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Enlargement of the Earth's Shadow During the Lunar Eclipses Observed in the Years 1973—1975

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From the timing of crater entrances into the umbra and their exits from the umbra obtained during the last four lunar eclipses in Czechoslovakia the enlargement of the shadow was determined. During the eclipse of 9–10 December 1973 the enlargment was found to be 1/67, during the eclipse of 4–5 June 1974 – 1/42, during the eclipse of 29 November 1974 – 1/48 and during the eclipse of 18–19 November 1975 – 1/47.

Увеличение земной тени во время лунных затмений наблюданых в 1973—1975 гг. По моментам вступления в земную тень и вихода из нее кратеров, отмеченных во время последних 4 затмений наблюдаемых в Чехословакии, определены увеличение тени. Во время затмения 9—10 декабря 1973 г. было найдено увеличение тени 1/67, по наблюдениям затмения 4—5 июня 1974 г. 1/42, по наблюдениям затмения 29 ноября 1974 г. 1/48 и по наблюдениям затмения 18—19 ноября 1975 г. 1/47.

Zvětšení zemského stínu při měsíčních zatměních pozorovaných v letech 1973—1975. Z časových okamžiků vstupů kráterů do stínu a výstupů z něho, určených při posledních čtyřech zatměních pozorovaných v Československu, bylo počítáno zvětšení stínu. Při zatmění z 9./10. prosince 1973 bylo nalezeno zvětšení stínu 1/67, při zatmění z 4./5. června 1974 – 1/42, při zatmění z 29. listopadu 1974 – 1/48 a při zatmění z 18./19. listopadu 1975 – 1/47.

I. Introduction

Four lunar eclipses were observed during the years 1973–1975 in Czechoslovakia: partial eclipse of 9–10 December 1973, partial eclipse of 4–5 June 1974, total eclipse of 29 November 1974 and total eclipse of 18–19 November 1975. During these eclipses the times of crater entrances into the umbra and of crater exits from the umbra were obtained by various observers. The enlargement of the shadow was determined from these timings by Kozik's (1940) method. The sun's and the moon's equatorial coordinates and parallaxes, the sun's selenographic colongitudes and latitudes and position angles of the moon's axis were taken from the Astronomical Ephemeris. Used for the computation were the rectangular coordinates of the observed lunar formations published in catalogues by Bouška

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and Vanýsek (1963) and Kozik (1960). In some cases these rectangular coordinates were computed from selenographic longitudes and latitudes of the craters. All computations were carried out on the Minsk-22 computer in the Center of Numerical Mathematics, Charles University. The following differences between Ephemeris time and Universal time were accepted:

9–10 Dcember 1973	$\Delta T(A) = +44.6^s$
4– 5 June 1974	+45.2
29 November 1974	+45.6
18–19 November 1975	+46.6

2. Partial Eclipse of the Moon of 9 — 10 December 1973

This eclipse was observed by one observer only, M. Dujnič (Bratislava), with the 100-mm reflector at 30×. The magnitude of this eclipse was 0.107 only and only the extreme southern limb of the moon immersed into the umbra. The shadow was very bright and most lunar formations were good observable during the eclipse. During mid-eclipse the darkness of the umbra was $L = 4$ on Dajon's five-point scale. The boundary between the penumbra and the umbra was not very sharp, the edge of the umbra being very bright.

Table 1. Partial Eclipse of the Moon of 9—10 December 1973

No	Obs.	Formation	E. T.	x	y	ψ	r_0
1	D	Kircher	1.364^h	−0.1340	+0.7200	+79.5°W	0.7324
2	D	Bettinus	1.384	−0.1187	+0.7274	+81.0 W	0.7365
3	D	Zucchius	1.401	−0.0890	+0.7337	+83.1 W	0.7391
4	D	A*)	1.404	−0.1416	+0.7307	+79.0 W	0.7443
5	D	B**)	1.564	−0.0980	+0.7499	+82.8 W	0.7559
6	D	Hommel	1.629	−0.1477	+0.7438	+78.8 W	0.7583
7	D	Schiller	1.692	+0.0719	+0.7436	+84.8 E	0.7471
8	D	Short	2.156	+0.2546	+0.6989	+70.0 E	0.7438
9	D	Simpelius	2.229	+0.2648	+0.7033	+69.4 E	0.7515

*) A ($\lambda = -19.0°$, $\beta = -61.0°$) **) B ($\lambda = -0.5°$, $\beta = -52.5°$)

Dujnič (Obs. *D*) obtained 9 timing observations only (Tab. 1), 7 of which were crater entrances into the shadow (Nos 1 — 7) and 2 were exits from the umbra (Nos 8 and 9). These observations are summarized in Tab. 1, in which x and y are the rectangular and ψ and r_0 the polar coordinates of the edge of the umbra relative to the shadow center. The position angle ψ is the angle between the observed radius r_0 of the shadow (expressed in units of the earth's equatorial radius) and the east-west direction. It is $\psi > 0$ for the northern part of the umbra and $\psi < 0$ for the southern one.

