Czechoslovak Mathematical Journal

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Czechoslovak Mathematical Journal, Vol. 58 (2008), No. 3, 741–749

Persistent URL: `http://dml.cz/dmlcz/140418`

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A CHARACTERIZATION OF WEIGHTED (LB)-SPACES OF HOLOMORPHIC FUNCTIONS HAVING THE DUAL DENSITY CONDITION

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(Received July 6, 2006)

Abstract. We characterize when weighted (LB)-spaces of holomorphic functions have the dual density condition, when the weights are radial and grow logarithmically.

Keywords: (LB)-spaces, weighted spaces of holomorphic functions, dual density condition

MSC 2010: 46E10, 46A04, 46A13

I. Introduction and notation

Heinrich introduced his "density condition" in the context of ultraproducts of locally convex spaces and gave some basic facts, see [11]. In [3] Bierstedt and Bonet studied the density condition in the setting of Fréchet spaces. This restriction is natural since each (DF)-space has the density condition. One of their main results is that a Fréchet space satisfies the density condition if and only if each bounded subset of its strong dual is metrizable. Many proofs in [3] are based on a dual reformulation of the density condition. This started the research on "dual density conditions" in the setting of (DF)-spaces. With duality methods and polarity Bierstedt and Bonet formulated two slightly different dual versions of the density condition, the "dual density condition" and the "strong dual density condition" (see [4]). In many cases they are equivalent, but there are also examples of (DF)-spaces for which this is not true. By [4] we know that a (DF)-space has the dual density condition if and only if its bounded subsets are metrizable. These concepts were used by several authors in various contexts, see e.g. [2] [5], [16], [17], [18]. Countable locally convex inductive limits of spaces of holomorphic functions arise in linear partial differential equations, convolution equations, distribution theory and representation of distributions as boundary values of holomorphic functions, complex analysis in one and several variables and spectral theory and the holomorphic calculus. In this article we show that the characterization of the dual density condition we obtained in [17] also holds in the setting of condition (LOG) of Bonet, Engliš and Taskinen (see [10]).

Our notation for locally convex (l.c.) spaces is standard; see for example Jarchow [12], Köthe [13], Meise, Vogt [14] and Pérez Carreras, Bonet [15]. For a locally convex space E, $\mathscr{U}(E)$ denotes the family of all absolutely convex 0-neighborhoods in E.

A locally convex (DF)-space E with a fundamental sequence $(B_n)_{n\in\mathbb{N}}$ of bounded sets is said to satisfy the *dual density condition* (DDC) (resp. the *strong dual density condition* $(SDDC)$) if the following holds: for every sequence $(\lambda_k)_{k\in\mathbb{N}}$ of strictly positive numbers and for every $n\in\mathbb{N}$ there are $m>n$ and $\mathscr{U}\in\mathscr{U}(E)$ such that

$$B_n\cap U\subset\overline{\Gamma}\Big(\bigcup_{k=1}^{m}\lambda_kB_k\Big)\quad(\text{resp. }\Gamma\Big(\bigcup_{k=1}^{m}\lambda_kB_k\Big)),$$

where Γ (resp. $\overline{\Gamma}$) denotes the absolutely convex hull (resp. the closed absolutely convex hull).

In the sequel $\mathbb{D}$ denotes the open unit disk of the complex plane. The space $H(\mathbb{D})$ of all holomorphic functions on $\mathbb{D}$ will usually be endowed with the topology *co* of uniform convergence on the compact subsets of $\mathbb{D}$. For a decreasing sequence $\mathscr{V}=(v_n)_{n\in\mathbb{N}}$ of strictly positive continuous functions (weights) on $\mathbb{D}$ we define

$$\begin{aligned}Hv_n(\mathbb{D})&:=\{f\in H(\mathbb{D});\ \|f\|_n:=\sup_{z\in\mathbb{D}}v_n(z)|f(z)|<\infty\},\\ H(v_n)_0(\mathbb{D})&:=\{f\in H(\mathbb{D});\ \lim_{|z|\to1-}v_n(z)|f(z)|=0\},\\ \mathscr{V}H(\mathbb{D})&:=\mathrm{ind}_nHv_n(\mathbb{D})\text{ and }\mathscr{V}_0H(\mathbb{D}):=\mathrm{ind}_nH(v_n)_0(\mathbb{D}).\end{aligned}$$

