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Passivity based stabilization of non-minimum phase nonlinear systems

Terms of use:

PASSIVITY BASED STABILIZATION OF NON–MINIMUM PHASE NONLINEAR SYSTEMS

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A cascade scheme for passivity-based stabilization of a wide class of nonlinear systems is proposed in this paper. Starting from the definitions and basic concepts of passivity-based stabilization via feedback (which are applicable to minimum phase nonlinear systems expressed in their normal forms) a cascade stabilization scheme is proposed for minimum and non-minimum phase nonlinear systems where the constraint of stable zero dynamics imposed by previous stabilization approaches is abandoned. Simulation results of the proposed algorithm are presented to demonstrate its performance.

Keywords: nonlinear systems, stabilization, passivity, state feedback

AMS Subject Classification: 93C10, 93D15, 93D99

1. INTRODUCTION

The problem of stabilization of nonlinear systems has widely been studied in the last three decades [4, 13]. Several approaches have been proposed to deal with this problem like the small gain theorem, Popov's criterion, Lyapunov theory, feedback linearization and backstepping [15, 16], to mention a few of them.

One of the most interesting approaches is passivity-based stabilization (PBS). Passivity is a property between the input and the output of the system and plays a central role in the control literature (see e.g. [2, 12]). PBS has an advantage over other nonlinear control techniques since it takes into account the energy of the system, which gives a clear physical meaning to the system variables.

In [2], a synthesis based on the concepts of passivity and the conditions under which a nonlinear system with known parameters can be rendered passive via a smooth state feedback are derived. The case when the plant parameters are unknown (the adaptive case) has also been studied [9, 10, 11, 17]. In all cases (adaptive and non–adaptive cases) the obtained results are only applicable to a certain class of weakly minimum phase nonlinear systems that can be represented in the so-called normal form, but they can not be applied to non-minimum phase nonlinear systems.

In the linear case the output feedback control of non-minimum phase systems is usually solved by factorizing the system dynamics into a minimum phase and non-

minimum phase parts [19] where only the former part is considered in the controller design while the latter one is ignored. Some attempts have been made to establish such a factorization in the nonlinear case, too, but the problem still remains open [23].

A semi-global practical stabilization design tool is proposed in [14] for a general class of uncertain non-minimum phase nonlinear systems. This approach is based on the assumption of global stabilizability of an auxiliary system and was extended in [7].

As far as the output tracking of non-minimum phase nonlinear systems is concerned, there are two major approaches. In the first approach, which has been proposed in [5, 6] and further modified in [20], the stabilizing control consists of a feedforward component that generates the zero dynamics trajectory and a feedback component that stabilizes the whole system. The second approach concerns the nonlinear output regulator problem [13]. This approach uses the center manifold theory and provides necessary and sufficient conditions under which the closed-loop system can be driven to a center manifold contained in the output zeroing manifold. The approach gives a local result around the equilibrium point. Another two local stabilization techniques using the concept of center manifold can be found in [1, 3]. In the former citation, the problem of local stabilization of nonlinear control systems with linearizations containing uncontrollable modes on the imaginary axis is considered. It reduces the stability problem to the stability of the center manifold system. In [3], the problem of the local state feedback stabilization of a class of nonlinear systems with nonminimum phase zero dynamics is addressed by using the so-called Lyapunov function with homogeneous derivative along the solution curves in order to test the approximate stability of the dynamics on the center manifold. Other approaches, like those based on backstepping and neural networks, have also been used in order to face this problem; see for instance [8, 24].

In this paper the stabilization of non-minimum phase systems is approached using the theory of passivity in order to show that this method can successfully be used when solving such problems. The achieved results can be viewed as an extension of the results proposed in [2] since the assumptions under which the proposed scheme is valid are the same as those in [2]. Thus, a passivity-based cascade scheme is suggested to stabilize a wide class of nonlinear systems (in their normal forms) without the constraint of being weakly minimum phase.