During this eclipse the observed points of the edge of the shadow were situated between position angles $+79° < \psi < +83°$ W and $+69° < \psi < +85°$ E (Fig. 1), also in the extreme northern part of the umbra. The mean values of the observed radius $\bar{r}_o$ and of the position angle $\bar{\psi}$ are

$$\bar{r}_o = 0.7454 \qquad \bar{\psi} = +78.7°.$$

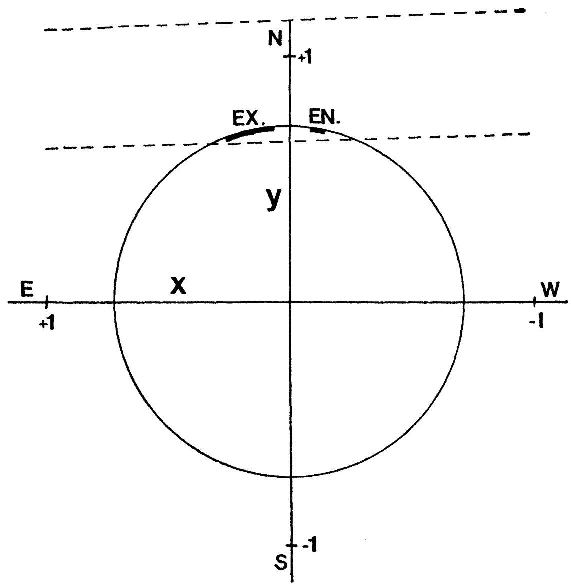

Fig. 1. Lunar eclipse of 9–10 December 1973. Path of the moon through the earth's shadow. The parts of the boundary of the umbra in which the contacts of lunar formations were observed are represented by thick arcs. (EN. – crater entrances into the shadow, EX. – exits from the shadow)

The theoretical radius of the umbra may be expressed for this eclipse by the equation

$$r_c = 0.7368 - 0.0029 \sin^2\psi$$

and thus the enlargement E of the shadow is

$$E = (\bar{r}_o - r_c) / \bar{r}_o = 0.0153\,.$$

The enlargement of the umbra was very small, 1.5 percent only, during this eclipse. It is one of the smallest values of the shadow enlargement observed up to this time. It is interesting that also during the last partial lunar eclipse of 26 July 1972 a similar small value of the enlargement was found (1/66) (Bouška 1972). It cannot be excluded that the very small enlargement of the shadow during the eclipse of 9–10 December 1973 was due to the very large oblateness of the umbra.

3. Partial Eclipse of the Moon of 4—5 June 1974

The observation conditions were not very favourable during this eclipse. The moon entered the umbra less than 2 hours after the moonrise and the whole eclipse occured low over the horizon. At the mid-eclipse the zenith distance of the moon was about 73°. Also the weather conditions were not very good for observation, the crater timings being interrupted by clouds many times. The brightness of the eclipse was $L = 3$ on Danjon's scale.

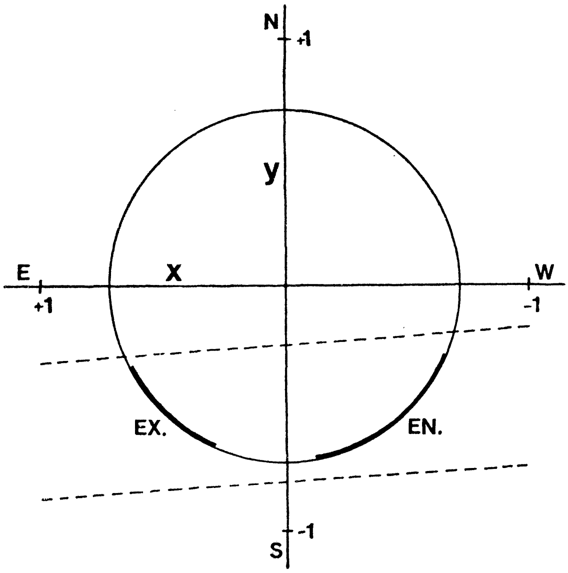

Fig. 2. Lunar eclipse of 4–5 June 1974. (See Fig. 1)

