Czechoslovak Mathematical Journal

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Czechoslovak Mathematical Journal, Vol. 62 (2012), No. 3, 645–654

Persistent URL: `http://dml.cz/dmlcz/143016`

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THE DIOPHANTINE EQUATION $x^2 + 2^a \cdot 17^b = y^n$

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(Received November 24, 2010)

Abstract. Let $\mathbb{Z}$, $\mathbb{N}$ be the sets of all integers and positive integers, respectively. Let p be a fixed odd prime. Recently, there have been many papers concerned with solutions (x, y, n, a, b) of the equation $x^2 + 2^a p^b = y^n$, $x, y, n \in \mathbb{N}$, $\gcd(x, y) = 1$, $n \geqslant 3$, $a, b \in \mathbb{Z}$, $a \geqslant 0$, $b \geqslant 0$. And all solutions of it have been determined for the cases $p = 3$, $p = 5$, $p = 11$ and $p = 13$. In this paper, we mainly concentrate on the case $p = 3$, and using certain recent results on exponential diophantine equations including the famous Catalan equation, all solutions (x, y, n, a, b) of the equation $x^2 + 2^a \cdot 17^b = y^n$, $x, y, n \in \mathbb{N}$, $\gcd(x, y) = 1$, $n \geqslant 3$, $a, b \in \mathbb{Z}$, $a \geqslant 0$, $b \geqslant 0$, are determined.

Keywords: exponential diophantine equation, modular approach, arithmetic properties of Lucas numbers

MSC 2010: 11D61

1. Introduction

Let $\mathbb{Z}, \mathbb{N}$ be the sets of all integers and positive integers respectively. Let p be a fixed odd prime. Recently, there have been many papers concerned with solutions (x, y, n, a, b) of the equation

$$(1.1) \quad x^2 + 2^a p^b = y^n, \quad x, y, n \in \mathbb{N},\ \gcd(x, y) = 1,\ n \geqslant 3,\ a, b \in \mathbb{Z},\ a \geqslant 0,\ b \geqslant 0.$$

All solutions of (1.1) have been determined for the following cases:

1. (F. Luca [12]) $p = 3$.
2. (F. Luca and A. Togbé [13]) $p = 5$.
3. (I. N. Cangul, M. Demirci, F. Luca, A. Pintér and G. Soydan [5]) $p = 11$.
4. (F. Luca and A. Togbé [14]) $p = 13$.

In this paper, using certain recent results on exponential diophantine equations including the famous Catalan equation, we solve (1.1) for $p = 17$. We prove the following result:

Theorem. *The equation*

$$(1.2)\quad x^2 + 2^a \cdot 17^b = y^n, \quad x, y, n \in \mathbb{N},\ \gcd(x, y) = 1,\ n \geqslant 3,\ a, b \in \mathbb{Z},\ a \geqslant 0,\ b \geqslant 0$$

has only the solutions $(x, y, n, a, b) = (5, 3, 3, 1, 0)$, $(7, 3, 4, 5, 0)$, $(11, 5, 3, 2, 0)$, $(8, 3, 4, 0, 1)$, $(1087, 33, 4, 8, 1)$, $(5, 7, 4, 7, 1)$, $(9, 5, 4, 5, 1)$, $(47, 9, 4, 8, 1)$, $(47, 3, 8, 8, 1)$ *and* $(495, 23, 4, 11, 1)$.

We notice that if $p \equiv 7 \pmod 8$, then (1.1) probably has solutions (x, y, n, a, b) with $a = 0$ and y is even. Thus it can be seen that this case is very hard.

