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Distinguished Riemann-Hamilton geometry in the polymomentum electrodynamics

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Abstract. In this paper we develop the distinguished (d-) Riemannian differential geometry (in the sense of d-connections, d-torsions, d-curvatures and some geometrical Maxwell-like and Einstein-like equations) for the polymomentum Hamiltonian which governs the multi-time electrodynamics.

1 Introduction

Let M^n be a smooth real manifold of dimension n, whose local coordinates are $x = (x^i)_{i=\overline{1,n}}$, having the physical meaning of *"space of events"*. In order to justify the *"electrodynamics"* terminology used in this paper, we recall that, in the study of classical electrodynamics, the Lagrangian function $L\colon TM \to \mathbb{R}$ that governs the movement law of a particule of mass $m \neq 0$ and electric charge e, placed concomitantly into a gravitational field and an electromagnetic one, is expressed by

$$L(x, y) = mc\varphi_{ij}(x)y^i y^j + \frac{2e}{m} A_i(x)y^i + \mathcal{P}(x)\,, \tag{1}$$

where the semi-Riemannian metric $\varphi_{ij}(x)$ represents the *gravitational potentials* of the space M, $A_i(x)$ are the components of an 1-form on M representing the *electromagnetic potential*, $\mathcal{P}(x)$ is a smooth *potential function* on M and c is the velocity of light in vacuum. The Lagrange space $L^n = (M, L(x, y))$, where L is given by (1), is known in the literature of specialty as the *autonomous Lagrange space of electrodynamics*. A deep geometrical study of the Lagrange space L^n is now completely done in Miron-Anastasiei's book [15]. More general, in the study of classical *time-dependent* electrodynamics, a central role is played by the

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autonomous time-dependent Lagrangian function of electrodynamics:

$$L(t,x,y) = mc\varphi_{ij}(x)y^i y^j + \frac{2e}{m}A_i(t,x)y^i + \mathcal{P}(t,x)\,, \tag{2}$$

where $L\colon \mathbb{R}\times TM \to \mathbb{R}$. Note that the non-dynamical character (i.e., the independence on the temporal coordinate t) of the spatial semi-Riemannian metric $\varphi_{ij}(x)$ determines the usage of the term *"autonomous"* in the preceding definition.

Let $(\mathcal{T}^m, h_{ab}(t))$ be a *"multi-time"* smooth Riemannian manifold of dimension m (please do not confuse with the mass $m \neq 0$), having the local coordinates $t = (t^c)_{c=\overline{1,m}}$, and let $J^1(\mathcal{T}, M)$ be the 1-jet space produced by the manifolds $\mathcal{T}$ and M.

Remark 1. The use in our work of the *"multi-time"* terminology was lent by us from Dickey's monograph [6]. However, it is important to note that "multi-time" does not mean a "multidimensional time", but has the sense of a *"multi-parameter"* or *"many parameters"*.

By a natural extension of the preceding examples of electrodynamics Lagrangian functions, we can consider the jet multi-time Lagrangian function

$$L(t^c, x^k, x^k_c) = mch^{ab}(t)\varphi_{ij}(x)x^i_a x^j_b + \frac{2e}{m}A^{(a)}_{(i)}(t,x)x^i_a + \mathcal{P}(t,x)\,, \tag{3}$$

where $A^{(a)}_{(i)}(t,x)$ is a d-tensor on $J^1(\mathcal{T}, M)$ and $\mathcal{P}(t,x)$ is a smooth function on the product manifold $\mathcal{T}\times M$.

Remark 2. Throughout this paper, the indices $a, b, c, \dots$ run from 1 to m, while the indices $i, j, k, \dots$ run from 1 to n. The Einstein convention of summation is also adopted all over this work.

The pair $\mathcal{EDML}^n_m = (J^1(\mathcal{T}, M), L)$, where L is given by (3), is called the *autonomous multi-time Lagrange space of electrodynamics*. The distinguished Riemannian geometrization of the multi-time Lagrange space $\mathcal{EDML}^n_m$ is now completely developed in the Neagu's works [17] and [18].