The crater entrances and exits have been timed by the following observers:

D – M. Dujnič (Rimavská Sobota), 80-mm double refractor (10×)
P – V. Přibyl (Kladno), 60-mm double refractor (12×)
S – V. Strnad (Kladno), 60-mm refractor (60×)
Z – M. Zadražil et al. (Turnov), 160-mm reflector.

The timing observations are summarized in Tab. 2. Nos 1–41 are crater entrances, Nos 42–56 crater exits. The entrances were observed between the position angles $-24° > \psi > -80°$ W, the exits between $-27° > \psi > -65°\ E$ (Fig. 2). The theoretical radius r_c of the umbra is given for this eclipse by the equation

$$r_c = 0.7173 - 0.0029 \sin^2\psi .$$

The following Tab. 3 contains the mean values of the position angles $\bar{\psi}$ and the observed radii $\bar{r}_o$, the computed values of the theoretical radii r_c for the angles $\bar{\psi}$ and the values of the enlargement of the shadow E (*No* means the number of observed contacts).

The mean value of E computed from crater entrances (enlargement of the western part of the umbra) is 2.6 percent, from crater exits (enlargement of the eastern part of the umbra) 1.9 percent. The mean value of the enlargement determined from crater entrances and exits together is 2.4 percent.

From values ψ and r_o the oblateness of the umbra can be computed, which may be represented by the equation

$$r_o = 0.7396 - 0.0191 \sin^2\psi .$$

Table 2. Partial Eclipse of the Moon of 4—5 June 1974

No	Obs.	Formation	E.T.	x	y	ψ	r_o
1	D	Seleucus	20.734^h	−0.5589	−0.4725	−40.2°W	0.7318
2	P	Aristarchus	20.751	−0.5907	−0.4541	−37.6	0.7451
3	S	Aristarchus	20.751	−0.5907	−0.4541	−37.6	0.7451
4	D	Harpalus	20.764	−0.6400	−0.3376	−27.8	0.7236
5	D	Aristarchus	20.766	−0.5830	−0.5443	−37.9	0.7391
6	D	Cap Laplace	20.806	−0.6802	−0.3556	−28.7 W	0.7411
7	D	Reiner	20.843	−0.5106	−0.5359	−46.4	0.7402
8	D	Euler	20.878	−0.5833	−0.4500	−37.6	0.7367
9	P	Plato	20.889	−0.6588	−0.3301	−26.6	0.7369
10	S	Plato	20.899	−0.6537	−0.3302	−26.8	0.7324
11	D	Plato	20.903	−0.6519	−0.3303	−26.9 W	0.7308
12	D	Timocharis	20.973	−0.5966	−0.4250	−35.6	0.7325
13	P	Grimaldi	20.979	−0.4277	−0.5975	−54.4	0.7348
14	P	Copernicus	21.019	−0.5460	−0.5071	−42.9	0.7452
15	D	Grimaldi	21.021	−0.4062	−0.5979	−55.8	0.7228
16	Z	Copernicus	21.029	−0.5408	−0.5072	−43.2 W	0.7415
17	S	Grimaldi	21.038	−0.3975	−0.5980	−56.4	0.7181
18	D	Copernicus	21.046	−0.5322	−0.5074	−43.6	0.7353
19	D	Gambart	21.133	−0.5128	−0.5459	−46.8	0.7490
20	D	Endymion	21.188	−0.6690	−0.3014	−24.1	0.7337
21	D	Manilius	21.239	−0.5599	−0.4679	−39.9 W	0.7297
22	P	Menelaus	21.276	−0.5712	−0.4552	−38.5	0.7304
23	S	Menelaus	21.276	−0.5712	−0.4552	−38.5	0.7304
24	P	Plinius	21.309	−0.5879	−0.4545	−37.7	0.7431
25	D	C*	21.358	−0.3491	−0.6408	−61.2	0.7297
26	D	Dionysius	21.399	−0.5282	−0.5175	−44.6 W	0.7373
27	D	Macrobius A	21.411	−0.6117	−0.4191	−34.4	0.7415
28	Z	Ptolemaius	21.419	−0.4320	−0.5867	−53.6	0.7285

Cont. Table 2.