A comparison of the proposed method with those known up to now is out of the scope of the paper and constitutes a topic for further investigation. However, the fact that an application of the proposed method leads to a first order behavior of the zero dynamics could be of some interest.

The paper is organized as follows. In Section 2 the basic concepts of passivity are introduced and PBS of nonlinear systems in the normal form is studied. The proposed cascade scheme is introduced in Section 3. In Section 4, simulation results are exposed to demonstrate the performance of the used method. Finally, some conclusions are drawn in Section 5.

2. DEFINITIONS AND BASIC CONCEPTS

Consider a nonlinear system of the form

$$
\begin{aligned}
\dot{x} &= f(x) + g(x)u, & x, f(x) \in \mathscr{X}, \; g(x) \in \mathscr{X} \times \mathscr{U} \\
y &= h(x), & y(t), \; u(t), \; h(x) \in \mathscr{Y}
\end{aligned}
\tag{1}
$$

where the symbol $\mathscr{X}$ denotes the state space of (1) ($\mathscr{X} = \mathbb{R}^n$), $\mathscr{U}$ stands for the input space, and $\mathscr{Y}$ for the output space of (1). Let $\mathscr{U} = \mathscr{Y} = \mathbb{R}^m$. The functions f and the columns g_i of g are supposed to be smooth vector fields (i. e. $f, g_i \in C^\infty$) and h is a smooth mapping ($h \in C^\infty$). Without any loss of generality it can also be assumed that the vector field f has at least one equilibrium point and (possibly after a coordinates shift) that $f(0) = h(0) = 0$. All the states of (1) are further assumed to be accessible.

For the reader's convenience some basic definitions and assumptions are introduced below; see [2] for more details.

Definition 1. (Byrnes et al. [2]) The system (1) is said to be C^r-*passive* if there exists a C^r non-negative function $V : \mathbb{R}^n \to \mathbb{R}, V(0) = 0$, called the storage function, such that for all $u \in \mathscr{U}, x(0) = x_0 \in \mathscr{X}, \; y \in \mathscr{Y}$, and $t \geq 0$.

$$
V(x(t)) - V(x_0) \leq \int_0^t y^T(s)u(s)\,\mathrm{d}s.
\tag{2}
$$

If V is continuous with continuous derivatives of order r ($V \in C^r$), the condition (2) will also be written in the form $\dot{V} \leq y^T u$. The system (1) is said to be *lossless* for the case when $\dot{V} = y^T u$.

If the system (1) is of relative degree $\{1, 1, \ldots, 1\}$ at $x = 0$ and the distribution spanned by vector fields $g_1(x), \cdots, g_m(x)$ is involutive - see [13], then it is possible to find a new set of local coordinates $z(x) \in \mathbb{R}^{n-m}, m \leq n$, defined around the point $x = 0$ with $z(0) = 0$, under which the system (1) can be represented in the following *normal form* [2]

$$
\begin{aligned}
\dot{y} &= a(y, z) + b(y, z)u \\
\dot{z} &= c(y, z)
\end{aligned}
\tag{3}
$$

where $a(y, z)) \in \mathbb{R}^m, \; b(y, z)) \in \mathbb{R}^{m \times m}, \; c(y, z)) \in \mathbb{R}^{n-m}$, and $b(y, z)$ is invertible for all (y, z) around the point $(0, 0)$.

Writing the system as a function of its zero dynamics, which is defined by the internal dynamic terms that are consistent with the external constraint $y = 0$, i. e. $c(y, z) = c(0, z) + p(y, z)y$, we obtain

$$
\begin{aligned}
\dot{y} &= a(y, z) + b(y, z)u \\
\dot{z} &= f_0(z) + p(y, z)y
\end{aligned}
\tag{4}
$$

where $f_0(z) = c(0, z) \in \mathbb{R}^{n-m}$ and $p(y, z)$ is a smooth function of dimension $(n - m) \times m$ (cf. [2]).

Under the above mentioned assumptions, two important concepts involving the zero dynamics of system (1) will now be recalled.