Equation (1.1) is a special case of the general exponential diophantine equation $Ax^m + Bz^r = Cy^n$ and such equations can be thought of as generalized Fermat equations. These equations can be attacked using the abc conjecture, which is still a famous unsolved problem. The abc conjecture says that for any fixed $\varepsilon > 0$, there is a constant $K(\varepsilon)$ such that if $a + b = c$ are three mutually coprime integers, then

$$\max(|a|, |b|, |c|) \leqslant K(\varepsilon)(\mathrm{rad}(abc))^{1+\varepsilon},$$

where (rad(abc) is the product of the distinct primes dividing abc. Applying this conjeture to our equation $x^2 + 2^a p^b = y^n$ shows that for any fixed prime p there are only finitely many (x, y, n, a, b). Since the abc conjeture is still unsolved, it is of interest to try to show this directly using other methods. This is why we look at this equation.

2. Preliminaries

Lemma 2.1 ([7]). *The equation*

$$(2.1)\qquad X^3 + 1 = 2Y^2, \quad X, Y \in \mathbb{N}$$

has only the solutions $(X, Y) = (1, 1)$ *and* $(23, 78)$.

Let q be an odd prime, let D be a positive integer.

Lemma 2.2 ([2]). *If* $q \geqslant 5$, *then the equation*

$$(2.2)\qquad X^q + 1 = 2Y^2, \quad X, Y \in \mathbb{N}$$

has only the solution $(X, Y) = (1, 1)$.

Lemma 2.3 ([11]). *The equation*

$$X^4 - DY^2 = -1, \quad X, Y \in \mathbb{N} \tag{2.3}$$

has at most one solution (X, Y).

Applying Lemma 2.3, we can immediately obtain the following lemma.

Lemma 2.4. *The equation*

$$X^4 - 2Y^2 = -1, \quad X, Y \in \mathbb{N} \tag{2.4}$$

has only the solution $(X, Y) = (1, 1)$.

Lemma 2.5 ([8], [10]). *The equation*

$$X^2 + 2^m = Y^n, \quad X, Y, m, n \in \mathbb{N},\ \gcd(X, Y) = 1,\ n \geqslant 2 \tag{2.5}$$

has only the solutions $(X, Y, m, n) = (3, 5, 4, 2)$, $(5, 3, 1, 3)$, $(7, 3, 5, 4)$ *and* $(11, 5, 2, 3)$.

Lemma 2.6 ([16]). *The equation*

$$X^2 - 2^m = Y^3, \quad X, Y, m \in \mathbb{N},\ \gcd(X, Y) = 1,\ Y > 1 \tag{2.6}$$

has only the solution $(X, Y, m) = (71, 17, 7)$.

Lemma 2.7 ([17]). *If* $q \geqslant 5$, *then the equation*

$$X^2 - 2^m = Y^q, \quad X, Y, m \in \mathbb{N},\ \gcd(X, Y) = 1,\ Y > 1,\ m > 1 \tag{2.7}$$

has no solution (X, Y, m).

Lemma 2.8 ([3]). *If* $D = 2^{2r} - 3 \cdot 2^{r+1} + 1$, *where* r *is a positive integer with* $r > 1$, *then the equation*

$$X^2 - D = 2^n, \quad X, n \in \mathbb{N} \tag{2.8}$$

has only the solutions $(X, n) = (2^r - 3, 3)$, $(2^r - 1, r + 2)$, $(2^r + 1, r + 3)$ *and* $(3 \cdot 2^r - 1, 2r + 3)$.

Put $r = 3$. Applying Lemma 2.8, we can obtain the following lemma.

Lemma 2.9. *The equation*

$$X^2 - 17 = 2^m, \quad X, m \in \mathbb{N} \tag{2.9}$$

has only the solutions $(X, n) = (5, 3)$, $(7, 5)$, $(9, 6)$ *and* $(23, 9)$.

Lemma 2.10 ([15]). *The equation*

$$X^m - Y^n = 1, \quad X, Y, m, n \in \mathbb{N},\ \min(X, Y, m, n) > 1 \tag{2.10}$$

has only the solution $(X, Y, m, n) = (3, 2, 2, 3)$.