No	Obs.	Formation	E.T.	x	y	ψ	r_o
29	D	Hipparchus C	21.466^h	−0.4587	−0.5724	−51.5°W	0.7317
30	Z	M. Crisium (NE)	21.471	−0.5991	−0.4364	−36.1	0.7412
31	D	Censorinus	21.551	−0.5166	−0.5238	−45.4 W	0.7357
32	P	Censorinus	21.554	−0.5149	−0.5238	−45.5	0.7345
33	D	Wolf	21.559	−0.3114	−0.6590	−64.7	0.7289
34	S	Censorinus	21.563	−0.5106	−0.5239	−45.7	0.7316
35	Z	M. Crisium (NW)	21.608	−0.5808	−0.4299	−36.5	0.7225
36	Z	Theophylus	21.609	−0.4651	−0.5800	−51.3 W	0.7434
37	D	Stevinus	22.101	−0.2971	−0.6642	−65.9	0.7277
38	D	Furnerius	22.144	−0.2838	−0.6672	−67.0	0.7250
39	D	Palmieri	22.233	+0.1242	−0.7192	−80.5	0.7299
40	P	Tycho	22.233	−0.0112	−0.7413	−89.1	0.7414
41	S	Tycho	22.246	−0.0043	−0.7414	−89.7 W	0.7414
42	D	Grimaldi B	22.591	+0.4062	−0.6130	−56.5 E	0.7353
43	D	C*)	22.619	+0.3023	−0.6532	−65.6	0.7197
44	D	D**)	22.661	+0.3633	−0.6266	−59.9	0.7243
45	D	Reiner γ	22.749	+0.4746	−0.5542	−49.4	0.7296
46	D	Guericke C	22.888	+0.3650	−0.6196	−59.5 E	0.7191
47	D	Aristarchus	22.961	+0.5499	−0.4754	−40.8	0.7269
48	D	Timocharis	23.253	+0.5783	−0.4478	−37.7	0.7314
49	D	Cap Laplace	23.258	+0.6139	−0.3801	−31.8	0.7220
50	D	Manilius	23.376	+0.5409	−0.4899	−42.2	0.7297
51	D	Menelaus	23.456	+0.5519	−0.4779	−40.9 E	0.7300
52	D	Possidonius A	23.639	+0.6136	−0.3992	−33.0	0.7321
53	D	Proclus	23.736	+0.5742	−0.4640	−38.8	0.7383
54	D	Macrobius	23.743	+0.5905	−0.4444	−37.0	0.7390
55	D	Endymion	23.754	+0.6552	−0.3290	−26.7	0.7332
56	D	Picard	23.783	+0.5731	−0.4696	−39.3 E	0.7409

* C ($\lambda = -31.5°$, $\beta = -17.0°$)　　**) D ($\lambda = -40{,}0°$, $\beta = -10°$)

Table 3.

Obs.	No	$\overline{\psi}$	$\overline{r_o}$	r_c	E
Entrances					
D	22	−42.1°W	0.7334	0.7160	0.0237
P	8	−46.6 W	0.7389	0.7158	0.0313
S	6	−49.1 W	0.7332	0.7156	0.0240
Z	5	−44.1 W	0.7354	0.7159	0.0265
Exits					
D	15	−43.9 E	0.7301	0.7159	0.0194

The difference $\Delta r = r_o - r_c$ is thus

$$\Delta r = 0.0223 - 0.0162 \sin^2\psi .$$

The oblateness of the umbra was much larger during this eclipse than that of the earth and somewhat larger than usual. The last equation gives the enlargement of the shadow to be $E = 0.0302$. This value is larger than the enlargement computed from individual contacts due to the large oblateness of the shadow. Naturarly, it must be taken into account that the value of the oblateness of the umbra may be influenced by a relatively great error.