Definition 2. (Byrnes et al. [2]) The system (1) is said to be locally weakly minimum phase if there exists a positive definite C^r-function $W_0(z)$, locally defined near 0, with $r \geq 2$ and $W_0(0) = 0$, satisfying

$$L_{f_0(z)} W_0(z) = \frac{\partial W_0(x)}{\partial z} f_0(z) \leq 0 \tag{5}$$

for all z in a neighborhood of $z = 0$.

Definition 3. (Byrnes et al. [2]) The locally weakly minimum phase system (1) is said to be C^r-passive equivalent via feedback if there exists a state feedback of the form $u = \alpha(x) + \beta(x)u^p$, where $\alpha(x) \in \mathbb{R}^m$ and $\beta(x) \in \mathbb{R}^{m \times m}$ are smooth functions defined either locally near $x = 0$ or globally with $\beta(x)$ invertible for all x, such that for the closed-loop system the following inequality is satisfied.

$$\dot{V}(x) = \left(\frac{\partial V(x)}{\partial x} \right)^T [f(x) + g(x)\alpha(x) + g(x)\beta(x)u^p] \leq y^T u^p \tag{6}$$

where V is a C^r-storage function and $u^p \in \mathbb{R}^m$ is a new control input.

The following definitions can be found in [5] where stabilization of passive systems is studied.

Definition 4. (Byrnes et al. [2]) A passive system is locally zero-state detectable if there exists a neighborhood $\mathscr{N}$ of 0 such that for all $x_0 \in \mathscr{N}$,

$$h(\Phi(t, x_0, 0)) = 0 \ \ \forall t > 0 \Rightarrow \lim_{t \to \infty} \Phi(t, x_0, 0) = 0 \tag{7}$$

where $\Phi(t, x_0, 0)$ denotes a state response upon the zero input u and initial condition x_0. If $\mathscr{N} = \mathbb{R}^n$, the system is called *zero-state detectable*.

For passive systems of the type (1) that are locally weakly minimum phase and locally zero-state detectable, the following theorem holds.

Theorem 1. (Byrnes et al. [5]) Let the system (1) be passive with C^r, $r \geq 1$ and with a proper storage function V that is positive definite. If the system is also locally zero-state detectable, then the control law

$$u = -Ky, \tag{8}$$

where K is a positive definite matrix, locally asymptotically stabilizes the equilibrium $x = 0$. Moreover, if the system is zero-state detectable and V is proper, then the control law (8) globally asymptotically stabilizes the equilibrium.

Example 1. Consider the system (1) with $n = 4$ and $m = 2$ defined as

$$f(x) = \begin{bmatrix} 3x_1 + x_2 - x_3 \\ -x_1 + 3x_2 - x_4 \\ x_1 + x_3 \left(x_3^2 + x_4^2 \right) \\ x_2 + x_4 \left(x_3^2 + x_4^2 \right) \end{bmatrix}, \ g(x) = \begin{bmatrix} I_2 \\ 0_2 \end{bmatrix}, \ h(x) = [x_1, x_2, 0, 0]^T \tag{9}$$

with $x = [x_1, x_2, x_3, x_4]^T \in \mathbb{R}^4$ and $y(t)$, $u(t) \in \mathbb{R}^2$. The system has zero equilibrium point. Since $L_g h(x) = \frac{\partial h(x)}{\partial x} g(x) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ the relative degree is unity around zero and the distribution spanned by the columns of $g(x)$ is involutive. Thus, the system (9) can be represented in the form (4) with

$$y = [x_1, x_2]^T, \quad z = [x_3, x_4]^T$$

$$a(y, z) = \begin{bmatrix} 3 & 1 \\ -1 & 3 \end{bmatrix} y - z, \ b(y, z) = I_2, \ f_0(z) = z\left(z^T z\right), \ p(y, z) = I_2. \tag{10}$$

It can easily be verified that the system is not locally weakly minimum phase. The associated positive definite function $W_0 = \frac{1}{2} z^T z$, $W_0(0) = 0$, is such that $L_{f_0} W_0(z) = \frac{\partial W_0(x)}{\partial z} f_0(z) = z^T z(z^T z) \geq 0$.