B_n (resp. $B_{n,0}$) denotes the closed unit ball of $Hv_n(\mathbb{D})$ (resp. $H(v_n)_0(\mathbb{D})$). By $\overline{B_n}$ and $\overline{B_{n,0}}$ we denote the *co*-closures of the corresponding sets. Note that $\overline{B_n}=B_n$. If we put now $C_n:=B_n\cap\mathscr{V}H(\mathbb{D})$ resp. $C_{n,0}:=C_{n,0}\cap\mathscr{V}H(\mathbb{D})$, $n\in\mathbb{N}$, by $(C_n)_{n\in\mathbb{N}}$ resp. $(C_{n,0})_{n\in\mathbb{N}}$ we obtain a fundamental sequence of the bounded subsets of $\mathscr{V}H(\mathbb{D})$ resp. $\mathscr{V}H_0(\mathbb{D})$. The system $\overline{V}$ of weights was introduced in [9] as

$$\overline{V}:=\{\overline{v}\colon\ \mathbb{D}\to]0,\infty[\,\overline{v}\text{ continuous, }\forall k\ \exists r_k>0\colon\ \overline{v}\leqslant\inf_k r_kv_k\text{ on }\mathbb{D}\}.$$

The corresponding weighted spaces for $\overline{V}$ are called *projective hulls* and are given by

$$\begin{aligned}H\overline{V}(\mathbb{D})&:=\{f\in H(\mathbb{D});\ \|f\|_{\overline{v}}:=\sup_{z\in\mathbb{D}}\overline{v}(z)|f(z)|<\infty\ \forall\overline{v}\in\overline{V}\},\\ H\overline{V}_0(\mathbb{D})&:=\{f\in H(\mathbb{D});\ \overline{v}|f|\text{ vanishes at the boundary of }\ \mathbb{D}\ \forall\overline{v}\in\overline{V}\}.\end{aligned}$$

The system $(C_{\overline{v}})_{\overline{v}\in\overline{V}}$ (resp. $(C_{\overline{v},0})_{\overline{v}\in\overline{V}}$) , where

$$C_{\overline{v}} := \{f \in H\overline{V}(\mathbb{D});\ \|f\|_{\overline{v}} \leqslant 1\} \text{ and } C_{\overline{v},0} := \{f \in H\overline{V}_0(\mathbb{D});\ \|f\|_{\overline{v}} \leqslant 1\},$$

gives a 0-neighborhood base of $H\overline{V}(\mathbb{D})$ (resp. $H\overline{V}_0(\mathbb{D})$). We write $\overline{C_{\overline{v}}}$ and $\overline{C_{\overline{v},0}}$ to refer to the *co*-closure. A fundamental sequence of the bounded sets of $H\overline{V}(\mathbb{D})$ resp. $H\overline{V}_0(\mathbb{D})$ is given by $C'_n = B_n \cap H\overline{V}(\mathbb{D})$ and $C'_{n,0} := B_{n,0} \cap H\overline{V}_0(D)$, $n \in \mathbb{N}$.

An important tool to handle weighted spaces of holomorphic functions are the so called *associated growth conditions* mentioned by Andersen and Duncan in [1] and studied thoroughly by Bierstedt, Bonet and Taskinen in [8]. Let v be a weight on $\mathbb{D}$. Its *associated growth* condition is defined by

$$\tilde{v}(z) := \sup\{|g(z)|;\ g \in H(\mathbb{D}),\ |g| \leqslant v\}, \quad z \in \mathbb{D}.$$

A weight v on $\mathbb{D}$, is said to be *radial* if $v(z) = v(\lambda z)$ holds for every $\lambda \in \mathbb{C}$ such that $|\lambda| = 1$.

II. Main result

All the weights in this section are defined on the unit disc $\mathbb{D}$ of the complex plane. For every $n \in \mathbb{N}$ we denote $r_n := 1-2^{-2^n}$, $r_0 = 0$, and $I_n := [r_n, r_{n+1}]$. The following definition is inspired by a condition introduced by Bonet, Engliš and Taskinen [10, Section 4].