Let D, k be positive integers such that $k > 1$, $2 \nmid k$ and $\gcd(D, k) = 1$. Further let $h(-4D)$ denote the class number of positive binary quadratic forms of discriminant $-4D$.

Lemma 2.11 ([9]). *Every solution* (X, Y, Z) *of the equation*

$$X^2 + DY^2 = k^Z, \quad X, Y, Z \in \mathbb{Z},\ \gcd(X, Y) = 1,\ Z > 0 \tag{2.11}$$

can be expressed as

$$Z = Z_1 t, \quad t \in \mathbb{N},$$
$$X + Y\sqrt{-D} = \lambda_1 (X_1 + \lambda_2 Y_1 \sqrt{-D})^t, \quad \lambda_1, \lambda_2 \in \{\pm 1\},$$

where (X_1, Y_1, Z_1) *is a positive integer solution of* (2.11) *with* $Z_1 \mid h(-4D)$.

Let α, β be algebraic integers. If $\alpha + \beta$ and $\alpha\beta$ are nonzero co-prime integers and α/β is not a root of unity, then (α, β) is called a Lucas pair. Further, let $f = \alpha + \beta$ and $g = \alpha\beta$. Then we have

$$\alpha = \frac{1}{2}\bigl(f + \lambda\sqrt{d}\bigr),\ \beta = \frac{1}{2}\bigl(f - \lambda\sqrt{d}\bigr), \quad \lambda \in \{\pm 1\},$$

where $d = f^2 - 4g$. We call (f, d) the parameters of the Lucas pair (α, β). Two Lucas pairs (α_1, β_1) and (α_2, β_2) are equivalent if $\alpha_1/\alpha_2 = \beta_1/\beta_2 = \pm 1$. Given a Lucas pair (α, β), one defines the corresponding sequence of Lucas numbers by

$$L_n(\alpha, \beta) = \frac{\alpha^n - \beta^n}{\alpha - \beta}, \quad n = 0, 1, 2, \ldots. \tag{2.12}$$

For equivalent Lucas pairs (α_1, β_1) and (α_2, β_2), we have $L_n(\alpha_1, \beta_1) = \pm L_n(\alpha_2, \beta_2)$ for any $n \geqslant 0$. A prime l is called a primitive divisor of $L_n(\alpha, \beta)$ $(n > 1)$ if

$$l \mid L_n(\alpha, \beta) \quad \text{and} \quad l \nmid dL_1(\alpha, \beta) \ldots L_{n-1}(\alpha, \beta).$$

Lemma 2.12 ([6]). *If l is a primitive divisor of $L_n(\alpha,\beta)$, then $l \equiv \pm 1 \pmod{n}$.*

A Lucas pair (α,β) such that $L_n(\alpha,\beta)$ has no primitive divisor will be called an n-defective Lucas pair.

Lemma 2.13 ([1], [18]). *Let n satisfy $4 < n \leqslant 30$ and $n \neq 6$. Then, up to equivalence, all parameters of n-defective Lucas pairs are given as follows:*

(i) $n = 5$, $(f,d) = (1,5)$, $(1,-7)$, $(2,-40)$, $(1,-11)$, $(1,-15)$, $(12,-76)$ *and* $(12,-1364)$.
(ii) $n = 7$, $(f,d) = (1,-7)$ *and* $(1,-19)$.
(iii) $n = 8$, $(f,d) = (2,-24)$ *and* $(1,-7)$.
(iv) $n = 10$, $(f,d) = (2,-8)$, $(5,-3)$ *and* $(5,-47)$.
(v) $n = 12$, $(f,d) = (1,5)$, $(1,-7)$, $(1,-11)$, $(2,-56)$, $(1,-15)$ *and* $(1,-19)$.
(vi) $n \in \{13,18,30\}$, $(f,d) = (1,-7)$.

A positive integer n is called totally non-defective if no Lucas pair is n-defective.