Previous definitions and procedures for PBS can not be directly applied to non-weakly minimum phase systems and thus the system in Example 1 can not be stabilized according to Theorem 1. In order to apply PBS to this case, a cascade scheme is proposed in the next section that stabilizes via passivity feedback non-minimum phase nonlinear systems.

3. PASSIVITY BASED STABILIZATION OF MINIMUM AND NON–MINIMUM PHASE NONLINEAR SYSTEMS

In order to apply PBS to a wider class of nonlinear systems (1) expressed in the normal form (4) and possibly non-weakly minimum phase, a cascade control system with an inner and an outer loop is proposed. The output variable y is now controlled trough the inner loop and internal dynamic variable z is controlled via the outer loop. Now the following theorem can be stated.

Theorem 2. Consider the system (1) with the relative degree $\{1, 1, \ldots, 1\}$ at $x = 0$ and assume that the distribution spanned by vector fields $g_1(x), \cdots, g_m(x)$ is involutive. Then, if the system represented in the normal form (4) (which can be weakly or non-weakly minimum phase) with $p(y, z)$ invertible, the cascade control law

$$u = b(y, z)^1 \left[-a(y, z) - \left(L_{p(z, y)} W_0 \right)^T + \dot{u}^* + u^p \right], \tag{11}$$

where

$$u^* = p(y, z)^{-1} \left[-f_0(z) + z^p \right], \tag{12}$$

$$u^p = -K_i(y - u^*), \tag{13}$$

$$z^p = -K_0 z, \tag{14}$$

with $K_i, K_0 \in \mathbb{R}^{m \times m}$ being positive definite matrices, stabilizes the system and guarantees the first order system behavior for the zero dynamics.

Proof of Theorem 2. After applying the relationships (11) and (12) to (4) and defining the variable $\tilde{y} = y - u^*$, the following system is obtained

$$\dot{\tilde{y}} = -\left(L_{p(z,y)} W_0\right)^T + u^p \tag{15}$$

$$\dot{z} = p(y,z)\tilde{y} + z^p. \tag{16}$$

The system $(15) - (16)$ is a lossless system according to Definition 1. In fact, considering a positive definite C^2-storage function of the form $V(z,\tilde{y}) = \frac{1}{2}\tilde{y}^T \tilde{y} + W_0(z)$, its first time derivative evaluated for (15) and (16) yields

$$\dot{V}(z,\tilde{y}) = -\tilde{y}^T u^p - z^T z^p = -\left[\tilde{y}^T z^T\right]^T \left[u^p z^p\right].$$

Using (13) and (14) equations (15) and (16) can be rewritten as follows

$$\dot{\tilde{y}} \;=\; -K_i \tilde{y} - \left(L_{p(z,y)} W_0\right)^T \tag{17}$$

$$\dot{z} \;=\; -K_0 z + p(y,z)\tilde{y}. \tag{18}$$

Consider again the positive definite C^2-storage function of the form $V(z,\tilde{y}) = \frac{1}{2}\tilde{y}^T \tilde{y} + W_0(z)$. Its first time derivative evaluated for (17) and (18) yields $\dot{V}(z,y) = -\tilde{y}^T K_i \tilde{y} - z^T K_0 z - L_{p(z,y)\tilde{y}} W_0 + L_{p(z,y)\tilde{y}} W_0$. Then

$$\dot{V}(z,\tilde{y}) = -\tilde{y}^T K_i \tilde{y} - z^T K_0 z \tag{19}$$

and integrating both sides of the previous equations leads to

$$\int_0^\infty \dot{V}(z,\tilde{y})\, \mathrm{d}t = -\int_0^\infty \tilde{y}^T K_i \tilde{y}\, \mathrm{d}t - \int_0^\infty z^T K_0 z\, \mathrm{d}t,$$

or equivalently

$$[V(\infty) - V(0)] = -\int_0^\infty \tilde{y}^T K_i \tilde{y}\, \mathrm{d}t - \int_0^\infty z^T K_0 z\, \mathrm{d}t. \tag{20}$$