Via the classical Legendre transformation, the jet multi-time Lagrangian function of electrodynamics (3) leads us to the Hamiltonian function of polymomenta

$$H = \frac{1}{4mc}h_{ab}\varphi^{ij}p^a_i p^b_j - \frac{e}{m^2c}h_{ab}\varphi^{ij}A^{(b)}_{(j)}p^a_i + \frac{e^2}{m^3c}\left\|A\right\|^2 - \mathcal{P}\,, \tag{4}$$

where $H\colon J^{1*}(\mathcal{T}, M) \to \mathbb{R}$, and

$$\left\|A\right\|^2(t,x) = h_{ab}\varphi^{ij}A^{(a)}_{(i)}A^{(b)}_{(j)}\,.$$

Definition 1. The pair $\mathcal{EDMH}^n_m = (J^{1*}(\mathcal{T}, M), H)$, where H is given by (4), is called the *autonomous multi-time Hamilton space of electrodynamics*.

But, using as a pattern the Miron's geometrical ideas from [16], the distinguished Riemannian geometry for quadratic Hamiltonians of polymomenta (geometry in the sense of d-connections, d-torsions, d-curvatures and geometrical Maxwell-like and Einstein-like equations) is constructed on dual 1-jet spaces in the Oană-Neagu's paper [21]. Consequently, in what follows, we apply the general geometrical result from [21] for the particular Hamiltonian function of polymomenta (4), which governs the multi-time electrodynamics.

2 The geometry of the autonomous multi-time Hamilton space of electrodynamics $\mathcal{EDMH}^n_m$

To initiate our Hamiltonian geometrical development for multi-time electrodynamics, let us consider on the dual 1-jet space $E^* = J^{1*}(\mathcal{T}, M)$ the *fundamental vertical metrical d-tensor*

$$\Phi^{(i)(j)}_{(a)(b)} = \frac{1}{2}\frac{\partial^2 H}{\partial p^a_i \partial p^b_j} = h^*_{ab}(t^c)\varphi^{ij}(x^k)\,,$$

where $h^*_{ab}(t) := (4mc)^{-1} \cdot h_{ab}(t)$. Let $\chi^a_{bc}(t)$ (respectively $\gamma^k_{ij}(x)$) be the Christoffel symbols of the metric $h_{ab}(t)$ (respectively $\varphi_{ij}(x)$). Obviously, if $\overset{*}{\chi}{}^a_{bc}$ are the Christoffel symbols of the Riemannian metric $h^*_{ab}(t)$, then we have $\overset{*}{\chi}{}^a_{bc} = \chi^a_{bc}$.

Using a general result from the geometrical theory of multi-time Hamilton spaces (see [2] and [21]), by direct computations, we find

Theorem 1. *The pair of local functions $N_{\mathcal{ED}} = \left(\underset{1}{N}{}^{(a)}_{(i)b},\ \underset{2}{N}{}^{(a)}_{(i)j}\right)$ on the dual 1-jet space E^*, which are given by*

$$\underset{1}{N}{}^{(a)}_{(i)b} = \chi^a_{bf}p^f_i\,, \qquad \underset{2}{N}{}^{(a)}_{(i)j} = \gamma^r_{ij}\left[\frac{2e}{m}A^{(a)}_{(r)} - p^a_r\right] - \frac{e}{m}\left[\frac{\partial A^{(a)}_{(i)}}{\partial x^j} + \frac{\partial A^{(a)}_{(j)}}{\partial x^i}\right],$$

represents a nonlinear connection on E^*. This nonlinear connection is called the canonical nonlinear connection of the multi-time Hamilton space of electrodynamics $\mathcal{EDMH}^n_m$.