Lemma 2.14 ([4]). *If $n > 30$, then n is totally non-defective.*

Lemma 2.15. *Let q be an odd prime with $q \geqslant 5$, and let (X,Y) be a solution of the equation*

$$X^2 + DY^2 = k^q, \quad X, Y \in \mathbb{Z},\ \gcd(X,Y) = 1. \tag{2.13}$$

If $q \nmid h(-4D)$, then Y must have an odd prime divisor l satisfying $l \equiv \pm 1 \pmod{q}$, except for $(D,k,q) = (19,55,5)$ and $(341,377,5)$.

Proof. Since $q \nmid h(-4D)$, applying Lemma 2.11 we have

$$X + Y\sqrt{-D} = \lambda_1(X_1 + \lambda_2 Y_1\sqrt{-D})^q, \quad \lambda_1, \lambda_2 \in \{\pm 1\}, \tag{2.14}$$

where X_1 and Y_1 satisfy

$$X_1^2 + DY_1^2 = k, \quad X_1, Y_1 \in \mathbb{N},\ \gcd(X_1,Y_1) = 1. \tag{2.15}$$

Let

$$\alpha = X_1 + Y_1\sqrt{-D}, \quad \beta = X_1 - Y_1\sqrt{-D}. \tag{2.16}$$

By (2.14) and (2.15), $\alpha + \beta = 2X_1$ and $\alpha\beta = k$ are co-prime positive integers. Further, since α/β satisfies $k(\alpha/\beta)^2 - (X_1^2 - DY_1^2)(\alpha/\beta) + k = 0$, α/β is not a root of unity. Therefore, (α,β) is a Lucas pair with parameters $(f,d) = (2X_1, -4DY_1^2)$.

Let $L_n(\alpha, \beta)$ denote the n-th corresponding Lucas numbers. By (2.12), (2.14) and (2.16), we get

$$Y = Y_1 |L_q(\alpha, \beta)|. \tag{2.17}$$

Since q is an odd prime with $q \geqslant 5$, applying Lemma 2.13 and 2.14 we find from (2.15) and (2.16) that, if $(D, k, q) \neq (19, 55, 5)$ and $(341, 377, 5)$, then $L_q(\alpha, \beta)$ has a primitive divisor l. Further, by (2.17) we have $l \mid Y$. Thus, since $2 \nmid L_q(\alpha, \beta)$, by Lemma 2.12, Y has an odd primitive divisor l with $l \equiv \pm 1 \pmod q$. The lemma is proved. □

3. Proof of the theorem

Let (x, y, n, a, b) be a solution of (1.2). We now proceed to prove the theorem in the following four cases separately.

Case I. $a = b = 0$.

By (1.2), we get $x^2 + 1 = y^n$. But, since $n \geqslant 3$, by Lemma 2.10 this is impossible.

Case II. $a > 0$ and $b = 0$.

Then $(X, Y, m, n) = (x, y, a, n)$ is a solution of (2.5).

Thus, applying Lemma 2.5, we obtain

$$(x, y, n, a, b) = (5, 3, 3, 1, 0),\ (7, 3, 4, 5, 0),\ (11, 5, 3, 2, 0). \tag{3.1}$$

Case III. $a = 0$ and $b > 0$.

Then we have

$$x^2 + 17^b = y^n, \quad x, y, n, b \in \mathbb{N},\ \gcd(x, y) = 1,\ n \geqslant 3. \tag{3.2}$$

We first consider the case of $4 \mid n$. From (3.2), we get $y^{n/2} + x = 17^b$ and $y^{n/2} - x = 1$. This implies that

$$2y^{n/2} = 17^b + 1, \tag{3.3}$$

and

$$2x = 17^b - 1. \tag{3.4}$$

If $b = 1$, then from (3.3) we get $y = 3$ and $n = 4$. Hence, by (3.4), we obtain the solution

$$(x, y, n, a, b) = (8, 3, 4, 0, 1). \tag{3.5}$$

If b is a power of 2, then $b = 2^r$, where r is a positive integer. Since $n/2$ is even, (3.3) is false for $r = 1$. When $r > 1$, $(X, Y) = (17^{b/4}, y^{n/4})$ is a solution of (2.4). But, by Lemma 2.4, this is impossible.