Since V is positive definite and its derivative $\dot{V}$ is negative definite, the left hand side of (19) is finite and therefore $\tilde{y} \in l^2$, $z \in l^2$. From the characteristics of system (17) and (17) it can be concluded that $\dot{\tilde{y}} \in l^\infty$, $\dot{z} \in l^\infty$. Using the Barbalat Lemma given in [8], we have that $\tilde{y} \in l^2$, $z \in l^2$ and $\dot{\tilde{y}} \in l^\infty$, $\dot{z} \in l^\infty \Rightarrow \lim_{t\to\infty} \tilde{y}(t) = 0$, $\lim_{t\to\infty} z(t) = 0$. □

It is important to remark that the assumptions under which Theorem 2 is valid are the same as those used in [5] where passivity is used to stabilize minimum phase nonlinear systems.

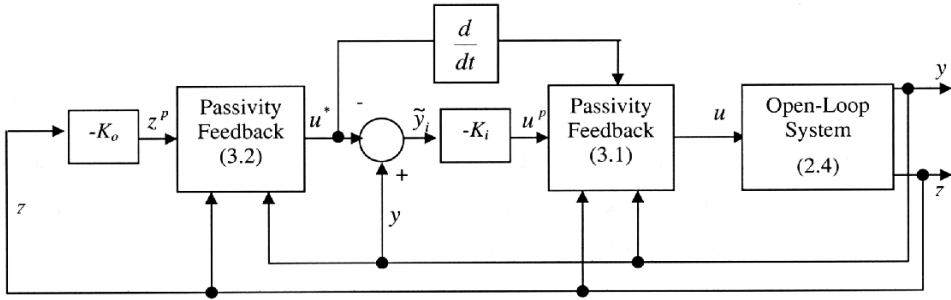The block diagram of the proposed method is shown in Figure 1.

Fig. 1. Block diagram of the proposed nonlinear cascade control.

Consider again the system of the Example 1 defined by (9) and (10). With a feedback control laws of the form

$$u = -\begin{bmatrix} 3 & 1 \\ -1 & 3 \end{bmatrix} y + z - I_2 y + \dot{u}^* - K_i(y - u^*)$$
$$u^* = z(z^T z) - K_0 z$$

the system is stabilized for any positive definite K_i and K_0.

4. SIMULATION RESULTS

In this Section, simulation results for Example 1 are exposed. The initial conditions for the simulation presented here are the following: $y_1(0) = 2$, $y_2(0) = -2$, $z_1(0) = 0.1$, $z_2(0) = 0.3$. The controller (3.11) and (3.12) is given by $K_i = 15I_2$ and $K_o = I_2$ and the saturation block is limited by 10 and -10.

The simulation results are presented in Figure 2. It can be seen that the system was stabilized at $x = [y_1 y_2 z_1 z_2]^T = 0$. The variables $y = [y_1 y_2]^T$ and $z = [z_1 z_2]^T$ tends to zero even if the input $u = [u_1 u_2]^T$ is saturated at $t = 0$.

5. CONCLUSIONS

A new passivity-based stabilization method for nonlinear dynamical systems has been presented in the paper. The proposed controller can be applied to a wide class of nonlinear systems including minimum and non-minimum phase systems given in their normal forms, under the assumption that all the parameters are known. The method uses the same amount of information that the method proposed earlier in [2] for stabilizing minimum phase systems. Its contribution lies in the fact that the presented method covers also the weakly non-minimum phase systems.