4. Total Eclipse of the Moon of 29 November 1974

Conditions for the observation of this eclipse were very unfavourable in Czechoslovakia. At the time of the beginning of the total eclipse the moon was still under the horizon and the sun over the horizon. Mid-eclipse followed 10 mi-

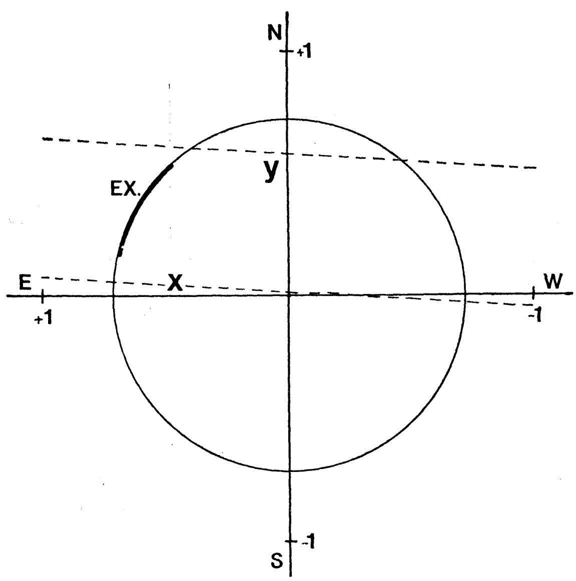

Fig. 3. Lunar eclipse of 29 November 1974. (See Fig. 1)

nutes after the sunset and 18 minutes after the moonrise. For these reasons it was possible to observe the crater exits only. During the observation the moon was very low over the horizon and the crater timing was often interrupted by clouds. The brightness of the eclipse was $L = 3$ on Danjon's scale.

The following observers timed the crater exits from the umbra:

D – M. Dujnič (Spišská Nová Ves), 130-mm reflector (50×)
L – I. Dupal (Hustopeče u Brna), 63-mm refractor (52×)
M – Z. Machovský (Domradovice), 83-mm refractor (30×)
R – P. Rapavý (Hlohovec), 100-mm double refractor (25×)
S – J. Stuchlík (Bučovice), 130-mm reflector (50×).

The results of the observations are given in Tab. 4, the mean values in Tab. 5. The theoretical radius of the umbra can be given by the equation

$$r_c = 0.7261 - 0.0029 \sin^2\psi .$$

The crater exits were observed between the position angles

Table 4. Total Eclipse of the Moon of 29 November 1974

No	Obs.	Formation	E.T.	x	y	ψ	r_o
1	M	Herodotus	16.021^h	+0.5739	+0.4667	+39.1°E	0.7397
2	L	Aristarchus	16.033	+0.5619	+0.4747	+40.2	0.7355
3	M	Aristarchus	16.034	+0.5625	+0.4748	+40.2	0.7361
4	L	Sin. Iridum (W)	16.099	+0.4925	+0.5381	+47.5	0.7395
5	R	Prom. Heraclides	16.099	+0.5169	+0.5325	+45.9	0.7422
6	R	Prom. Laplace	16.129	+0.4978	+0.5419	+47.4 E	0.7358
7	M	Kepler	16.146	+0.6161	+0.4049	+33.3	0.7372
8	M	Gassendi	16.204	+0.6712	+0.2914	+23.5	0.7317
9	S	Plato (W)	16.209	+0.4852	+0.5524	+48.7	0.7352
10	L	Plato	16.213	+0.4875	+0.5525	+48.6	0.7367
11	M	Doppelmayer	16.253	+0.7016	+0.2445	+19.2 E	0.7430
12	M	Copernicus	16.291	+0.6187	+0.4031	+33.1	0.7384
13	L	Copernicus	16.296	+0.6215	+0.4033	+33.0	0.7409
14	L	Eudoxus	16.431	+0.5316	+0.5231	+44.5	0.7458
15	R	Tycho (E)	16.481	+0.7224	+0.1719	+13.4	0.7425
16	L	Tycho	16.486	+0.7230	+0.1721	+13.4 E	0.7433
17	S	Tycho (W)	16.498	+0.7265	+0.1725	+13.3	0.7467
18	D	Manilius	16.529	+0.6144	+0.4082	+33.6	0.7376
19	R	Manilius	16.531	+0.6154	+0.4083	+33.6	0.7385
20	R	Menelaus	16.589	+0.6161	+0.4129	+33.8	0.7417
21	D	Menelaus	16.589	+0.6161	+0.4129	+33.8 E	0.7417
22	L	Possidonius	16.603	+0.5747	+0.4803	+39.9	0.7490
23	R	Delambre	16.679	+0.6724	+0.3314	+26.2	0.7496
24	L	M. Crisium (NE)	16.798	+0.6162	+0.4000	+33.0	0.7347
25	R	Picard	16.851	+0.6318	+0.3885	+31.5	0.7417
26	D	Picard	16.851	+0.6318	+0.3885	+31.6 E	0.7417
27	L	M. Crisium (SW)	16.889	+0.6275	+0.3959	+32.2	0.7420
28	D	Firmicus	16.896	+0.6412	+0.3585	+29.2	0.7346
29	D	Prom. Agarum	16.913	+0.6439	+0.3858	+31.0	0.7506
30	R	Prom. Agarum	16.913	+0.6439	+0.3858	+31.0	0.7506

Table 5.