Definition 1. A sequence $W = (w_n)_{n\in\mathbb{N}}$ of weights on $\mathbb{D}$ satisfies the condition (LOG) if each weight in the sequence is radial and approaches monotonically 0 as $r \to 1-$ and there exist constants $0 < a < 1 < A$ such that

(a) $Aw_k(r_{n+1}) \geqslant w_k(r_n)$ and
(b) $w_k(r_{n+1}) \leqslant aw_k(r_n)$.

for every k and n.

For examples of systems of weights with (LOG) we refer the reader to [10, Examples 5 and 6].

First we need an auxiliary result.

Lemma 2. *Let E be a l.c. space with the fundamental sequence $(B_n)_{n\in\mathbb{N}}$ of bounded subsets. E has (DDC) (resp. $(SDDC)$) if and only if for every sequence $(\lambda_j)_{j\in\mathbb{N}}$ of strictly positive numbers and for every $n \in \mathbb{N}$ there are $m > n$ and $U \in \mathscr{U}(E)$ such that*

$$B_n \cap U \subset \overline{\sum_{j=1}^{m} \lambda_j B_j} \quad (\text{resp. } B_n \cap U \subset \sum_{j=1}^{m} \lambda_j B_j) \tag{1}$$

Proof. If E has (DDC), for every sequence $(\lambda_j)_{j\in\mathbb{N}}$ of strictly positive numbers and for every $n \in \mathbb{N}$ there are $m > n$ and $U \in \mathscr{U}(E)$ such that $B_n \cap U \subset \overline{\Gamma}\Big(\bigcup\limits_{j=1}^{m} \lambda_j B_j\Big) \subset \overline{\sum\limits_{j=1}^{m} \lambda_j B_j}$.

Conversely, we fix a sequence $(\mu_j)_{j\in\mathbb{N}}$ of strictly positive numbers and $n \in \mathbb{N}$. Put $\lambda_j := \mu_j 2^{-j}$ for every $j \in \mathbb{N}$ and apply (1). Then there are $m > n$ and $U \in \mathscr{U}(E)$ with

$$B_n \cap U \subset \overline{\sum_{j=1}^{m} \lambda_j B_j} = \overline{\sum_{j=1}^{m} \frac{\mu_j}{2^j} B_j} \subset \overline{\Gamma}\Big(\bigcup_{j=1}^{m} \mu_j B_j\Big).$$

Hence, E has the dual density condition.

For the strong dual density condition the proof is analogous. $\square$

Theorem 3. *Let $\mathscr{V} = (v_n)_{n\in\mathbb{N}}$ be a decreasing sequence of strictly positive continuous radial functions on the unit disc $\mathbb{D}$ such that each v_n approaches monotonically 0 as $r \to 1-$ and such that (LOG) is satisfied. The following are equivalent:*

(a) *$\mathscr{V}_0 H(\mathbb{D})$ has the dual density condition.*

(b) *$\mathscr{V} H(\mathbb{D})$ has the dual density condition.*

(c) *For every sequence $(\lambda_k)_{k\in\mathbb{N}}$ of strictly positive numbers and every $n \in \mathbb{N}$ there are $m > n$ and $\overline{w} \in \overline{V}$ such that*

$$C_{n,0} \cap C_{\overline{w},0} \subset \overline{\sum_{k=1}^{m} \lambda_k C_{k,0}}.$$

(d) *For every sequence $(\lambda_k)_{k\in\mathbb{N}}$ of strictly positive numbers and every $n \in \mathbb{N}$ there are $m > n$ and $\overline{v} \in \overline{V}$ such that*

$$\Big(\min\Big(\frac{1}{v_n}, \frac{1}{\overline{v}}\Big)\Big)^{\sim} \leqslant \sum_{k=1}^{m} \frac{\lambda_k}{v_k} \text{ on } \mathbb{D}.$$

Proof. First, we show the equivalence of (a) and (b). By [6] Proposition 4 we have $\mathscr{V} H(\mathbb{D}) = (\mathscr{V}_0 H(\mathbb{D})'_b)'_i$.