Now, let

$$\left\{\frac{\delta}{\delta t^a}, \frac{\delta}{\delta x^i}, \frac{\partial}{\partial p^a_i}\right\} \subset \chi\left(E^*\right), \qquad \left\{dt^a, dx^i, \delta p^a_i\right\} \subset \chi^*\left(E^*\right)$$

be the adapted bases produced by the nonlinear connection $N_{\mathcal{ED}}$, where

$$\frac{\delta}{\delta t^a} = \frac{\partial}{\partial t^a} - \underset{1}{N}{}^{(f)}_{(r)a}\frac{\partial}{\partial p^f_r}\,, \qquad \frac{\delta}{\delta x^i} = \frac{\partial}{\partial x^i} - \underset{2}{N}{}^{(f)}_{(r)i}\frac{\partial}{\partial p^f_r}\,,$$
$$\delta p^a_i = dp^a_i + \underset{1}{N}{}^{(a)}_{(i)f}dt^f + \underset{2}{N}{}^{(a)}_{(i)r}dx^r\,. \tag{5}$$

Working with these adapted bases, by direct computations, we can determine the adapted components of the *generalized Cartan canonical connection* of the space $\mathcal{EDMH}^n_m$, together with its local d-torsions and d-curvatures (for details, see the general formulas from [21]).

Theorem 2. (1) *The generalized Cartan canonical linear connection of the autonomous multi-time Hamilton space of electrodynamics $\mathcal{EDMH}^n_m$ is given by*

$$C\Gamma(N) = \left(\chi^a_{bc},\ A^i_{jc},\ H^i_{jk},\ C^{i(k)}_{j(c)}\right),$$

where its adapted components are

$$H^c_{ab} = \chi^c_{ab}\,, \quad A^i_{jc} = 0\,, \quad H^i_{jk} = \gamma^i_{jk}\,, \quad C^{i(k)}_{j(c)} = 0\,. \tag{6}$$

(2) *The torsion $\mathbb{T}$ of the generalized Cartan canonical linear connection of the space $\mathcal{EDMH}^n_m$ is determined by* three *effective adapted components:*

$$\begin{aligned} R^{(f)}_{(r)ab} &= \chi^f_{gab} p^g_r\,, \\ R^{(f)}_{(r)aj} &= -\frac{2e}{m}\gamma^s_{rj} A^{(f)}_{(s);a} + \frac{e}{m}\left[\frac{\partial A^{(f)}_{(r)}}{\partial x^j} + \frac{\partial A^{(f)}_{(j)}}{\partial x^r}\right]_{;a}, \\ R^{(f)}_{(r)ij} &= \mathfrak{R}^s_{rij}\left[\frac{2e}{m}A^{(f)}_{(s)} - p^f_s\right] - \frac{e}{m}\left[\frac{\partial A^{(f)}_{(i)}}{\partial x^j} - \frac{\partial A^{(f)}_{(j)}}{\partial x^i}\right]_{:r}, \end{aligned} \tag{7}$$

where $\chi^c_{dab}(t)$ (respectively $\mathfrak{R}^k_{rij}(x)$) are the classical local curvature tensors of the Riemannian metric $h_{ab}(t)$ (respectively semi-Riemannian metric $\varphi_{ij}(x)$), and "$_{;a}$" and "$_{:k}$" represent the following generalized Levi-Civita covariant derivatives*:*

- *the $\mathcal{T}$-generalized Levi-Civita covariant derivative:*

$$\begin{aligned} T^{bi(d)(r)\dots}_{cj(l)(f)\dots;a} \stackrel{\text{def}}{=} \frac{\partial T^{bi(d)(r)\dots}_{cj(l)(f)\dots}}{\partial t^a} &+ T^{gi(d)(r)\dots}_{cj(l)(f)\dots}\chi^b_{ga} + T^{bi(g)(r)\dots}_{cj(l)(f)\dots}\chi^d_{ga} \\ &+ \dots - T^{bi(d)(r)\dots}_{gj(l)(f)\dots}\chi^g_{ca} - T^{bi(d)(r)\dots}_{cj(l)(g)\dots}\chi^g_{fa} - \dots\,, \end{aligned}$$

- *the M-generalized Levi-Civita covariant derivative:*

$$\begin{aligned} T^{bi(d)(r)\dots}_{cj(l)(f)\dots:k} \stackrel{\text{def}}{=} \frac{\partial T^{bi(d)(r)\dots}_{cj(l)(f)\dots}}{\partial x^k} &+ T^{bs(d)(r)\dots}_{cj(l)(f)\dots}\gamma^i_{sk} + T^{bi(d)(s)\dots}_{cj(l)(f)\dots}\gamma^r_{sk} \\ &+ \dots - T^{bi(d)(r)\dots}_{cs(l)(f)\dots}\gamma^s_{jk} - T^{bi(d)(r)\dots}_{cj(s)(f)\dots}\gamma^s_{lk} - \dots\,. \end{aligned}$$