Obs.	No	$\overline{\psi}$	$\overline{r_o}$	r_c	E
D	5	+31.8°E	0.7412	0.7253	0.0214
L	9	+36.9 E	0.7408	0.7251	0.0212
M	6	+31.4 E	0.7377	0.7253	0.0169
R	8	+32.9 E	0.7428	0.7253	0.0236
S	2	+31.0 E	0.7410	0.7253	0.0211

$+13° < \psi < +49° E$ (Fig. 3). The mean value of the enlargement of the shadow from all the 30 timings is $E = 0.0209$, which is very close to its average value.

5. Total Eclipse of the Moon of 18—19 November 1975

This eclipse was observed by the following observers, mostly under very unfavourable weather conditions:

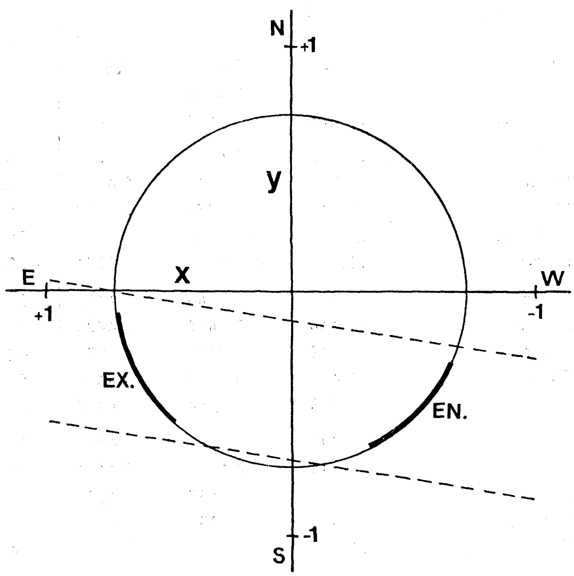

Fig. 4. Lunar eclipse of 18–19 November 1975. (See Fig. 1)

D – M. Dujnič (Spišská Nová Ves), 80-mm double refractor (10 ×)
V – V. and L. Kováč (Sered), 150-mm reflector (36 ×)
K – L. Kulčár (Hurbanovo), 120-mm refractor (66 ×)
W – V. Vagner (Havířov), 100-mm reflector (40 ×).