If $b > 1$ and b is not a power of 2, then b has an odd prime divisor q. By (3.3), $(X, Y) = (17^{b/q}, y^{n/4})$ is a solution of (2.2). But, by Lemmas 2.1 and 2.2, this is impossible.

We next consider the case of $4 \nmid n$. Since $n \geqslant 3$, n has an odd prime divisor q. Let $z = y^{n/q}$, then (3.2) can be written as

$$x^2 + 17^b = z^q, \quad x, z, b \in \mathbb{N},\ \gcd(x, z) = 1. \tag{3.6}$$

If $2 \mid b$ and $q = 3$, then since $h(-4) = 1$, applying Lemma 2.11, from (3.6) we get

$$x + 17^{b/2}\sqrt{-1} = \lambda_1\big(X_1 + \lambda_2 Y_1\sqrt{-1}\big)^3, \quad \lambda_1, \lambda_2 \in \{\pm 1\}, \tag{3.7}$$

where X_1 and Y_1 satisfy

$$X_1^2 + Y_1^2 = z, \quad X_1, Y_1 \in \mathbb{N},\ \gcd(X_1, Y_1) = 1. \tag{3.8}$$

By (3.7) we obtain

$$17^{b/2} = Y_1|3X_1^2 - Y_1^2|. \tag{3.9}$$

Since $(3/17) = -1$, where $(\star/\star)$ is the Legendre symbol, we have $17 \nmid 3X_1^2 - Y_1^2$. Therefore, by (3.9), we get $Y_1 = 17^{b/2}$ and

$$3X_1^2 - 17^b = -1. \tag{3.10}$$

But, since $(-3/17) = -1$, (3.10) is impossible.

If $2 \mid b$ and $q \geqslant 5$, then $(X, Y) = (x, 17^{b/2})$ is a solution of (2.13) for $(D, k) = (1, z)$. But, since $h(-4) = 1$ and $q \nmid 17 \pm 1$, by Lemma 2.15, this is impossible.

Since $h(-68) = 4$, using the same method, we can prove that if $2 \nmid b$, then (3.6) is false.

Case IV. $a > 0$ and $b > 0$.

Then (1.2) can be written as

$$x^2 + 2^a \cdot 17^b = y^n, \quad x, y, n, a, b \in \mathbb{N},\ \gcd(x, y) = 1,\ n \geqslant 3. \tag{3.11}$$

We first consider the case of $4 \mid n$. By (3.11), we get either

$$y^{n/2} + x = 2^{a-1} \cdot 17^b, \quad y^{n/2} - x = 2, \tag{3.12}$$

or

$$y^{n/2} + x = \begin{cases} 2^{a-1}, \\ 2 \cdot 17^b, \end{cases} \qquad y^{n/2} - x = \begin{cases} 2 \cdot 17^b, \\ 2^{a-1}. \end{cases} \tag{3.13}$$

If (3.12) holds, then we have

$$y^{n/2} = 2^{a-2} \cdot 17^b + 1 \tag{3.14}$$

and

$$x = 2^{a-2} \cdot 17^b - 1. \tag{3.15}$$

Further, since $n/2$ is even, by (3.14) we obtain either

$$y^{n/4} + 1 = 2^{a-3} \cdot 17^b, \quad y^{n/4} - 1 = 2, \tag{3.16}$$

or

$$y^{n/4} + 1 = \begin{cases} 2^{a-3}, \\ 2 \cdot 17^b, \end{cases} \qquad y^{n/4} - 1 = \begin{cases} 2 \cdot 17^b, \\ 2^{a-3}. \end{cases} \tag{3.17}$$

When (3.15) holds, we have $y^{n/4} = 3$ and $2^{a-3} \cdot 17^b = 4$, a contradiction. When (3.17) holds, we get

$$y^{n/4} = 2^{a-4} + 17^b \tag{3.18}$$

and

$$2^{a-1} - 17^b = \pm 1. \tag{3.19}$$

Applying Lemma 2.10 to (3.19), we obtain $b = 1$ and $a = 8$.