(a) $\Longrightarrow$ (b): Let $\mathscr{V}_0 H(\mathbb{D})$ have the dual density condition. Then each bounded subset of $\mathscr{V}_0 H(\mathbb{D})$ is metrizable by [4] Theorem 1.5.(a). [3] Theorem 1.4 yields that $\mathscr{V}_0 H(\mathbb{D})'_b$ has the density condition and is distinguished. Thus, $\mathscr{V} H(\mathbb{D}) = (\mathscr{V}_0 H(\mathbb{D})'_b)'_i = \mathscr{V} H(\mathbb{D})''_{bb}$, and $\mathscr{V} H(\mathbb{D})$ has the dual density condition.

(b) $\Longrightarrow$ (a): Let $\mathscr{V} H(\mathbb{D})$ have the dual density condition. Since $\mathscr{V} H(\mathbb{D}) = (\mathscr{V}_0 H(\mathbb{D})'_b)'_i$ holds, by [16, Corollary 2], $\mathscr{V}_0 H(\mathbb{D})'_b$ has the density condition. An application of [3, Theorem 1.4 and 1.5] implies that $\mathscr{V}_0 H(\mathbb{D})$ enjoys the dual density condition.

Next we want to show that (a) implies (d). By [10, Theorem 5] $\mathscr{V}_0 H(\mathbb{D})$ is a dense topological subspace of $H\overline{V}_0(\mathbb{D})$. In particular $H\overline{V}_0(\mathbb{D})$ is a (DF)-space (see [12, Theorem 12.4.8.(d)]. Using [4, Theorem 1.10.(a)]. By Lemma 2 we know that then for a sequence $(\lambda_j)_{j\in\mathbb{N}}$ of strictly positive numbers and every $n \in \mathbb{N}$ there are $m > n$ and $\overline{v} \in \overline{V}$ such that

$$C'_{n,0} \cap C_{\overline{v},0} \subset \sum_{j=1}^{m} \lambda_j C'_{j,0} \tag{2}$$

So we have to show that (2) implies (d). We fix $f \in H(G)$ such that $|f| \leqslant (\min(1/v_n, 1/\overline{v}))^{\sim}$ on G; hence $|f| \leqslant 1/v_n$ and $|f| \leqslant 1/\overline{v}$ on G. W.l.o.g. we can choose $\overline{v} \in \overline{V}$ strictly positive, continuous and radial (see [7]).

Now, we consider the sequence $(S_k f)_{k\in\mathbb{N}}$ of the Cesàro means (of the partial sums) of the Taylor series about 0. We obtain $|S_k f| \leqslant 1/v_n$ and $|S_k f| \leqslant 1/\overline{v}$ on G (see [7] Proposition 1.2.(c)). Moreover, each polynomial $S_k f$ is an element of $H\overline{V}_0(G)$ and hence of $C'_{n,0} \cap C_{\overline{v},0}$. (2) yields $S_k f \in \overline{\sum_{j=1}^{m} \lambda_j C'_{j,0}} = \sum_{j=1}^{m} \lambda_j C'_{j,0}$ for every $k \in \mathbb{N}$. Thus, each $S_k f$ can be written as $S_k f = \sum_{j=1}^{m} \lambda_j g_j$ where $g_j \in C_{j,0}$ for every $j \in \{1, \ldots, m\}$. We get

$$|S_k f| \leqslant \sum_{j=1}^{m} \frac{\lambda_j}{v_j} \text{ on } G \text{ for every } k \in \mathbb{N}.$$

Since $S_k f \to f$ pointwise, we obtain $|f| \leqslant \sum_{j=1}^{m} \lambda_j / v_j$ on G. Taking the supremum over all f, condition (d) follows.

By [17, Lemma 1] (a) and (c) are equivalent. It remains to prove that (d) yields (c). Our proof was inspired by [10, Theorem 5].

Let $M > 0$ denote a constant such that $M > \sum_{k\in\mathbb{N}} a^k$, $0 < a < 1$ as in (b) of Definition 1, and $M > 2A^t 2^{n-t} 2^{-2^{n-1}}$ if $n > t$, $1 < A$ as in (a) of Definition 1.