(3) *The curvature $\mathbb{R}$ of the Cartan canonical connection of the space $\mathcal{EDMH}^n_m$ is determined by the following* four *effective adapted components:*

$$H^d_{abc} = \chi^d_{abc}\,, \quad R^l_{ijk} = \mathfrak{R}^l_{ijk}$$

and

$$-R^{(d)(i)}_{(l)(a)bc} = \delta^i_l \chi^d_{abc}\,, \quad -R^{(d)(l)}_{(i)(a)jk} = -\delta^d_a \mathfrak{R}^l_{ijk}\,.$$

3 Electromagnetic-like model on the multi-time Hamilton space of electrodynamics $\mathcal{EDM}H^n_m$

In order to describe our geometrical electromagnetic-like theory (depending on polymomenta) on the multi-time Hamilton space of electrodynamics $\mathcal{EDM}H^n_m$, we underline that, by a simple direct calculation, we obtain (see [21]).

Proposition 1. *The* metrical deflection d-tensors *of the space* $\mathcal{EDM}H^n_m$ *are expressed by the formulas:*

$$\Delta^{(i)}_{(a)b} = \left[h^*_{af}\varphi^{ir}p^f_r\right]_{/b} = 0\,, \qquad \vartheta^{(i)(j)}_{(a)(b)} = [h^*_{af}\varphi^{ir}p^f_r]\Big|^{(j)}_{(b)} = \frac{1}{4mc}h_{ab}\varphi^{ij}\,, \\ \Delta^{(i)}_{(a)j} = [h^*_{af}\varphi^{ir}p^f_r]_{|j} = \frac{e}{4m^2c}h_{af}\varphi^{ir}\left[A^{(f)}_{(r):j} + A^{(f)}_{(j):r}\right], \tag{8}$$

where "$_{/b}$", "$_{|j}$" and "$\big|^{(b)}_{(j)}$" are the local covariant derivatives induced by the generalized Cartan canonical connection $C\Gamma(N)$ (see [20] and [21]).

Moreover, taking into account some general formulas from [21], we introduce

Definition 2. The distinguished 2-form on $J^{1*}(\mathcal{T}, M)$, locally defined by

$$\mathbb{F} = F^{(i)}_{(a)j}\delta p^a_i \wedge dx^j + f^{(i)(j)}_{(a)(b)}\delta p^a_i \wedge \delta p^b_j\,, \tag{9}$$

where

$$F^{(i)}_{(a)j} = \frac{1}{2}\left[\Delta^{(i)}_{(a)j} - \Delta^{(j)}_{(a)i}\right] = \frac{e}{8m^2c}\cdot \underset{\{i,j\}}{\mathcal{A}}\left\{h_{af}\varphi^{ir}\left[A^{(f)}_{(r):j} + A^{(f)}_{(j):r}\right]\right\}, \\ f^{(i)(j)}_{(a)(b)} = \frac{1}{2}\left[\vartheta^{(i)(j)}_{(a)(b)} - \vartheta^{(j)(i)}_{(a)(b)}\right] = 0\,, \tag{10}$$

is called the *polymomentum electromagnetic field attached to the multi-time Hamilton space of electrodynamics* $\mathcal{EDM}H^n_m$.

Now, particularizing the generalized Maxwell-like equations of the polymomentum electromagnetic field that govern a general multi-time Hamilton space MH^n_m, we obtain the main result of the polymomentum electromagnetism on the space $\mathcal{EDM}H^n_m$ (for more details, see [21]):