Table 6. Total Eclipse of the Moon of 18-19 November 1975

No	Obs.	Formation	E.T.	x	y	ψ	r_o
1	V	Herodotus	20.808^h	−0.6153	−0.3720	−31.2°W	0.7190
2	K	Aristarchus	20.813	−0.6204	−0.3717	−30.9	0.7232
3	K	Kepler	20.901	−0.5760	−0.4395	−37.3	0.7245
4	W	Sin. Iridum (E)	20.910	−0.6551	−0.3052	−25.0	0.7227
5	V	Kepler	20.911	−0.5709	−0.4388	−37.5	0.7200
6	V	Euler	20.928	−0.6238	−0.3827	−31.5 W	0.7318
7	V	Lambert	20.981	−0.6328	−0.3778	−30.8	0.7370
8	K	Copernicus	21.025	−0.5881	−0.4438	−37.0	0.7367
9	V	Timocharis	21.058	−0.6262	−0.3776	−31.1	0.7313
10	V	Copernicus	21.060	−0.5702	−0.4413	−37.7	0.7210
11	K	Plato (E)	21.071	−0.6597	−0.2943	−24.0 W	0.7224
12	K	Pico	21.081	−0.6555	−0.3116	−25.4	0.7258
13	W	Plato	21.084	−0.6602	−0.2944	−24.0	0.7228
14	V	Pico	21.090	−0.6512	−0.3110	−25.5	0.7217
15	V	Piton	21.116	−0.6593	−0.3334	−26.8	0.7388
16	V	Archimedes	21.126	−0.6320	−0.3705	−30.8 W	0.7326
17	K	Autolycus	21.160	−0.6373	−0.3704	−30.2	0.7371
18	V	Aristillus	21.165	−0.6366	−0.3574	−29.3	0.7301
19	W	Bullialdus	21.195	−0.4586	−0.5674	−51.1	0.7296
20	K	Aristoteles	21.228	−0.6614	−0.3088	−25.0	0.7299
21	K	Eudoxus	21.240	−0.6545	−0.3271	−26.6 W	0.7317
22	V	Manilius	21.296	−0.5844	−0.4388	−36.9	0.7308
23	V	Menelaus	21.348	−0.5888	−0.4350	−36.5	0.7320
24	K	Menelaus	21.353	−0.5862	−0.4346	−36.6	0.7297
25	V	Endymion	21.396	−0.6528	−0.3075	−25.2	0.7216
26	K	Plinius	21.411	−0.5865	−0.4430	−37.1 W	0.7350
27	V	Plinius	21.446	−0.5686	−0.4405	−37.8	0.7193
28	D	Ritter	21.448	−0.5381	−0.4975	−42.8	0.7328
29	K	Tycho	21.498	−0.3324	−0.6488	−62.9	0.7285
30	V	Tycho	21.510	−0.3264	−0.6474	−63.2	0.7250
31	K	Proclus	21.593	−0.5738	−0.4485	−38.0 W	0.7282
32	V	Proclus	21.595	−0.5729	−0.4484	−38.0	0.7275
33	K	Picard	21.646	−0.5657	−0.4572	−38.9	0.7274
34	W	Taruntius	21.663	−0.5292	−0.4913	−42.9	0.7221
35	W	Fracastrorius (E)	21.704	−0.4175	−0.5887	−54.7	0.7217
36	D	Grimaldi	23.068	+0.6404	−0.3242	−26.8 E	0.7178
37	K	Tycho (E)	23.068	+0.4839	−0.5353	−47.9	0.7216
38	K	Tycho (W)	23.121	+0.4976	−0.5334	−47.0	0.7294
39	D	Tycho (E)	23.130	+0.5018	−0.5328	−46.7	0.7319
40	D	Reiner	23.205	+0.6721	−0.2623	−21.3	0.7215
41	D	Hesiodus	23.220	+0.5580	−0.4680	−40.0 E	0.7283
42	D	Walter	23.288	+0.5302	−0.4924	−42.9	0.7236
43	D	Langsberg B	23.316	+0.6347	−0.3281	−27.3	0.7145

No	Obs.	Formation	E.T.	x	y	ψ	r_o
44	K	Kepler	23.353^h	+0.6789	−0.2666	−21.4°E	0.7294
45	D	Aristarchus	23.378	+0.6941	−0.1905	−15.4	0.7198
46	K	Aristarchus	23.386	+0.6984	−0.1900	−15.2 E	0.7237
47	D	Copernicus	23.483	+0.6694	−0.2701	−22.0	0.7218
48	D	Euler	23.486	+0.6862	−0.2018	−16.4	0.7152
49	K	Copernicus	23.488	+0.6720	−0.2697	−21.9	0.7241
50	D	Prom. Laplace	23.648	+0.7117	−0.1133	−9.0	0.7207
51	K	Pico	23.746	+0.7105	−0.1221	− 9.7 E	0.7209
52	D	McClure	23.750	+0.5630	−0.4390	−37.9	0.7139
53	K	Plato (E)	23.753	+0.7185	−0.1044	− 8.3	0.7261
54	K	Plato (C)	23.766	+0.7153	−0.1034	− 8.2	0.7228
55	K	Autolycus	23.770	+0.6989	−0.1849	−14.8	0.7229
56	D	Arago	23.776	+0.6394	−0.3146	−26.2 E	0.7126
57	K	Plato (W)	23.781	+0.7170	−0.1023	− 8.1	0.7243
58	D	Menelaus	23.831	+0.6817	−0.2582	−20.7	0.7289
59	K	Menelaus	23.836	+0.6843	−0.2578	−20.6	0.7312
60	K	Plinius	23.888	+0.6807	−0.2664	−21.4	0.7310
61	K	Eudoxus (W)	23.903	+0.7105	−0.1370	−10.9 E	0.7236
62	K	Aristoteles	23.908	+0.7128	−0.1174	− 9.3	0.7224
63	D	Dawes	23.916	+0.6837	−0.2587	−20.7	0.7310
64	D	Langrenus M	23.926	+0.6055	−0.4111	−34.2	0.7319
65	D	Possidonius A	23.976	+0.7033	−0.1930	−15.4	0.7283
66	K	Proclus	0.015	+0.6666	−0.2748	−22.4 E	0.7210
67	K	Picard	0.061	+0.6717	−0.2838	−22.9	0.7292
68	D	Geminus	0.081	+0.7013	−0.2035	−16.2	0.7303
69	D	Eimmart	0.098	+0.6738	−0.2417	−19.7	0.7159

Table 7.