Hence, by (3.18), we get the solution

$$(x, y, n, a, b) = (1087, 33, 4, 8, 1). \tag{3.20}$$

If (3.13) holds, then we have

$$y^{n/2} = 2^{a-2} + 17^b \tag{3.21}$$

and

$$x = |2^{a-2} - 17^b|. \tag{3.22}$$

When $b=1$, we see from (3.21) that $(X,m)=(y^{n/4},a-2)$ is a solution of (2.9). Therefore, by Lemma 2.9, we get from (3.21) and (3.22) that

$$(x,y,n,a,b)=(5,7,4,7,1),\ (9,5,4,5,1),\ (47,9,4,8,1),\ (47,3,8,8,1) \text{ and } (495,23,4,11,1). \tag{3.23}$$

When $2\mid b$, we get from (3.21) that $y^{n/4}+17^{b/2}=2^{a-3}$ and $y^{n/4}-17^{b/2}=2$. This implies that

$$17^{b/2}=2^{a-4}-1. \tag{3.24}$$

But, since $a>7$, by (3.24) we get $1\equiv 17^{b/2}\equiv 2^{a-4}-1\equiv -1 \pmod 8$, a contradiction.

Let $b>1$ and $2\nmid b$, where b has an odd prime divisor q. For $q=3$, applying Lemma 2.6 to (3.21), we obtain the solution

$$(x,y,n,a,b)=(4785,71,4,9,3) \tag{3.25}$$

by (3.22). For $q\geqslant 5$, by Lemma 2.7, (3.21) is impossible, since $a\geqslant 5$.

We next consider the case of $4\nmid n$. Since $n>2$, n has an odd prime divisor q. Let $z=y^{n/q}$ (3.11) can be written as

$$x^2+2^a\cdot 17^b=z^q,\quad x,z,a,b\in\mathbb{N},\ \gcd(x,z)=1. \tag{3.26}$$

If $2\mid a$, $2\mid b$ and $q=3$, then since $h(-4)=1$, by Lemma 2.11, from (3.26) we get

$$x+2^{a/2}\cdot 17^{b/2}\sqrt{-1}=\lambda_1(X_1+\lambda_2 Y_1\sqrt{-1})^3,\ \lambda_1,\lambda_2\in\{\pm 1\}, \tag{3.27}$$

where X_1 and Y_1 satisfy

$$X_1^2+Y_1^2=z,\ X_1,Y_1\in\mathbb{N},\ \gcd(X_1,Y_1)=1. \tag{3.28}$$

By (3.27), we have

$$2^{a/2}\cdot 17^{b/2}=Y_1|3X_1^2-Y_1^2|. \tag{3.29}$$

Since $2\mid X_1Y_1$ and $(3/17)=-1$, we see from (3.29) that $Y_1=2^{a/2}\cdot 17^{b/2}$ and

$$3X_1^2-2^a\cdot 17^b=\pm 1. \tag{3.30}$$

But, since $(\pm 3/17)=-1$, (3.30) is impossible.

Since $h(-8)=1$ and $h(-68)=h(-136)=4$, using the same method we can deal with the other cases for $q=3$.

If $2\mid a$, $2\mid b$ and $q\geqslant 5$, then since $h(-4)=1$ and $q\nmid 17\pm 1$, by Lemma 2.15, (3.26) is false. Using the same method, we can remove the other cases for $q\geqslant 5$.

Thus, the combination of solutions (3.1), (3.5), (3.20), (3.23) and (3.25), proves the theorem. □

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