Theorem 3. *The polymomentum electromagnetic components* (10) *of the autonomous multi-time Hamilton space of electrodynamics* $\mathcal{EDM}H^n_m$ *are governed by* *the following* geometrical Maxwell-like equations*:*

$$\begin{cases} F^{(i)}_{(a)j/b} = F^{(i)}_{(a)j;b} = \dfrac{e \cdot h_{af}}{8m^2c} \cdot \underset{\{i,j\}}{\mathcal{A}} \left\{ \varphi^{ir} \left[\dfrac{\partial A^{(f)}_{(r)}}{\partial x^j} + \dfrac{\partial A^{(f)}_{(j)}}{\partial x^r} \right]_{;b} -2\varphi^{ir}\gamma^s_{rj} A^{(f)}_{(s);b} \right\} \\ \sum_{\{i,j,k\}} F^{(i)}_{(a)j|k} = \sum_{\{i,j,k\}} F^{(i)}_{(a)j:k} = -\dfrac{h_{af}}{8mc} \cdot \sum_{\{i,j,k\}} \left\{ \left[\varphi^{sr}\mathfrak{R}^i_{rjk} - \varphi^{ir}\mathfrak{R}^s_{rjk} \right] p^f_s + \right. \\ \qquad \left. + \dfrac{e}{m}\varphi^{ir} \left[2\mathfrak{R}^s_{rjk} A^{(f)}_{(s)} - \left(\dfrac{\partial A^{(f)}_{(j)}}{\partial x^k} - \dfrac{\partial A^{(f)}_{(k)}}{\partial x^j} \right)_{:r} \right] \right\} \\ \sum_{\{i,j,k\}} F^{(i)}_{(a)j} \Big|^{(k)}_{(c)} = 0\,, \end{cases} \tag{11}$$

where $\mathcal{A}_{\{i,j\}}$ *represents an alternate sum,* $\sum_{\{i,j,k\}}$ *represents a cyclic sum, and we have*

$$F^{(i)}_{(a)j} \Big|^{(k)}_{(c)} = \frac{\partial F^{(i)}_{(a)j}}{\partial p^c_k} = 0\,.$$

4 Gravitational-like geometrical model on the multi-time Hamilton space of electrodynamics

To expose our geometrical Hamiltonian polymomentum gravitational theory on the autonomous multi-time Hamilton space of electrodynamics $\mathcal{EDMH}^n_m$, we recall that the fundamental vertical metrical d-tensor

$$\Phi^{(i)(j)}_{(a)(b)} = h^*_{ab}(t)\varphi^{ij}(x)$$

and the canonical nonlinear connection

$$N_{\mathcal{ED}} = \left(\underset{1}{N}{}^{(a)}_{(i)b},\ \underset{2}{N}{}^{(a)}_{(i)j} \right)$$

of the multi-time Hamilton space $\mathcal{EDMH}^n_m$ produce a polymomentum gravitational h^*-potential $\mathbb{G}$ on $E^* = J^{1*}(\mathcal{T}, M)$, locally expressed by

$$\mathbb{G} = h^*_{ab} dt^a \otimes dt^b + \varphi_{ij} dx^i \otimes dx^j + h^*_{ab}\varphi^{ij} \delta p^a_i \otimes \delta p^b_j\,. \tag{12}$$

We postulate that the *geometrical Einstein-like equations*, which govern the multi-time gravitational h^*-potential $\mathbb{G}$ of the multi-time Hamilton space of electrodynamics $\mathcal{EDMH}^n_m$, are the abstract geometrical Einstein equations attached to the Cartan canonical connection $C\Gamma(N)$ and to the adapted metric $\mathbb{G}$ on E^*, namely

$$\mathrm{Ric}(C\Gamma) - \frac{\mathrm{Sc}(C\Gamma)}{2}\mathbb{G} = \mathcal{K}\mathbb{T}\,, \tag{13}$$

where $\mathrm{Ric}(C\Gamma)$ represents the *Ricci tensor* of the Cartan connection, $\mathrm{Sc}(C\Gamma)$ is the *scalar curvature*, $\mathcal{K}$ is the *Einstein constant* and $\mathbb{T}$ is an intrinsic d-tensor of matter, which is called the *stress-energy d-tensor of polymomenta*.