Obs.	No	$\overline{\psi}$	$\overline{r_o}$	r_c	E
Entrances					
V	16	−34.3°W	0.7275	0.7101	0.0239
K	13	−34.6 W	0.7292	0.7101	0.0262
W	5	−39.4 W	0.7238	0.7099	0.0192
Exits					
D	18	−25.5 E	0.7227	0.7105	0.0169
K	16	−19.4 E	0.7252	0.7108	0.0198

Altogether 35 crater entrances (Tab. 6, Nos 1–35) and 34 crater exits (Nos 36–69) were timed. The mean values of the observation are summarized in Tab. 7.

The theoretical radius of the shadow can be given by the equation

$$r_c = 0.7111 - 0.0030 \sin^2 .$$

The mean value of the shadow enlargement from 35 crater entrances ($-24° > \psi > -63°$ *W*) is $E = 0.0243$ and from 34 exits ($-8° > \psi > -48°$ *E*) it is $E = 0.0182$. The mean value from all 69 timings is $E = 0.0213$, which is a value very close to the average enlargement of the umbra.

The brightness of this eclipse was $L = 3{,}5$ on Danjon's scale.

6. Discussion of Results

The mean value of the shadow enlargement was found to be $E = 2.25$ percent from 15 lunar eclipses observed between the years 1943 and 1972 and collected by the writer of this paper. From an analysis of 57 lunar eclipses observed between 1776 and 1936 Link (1969) obtained the mean value of the enlargement $E = 2.3$ percent. It is indubitable that the value of the enlargement of the shadow varies within small limits from eclipse to eclipse. These variations are in connection with the activity of the principal meteoric showers as first shown by Bouška and Švestka (1950).

Table 8.

Eclipse	Entrances, exits	Δt	Δr_o	ΔE
4—5 June 1974	entrances	$\pm 0.48^m$	± 0.0026	± 0.0017
29 Nov. 1974	exits	± 0.04	± 0.0005	± 0.0011
18—19 Nov. 1975	entrances	± 0.60	± 0.0033	± 0.0021
18—19 Nov. 1975	exits	± 0.20	± 0.0016	± 0.0014

Of course, the enlargement of the umbra is computed from visual crater timings which are influenced by systematical and accidental errors not exactly known. From some craters observed by various observers the average mean errors of one timing (Δt) can be computed; these errors are shown in Tab. 8. This Table also contains the average errors of one determination of the observed radius of the shadow (Δr_o) and the mean errors of the computed mean values of the enlargement of the shadow (ΔE). It is evident that the errors of Δt and Δr_o vary in relatively large limits up to ± 0.6 min. in timing and up to ± 0.003 in the radius of the umbra. On the contrary, the errors of ΔE are sufficiently small, lying within the limits ± 0.1 and ± 0.2 percent. For the enlargement of the shadow computed from the crater exits the mean error ΔE is somewhat smaller than that for the enlargement computed from crater entrances.

During the lunar eclipses of 4–5 June 1974 and 18–19 November 1975 timings of crater entrances as well as of crater exits were obtained. In both cases the enlargement of the western part of the shadow (computed from crater entrances) was somewhat larger (i.e. 2.6 and 2.4 percent) than that of the eastern part of the umbra (i.e. 1.9 and 1.8 percent) computed from the crater exits. This east-west asymmetry of the shadow was found many times during the past lunar eclipses and probably is only apparent. This asymmetry might be caused by the systematic errors in the two somewhat different kinds of observation, those of crater entrances into and of the exits from the shadow. On the other hand it is interesting that some lunar eclipses were observed during which the enlargement of the western part of the umbra was somewhat smaller than that of the eastern part (e.g. the partial lunar eclipse of 13–14 May 1957; Bouška 1958). It is also interesting that no east-west asymmetry of the shadow enlargement has been found during many lunar eclipses.

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