We fix $(\lambda_k)_{k\in\mathbb{N}}$ and $n \in \mathbb{N}$. Moreover we select $m > n$ and $\overline{v} \in \overline{V}$. Now we fix $f \in C_{n,0} \cap C_{\overline{v},0} \cap \mathscr{V}_0 H(\mathbb{D})$. Hence $f \in \mathscr{V}_0 H(\mathbb{D})$ and

$$|f| \leqslant \Big(\min\Big(\frac{1}{v_n}, \frac{1}{\overline{v}}\Big)\Big)^{\sim} \leqslant \sum_{k=1}^{m} \frac{\lambda_k}{v_k} \leqslant \max\Big(\frac{m\lambda_1}{v_1}, \dots, \frac{m\lambda_m}{v_m}\Big).$$

Put $u := \min(v_1/m\lambda_1, \dots, v_m/m\lambda_m)$. Hence

$$f \in D_{u,0} := \big\{f \in \mathscr{V}_0 H(\mathbb{D});\ \sup_{z\in\mathbb{D}} u(z)|f(z)| \leqslant 1\big\}.$$

We write $u = \min(a_1u_1, \dots, a_mu_m)$ where $a_i = 1/m\lambda_i$ and $u_i = v_i$ for every $1 \leqslant i \leqslant m$. u is a radial, continuous and non-increasing function. It is known that each $g \in \mathscr{V}_0 H(\mathbb{D})$ can be approximated in $\mathscr{V}_0 H(\mathbb{D})$ by the functions $g_{r_n}(z) = g(r_n z)$ for large n. (Taking a k such that $g \in H(v_k)_0(\mathbb{D})$ we have $\lim\limits_{|z|\to 1-} |g(z)|v_k(z) = 0$. This implies $g_{r_n} \to g$ in $H(v_k)_0(\mathbb{D})$ as $n \to \infty$. The topology of the inductive limit is coarser, hence, $g_{r_n} \to g$ also there.) Since the weight u is non-increasing, we get

$$\inf_{|z|\in I_n} u(z) = u(r_{n+1}) \geqslant u(r_{n+2}) = \inf_{|z|\in I_{n+1}} u(z) \geqslant A^{-2}u(r_n). \tag{3}$$

For every $n \in \mathbb{N}$ we can thus pick a $k(n) \in \{1, \dots, m\}$ such that

$$u(r_n) = a_{k(n)}u_{k(n)}(r_n) = a_{k(n)} \sup_{|z|\in I_n} u_{k(n)}(z). \tag{4}$$

For $\nu \in \mathbb{N}$ let $N_l = \{n \leqslant \nu;\ k(n) = l\}$ for each $1 \leqslant l \leqslant m$. Let us define, for all n, the function $g_n(z) := f(r_{n+1}z) - f(r_n z)$ and $g_0(z) := f(0)$, and, for $i \in \{1, \dots, m\}$,

$$h_i := \sum_{n\in N_i} g_n, \tag{5}$$

and $h_i := 0$ if $N_i = \emptyset$. Clearly $f_{r_\nu} = h_1 + \ldots + h_m + g_0$. The constant function g_0 belongs to $H(u_1)_0(\mathbb{D})$ and $|f(0)| \leqslant a_1^{-1}u_1^{-1}(0)$, hence $g_0 \in a_1^{-1}C_{k,0}$. Let us fix $i \in \{1, \dots, m\}$. We pick $n \in N_i$, and estimate $|g_n(z)|$ for different z.

1. Assume first $|z| \geqslant r_{n-1}$. Then

$$|r_n z| \geqslant (1 - 2^{-2^n})(1 - 2^{-2^{n-1}}) \geqslant (1 - 2 \cdot 2^{-2^{n-1}}) \in I_{n-2}$$

and similarly for $|r_{n+1}z|$; hence

$$r_{n-2} \leqslant |r_n z| \leqslant |r_{n+1}z| \leqslant r_{n+1}. \tag{6}$$

Since $f \in D_{u,0}$ we have for these z, by (3)

$$(7) \qquad |g_n(z)| \leqslant |f(r_n z)| + |f(r_{n+1} z)| \leqslant 2 \sup_{r_{n-2} \leqslant r \leqslant r_{n+1}} u(r)^{-1} = 2u(r_{n+1})^{-1}.$$