In order to describe the local geometrical Einstein-like equations (together with their generalized conservation laws) in the adapted basis

$$\{X_A\} = \left\{ \frac{\delta}{\delta t^a}\,,\ \frac{\delta}{\delta x^i}\,,\ \frac{\partial}{\partial p_i^a} \right\},$$

let $C\Gamma(N) = (\chi^c_{ab}, 0, \gamma^i_{jk}, 0)$ be the generalized Cartan canonical connection of the space $\mathcal{EDMH}^n_m$. Taking into account the expressions of its adapted curvature d-tensors on the space $\mathcal{EDMH}^n_m$, we immediately find (see [21]):

Theorem 4. *The Ricci tensor* $\mathrm{Ric}(C\Gamma)$ *of the autonomous multi-time Hamilton space of electrodynamics* $\mathcal{EDMH}^n_m$ *is characterized by* two *effective local Ricci d-tensors:*

$$\chi_{ab} = \chi^f_{abf}\,, \qquad \mathfrak{R}_{ij} = \mathfrak{R}^r_{ijr}\,.$$

These are exactly the classical Ricci tensors of the Riemannian temporal metric $h_{ab}(t)$ *and the semi-Riemannian spatial metric* $\varphi_{ij}(x)$.

Consequently, using the notations $\chi = h^{ab}\chi_{ab}$ and $\mathfrak{R} = \varphi^{ij}\mathfrak{R}_{ij}$, we get

Theorem 5. *The scalar curvature* $\mathrm{Sc}(C\Gamma)$ *of the generalized Cartan connection* $C\Gamma$ *of the space* $\mathcal{EDMH}^n_m$ *has the expression (for details, see [21])*

$$Sc(C\Gamma) = (4mc)\cdot\chi + \mathfrak{R}\,,$$

where χ *and* $\mathfrak{R}$ *are the classical scalar curvatures of the semi-Riemannian metrics* $h_{ab}(t)$ *and* $\varphi_{ij}(x)$.

Particularizing the generalized Einstein-like equations and the generalized conservation laws of an arbitrary multi-time Hamilton space MH^n_m, we can establish the main result of the geometrical polymomentum gravitational theory on the autonomous multi-time Hamilton space of electrodynamics $\mathcal{EDMH}^n_m$ (for more details, see [21]):

Theorem 6. (1) *The local* geometrical Einstein-like equations, *that govern the polymomentum gravitational potential of the space* $\mathcal{EDMH}^n_m$, *have the form*

$$\begin{cases} \chi_{ab} - \dfrac{(4mc)\cdot\chi + \mathfrak{R}}{8mc} h_{ab} = \mathcal{K}\mathbb{T}_{ab} \\[2mm] \mathfrak{R}_{ij} - \dfrac{(4mc)\cdot\chi + \mathfrak{R}}{2} \varphi_{ij} = \mathcal{K}\mathbb{T}_{ij} \\[2mm] -\dfrac{(4mc)\cdot\chi + \mathfrak{R}}{8mc} h_{ab}\varphi^{ij} = \mathcal{K}\mathbb{T}^{(i)(j)}_{(a)(b)}, \end{cases} \tag{14}$$

$$\begin{cases} 0 = \mathbb{T}_{ai}, \quad 0 = \mathbb{T}_{ia}, \quad 0 = \mathbb{T}^{(i)}_{(a)b} \\ 0 = \mathbb{T}^{\ (j)}_{a(b)}, \quad 0 = \mathbb{T}^{\ (j)}_{i(b)}, \quad 0 = \mathbb{T}^{(i)}_{(a)j}, \end{cases} \tag{15}$$

where $\mathbb{T}_{AB},\ A, B \in \left\{a, i, {(i) \atop (a)}\right\}$, *are the adapted components of the polymomentum stress-energy d-tensor of matter* $\mathbb{T}$.

(2) *The* polymomentum conservation laws *of the geometrical Einstein-like equations of the space $\mathcal{EDMH}^n_m$ are expressed by the formulas*

$$\begin{cases} \left[(4mc)\cdot\chi^f_b - \dfrac{(4mc)\cdot\chi+\mathfrak{R}}{2}\delta^f_b\right]_{/f} = 0 \\ \left[\mathfrak{R}^r_j - \dfrac{(4mc)\cdot\chi+\mathfrak{R}}{2}\delta^r_j\right]_{|r} = 0, \end{cases} \tag{16}$$

where $\chi^f_b = h^{fd}\chi_{db}$ *and* $\mathfrak{R}^r_j = \varphi^{rs}\mathfrak{R}_{sj}$.

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