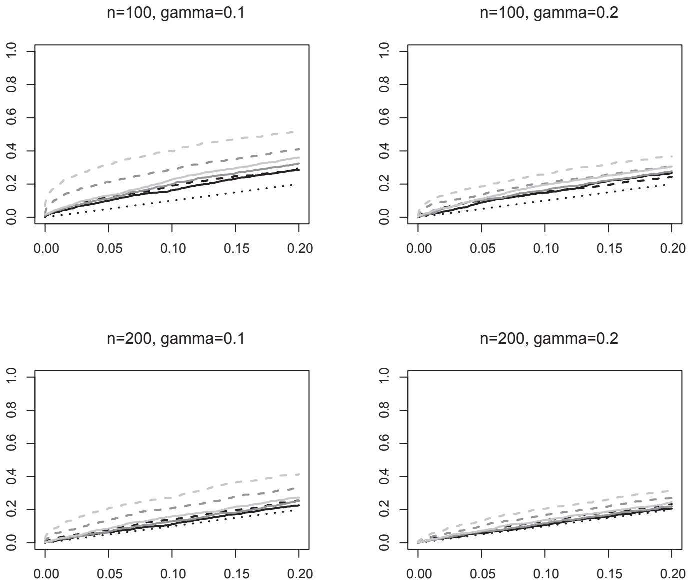

Figure 1. Null hypothesis, $N(0, 1)$-distributed innovations, L_2 version of the test statistic

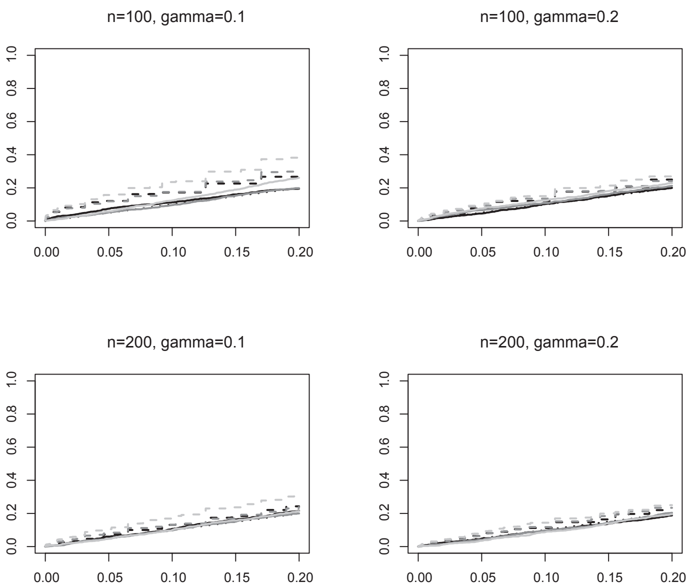

Figure 2. Null hypothesis, $N(0, 1)$-distributed innovations, L_1 version of the test statistic

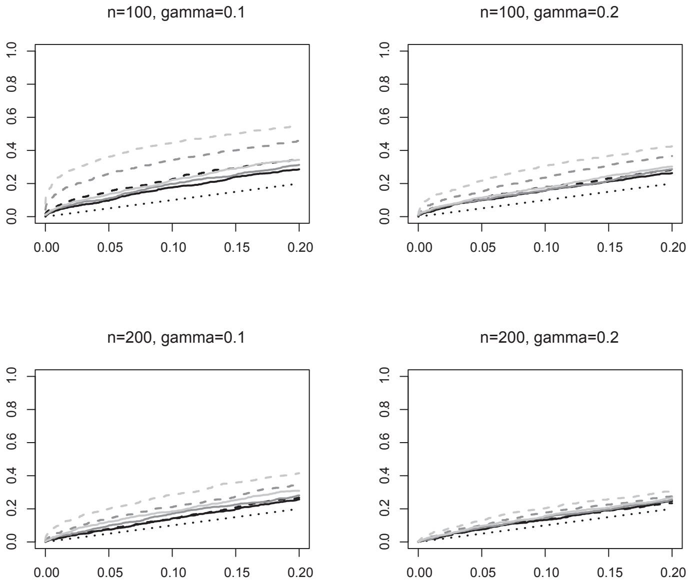

FIGURE 3. Null hypothesis, t_5-distributed innovations, L_2 version of the test statistic

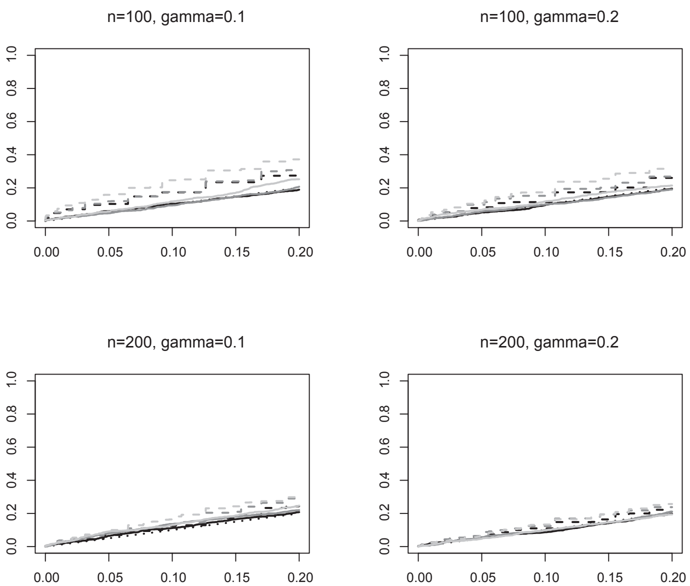

FIGURE 4. Null hypothesis, t_5-distributed innovations, L_1 version of the test statistic

location model with α-mixing random errors. Simulations showed that critical values obtained by (block-)bootstrapping seem to be more accurate than critical values obtained by simulation from the limiting distribution, especially for AR(1) sequences.

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