Now (7) can still be estimated using (4) by

$$(8) \qquad 2A^2 u(r_n)^{-1} = 2A^2 a_i^{-1} u_i(r_n)^{-1}.$$

2. Assume $2 \leqslant t \leqslant n$ and $|z| \in I_{n-t}$. We have

$$(9) \qquad \begin{aligned} |g_n(z)| = |f(r_n z) - f(r_{n+1} z)| &\leqslant \sup_{\xi \in I_{n-t} \cup I_{n-t-1}} |f'(\xi)||r_{n+1} - r_n| \\ &\leqslant \sup_{\xi \in I_{n-t} \cup I_{n-t-1}} |f'(\xi)| 2^{-2^n}. \end{aligned}$$

We estimate $|f'(\xi)|$ using the Cauchy formula

$$(10) \qquad |f'(\xi)| \leqslant \int_{|\eta| = r_n} \frac{|f(\eta)|}{|\eta - \xi|^2} \, \mathrm{d}\eta \leqslant u(r_n)^{-1} 2^{2^{n-t+1}},$$

since $|\eta - \xi| \geqslant 2^{-2^{n-t+1}} - 2^{-2^n} \geqslant 2^{-1} \cdot 2^{-2^{n-t+1}}$. We use $2^n - 2^{n-t+1} \geqslant 2^{n-1}$ and from (9) and (10) we obtain

$$(11) \qquad |g_n(z)| \leqslant 2^{-2^{n-1}} \cdot u(r_n)^{-1} \leqslant 2^{-2^{n-1}} \cdot a_{k(n)}^{-1} u_{k(n)}(r_n)^{-1}.$$

Here we used (4). Moreover, using (a) of Definition 1 t times, we can continue the estimate by

$$(12) \qquad \leqslant 2^{-2^{n-1}} \cdot A^t a_{k(n)}^{-1} u_{k(n)}(z)^{-1}.$$

Since $n > t$ we have $2^{-2^{n-1}} A^t \leqslant M 2^{-(n-t)}$ (for all n and t), hence (12) is bounded by

$$(13) \qquad M 2^{-(n-t)} a_{k(n)}^{-1} u_{k(n)}(z)^{-1} = M 2^{-(n-t)} a_i^{-1} u_i(z)^{-1}.$$

So altogether

$$(14) \qquad |g_n(z)| \leqslant M 2^{-(n-t)} a_i^{-1} u_i(z)^{-1}.$$

To complete the proof, let now $z \in \mathbb{D}$; we want to show that there is a constant $C > 0$ such that

$$(15) \qquad |h_i(z)| \leqslant (2MCA^2 + M) a_i^{-1}.$$

Let $t \in \mathbb{N}$ be such that $|z| \in I_t$, then

$$|h_i(z)| \leqslant \sum_{n \in N_i, n \leqslant t+1} |g_n(z)| + \sum_{n \in N_i, n > t+1} |g_n(z)| =: G_i(z) + H_i(z). \tag{16}$$

(a) Consider $G_i(z)$. In this case (8) of 1. implies

$$G_i(z) \leqslant \sum_{n \in N_i, n \leqslant t+1} 2A^2 a_i^{-1} u_i(r_n)^{-1}.$$

By using (b) of Definition 1 $(t-n)$ times, there is a constant $C > 0$ such that this is bounded by

$$C \sum_{n \leqslant t+1} 2A^2 a_i^{-1} u_i(r_t)^{-1} a^{-t+n} \leqslant 2CMA^2 a_i^{-1} u_i(z)^{-1}. \tag{17}$$

Remember that $a < 1$ and $M > \sum\limits_{k \in \mathbb{N}} a^k$.

(b) Consider $H_i(z)$. Then 2. (14) implies

$$H_i(z) \leqslant \sum_{n \in N_i, n > t+1} M 2^{-(n-t)} a_i^{-1} u_i(z)^{-1} \leqslant M a_i^{-1} u_i(z)^{-1}. \tag{18}$$

We obtain

$$\begin{aligned} f_{r_\nu} = g_0 + \sum_{l=1}^{m} h_l &\in (1 + 2CMA^2 + M) a_1^{-1} C_{1,0} + \sum_{k=2}^{m} (2CMA^2 + M) a_k^{-1} C_{k,0} \\ &\subset (1 + 2CMA^2 + M) m \sum_{k=1}^{m} \lambda_k C_{k,0}. \end{aligned}$$

Put $\overline{w} = \overline{v}/m(1 + 2CMA^2 + M)$ and obtain the assertion (c). □

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