In order to verify the performance of the proposed method, the new cascade scheme for PBS is applied to stabilizing a fourth order non-minimum phase system (given in its normal form). It should be noted that all the simulation results completely correspond to the theoretical results presented in Section 3.
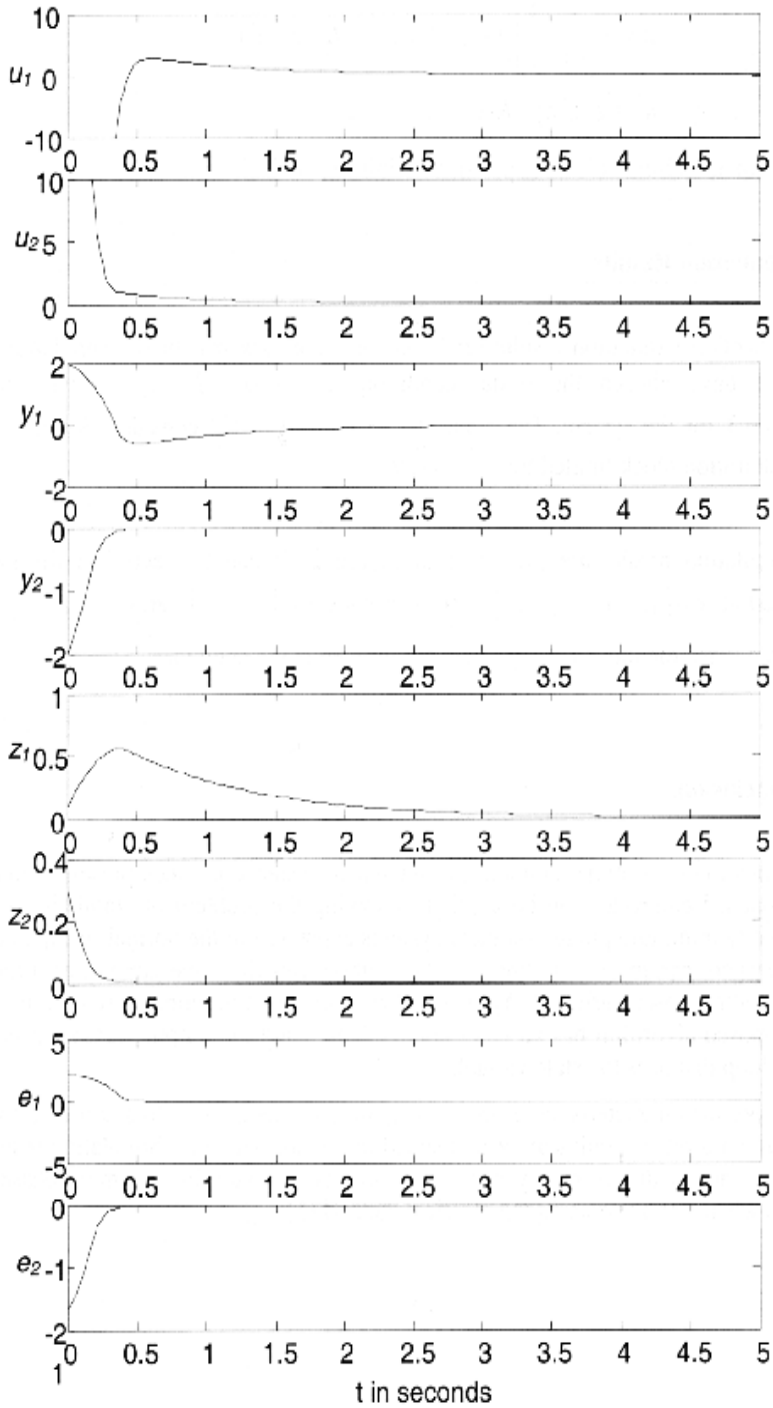
Fig. 2. Simulation results for the proposed cascade control scheme applied to Example 1.

The comparison of the proposed technique with other methodologies constitutes a topic of further investigation. However, the fact that applying the proposed method leads to a first order behavior of the zero dynamics is of major importance.

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