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STRONGLY ALMOST CONVERGENT GENERALIZED DIFFERENCE SEQUENCES ASSOCIATED WITH MULTIPLIER SEQUENCES

AYHAN ESI* — BINOD CHANDRA TRIPATHY**

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ABSTRACT. Let $\Lambda = (\lambda_k)$ be a sequence of non-zero complex numbers. In this paper we introduce the strongly almost convergent generalized difference sequence spaces associated with multiplier sequences i.e. $w_0[A, \Delta^m, \Lambda, p]$, $w_1[A, \Delta^m, \Lambda, p]$, $w_\infty[A, \Delta^m, \Lambda, p]$ and study their different properties. We also introduce Δ_Λ^m-statistically convergent sequences and give some inclusion relations between $w_1[\Delta^m, \lambda, p]$ convergence and Δ_Λ^m-statistical convergence.

1. Introduction

Throughout the article w, ℓ_∞, c, c_0 denote the spaces of *all*, *bounded*, *convergent* and *null* sequences respectively. The studies on difference sequence space was initiated by Kizmaz [8]. He studied the spaces

$$Z(\Delta) = \{x = (x_k) \in w : \ \Delta x = (\Delta x_k) \in Z\},$$

for $Z = \ell_\infty, c$ and c_0, where $\Delta x_k = x_k - x_{k+1}$, for all $k \in \mathbb{N}$.

It was shown by him that these spaces are Banach spaces, normed by

$$\|x\|_\Delta = |x_1| + \sup_k |\Delta x_k|.$$

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The notion was further generalized by Et and Colak [1] as follows:
Let $m \geq 0$ be an integer, then

$$Z(\Delta^m) = \{x = (x_k) \in w : \ \Delta^m x = (\Delta^m x_k) \in Z\},$$

for $Z = \ell_\infty, c$ and c_0, where $\Delta^m x_k = \Delta^{m-1} x_k - \Delta^{m-1} x_{k+1}$, $\Delta^0 x_k = x_k$, for all $k \in \mathbb{N}$.

The generalized difference $\Delta^m x_k$ has the following binomial representation

$$\Delta^m x_k = \sum_{i=0}^{m} (-1)^i \binom{m}{i} x_{k+i}, \tag{1)}$$

for all $k \in \mathbb{N}$.

Later on the notion was further investigated by Tripathy ([19], [20]), Et and Esi [2] and many others.

Let $A = (a_{nk})$ be an infinite matrix of complex numbers. Then A is said to be *regular* if and only if it satisfies the following well-known Siverman-Toeplitz conditions

(i) $\sup\limits_n \sum\limits_{k=1}^{\infty} |a_{nk}| < \infty$.

(ii) $\lim\limits_{n\to\infty} a_{nk} = 0$, for each $k \in \mathbb{N}$.

(iii) $\lim\limits_{n\to\infty} \sum\limits_{k=1}^{\infty} a_{nk} = 1$.

The scope for the studies on sequence spaces was extended by using the notion of associated multiplier sequenres. Goes and Goes [7] defined the differentiated sequence space $\mathrm{d}E$ and the integrated sequence space $\int E$ for a given sequence space E, using the multiplier sequences (k^{-1}) and (k) respectively. Some other authors took some particular type of multiplier sequences for their study. In this article we shall ronsider a general multiplier sequence $\Lambda \quad (\lambda_k)$ of non-zero scalars.

Let $\Lambda = (\lambda_k)$ be a sequence of non-zero scalars. Then for E a sequence space, the multiplier sequence space $E(\Lambda)$, associated with the multiplier sequence Λ is defined as

$$E(\Lambda) = \{(x_k) \in w : \ (\lambda_k x_k) \in E\}.$$

The notion of paranormed sequence space was studied at the initial stage by Nakano [12] and Simons [17]. It was further investigated from sequence space point of view and linked with summability theory by Maddox [10], Lascarides [9], Nanda [13], Rath and Tripathy [14], Tripathy and Sen [21], Tripathy [20] and many others.

2. Definitions and preliminaries

Throughout $A = (a_{nk})$ be an infinite regular matrix of non-negative complex numbers and $p = (p_k)$ be a sequence of real numbers such that $p_k > 0$, for all $k \in \mathbb{N}$ and $H = \sup_k p_k < \infty$. Let $m \geq 0$ be an integer and $\Lambda = (\lambda_k)$, be a multiplier sequence. Then we define

$$w_0[A, \Delta^m, \Lambda, p] = \Big\{x = (x_k) \in w : \lim_{n\to\infty} \sum_{k=1}^{\infty} a_{nk} |\lambda_k \Delta^m x_k|^{p_k} = 0\Big\},$$

$$w_1[A, \Delta^m, \Lambda, p] = \Big\{x = (x_k) \in w : \lim_{n\to\infty} \sum_{k=1}^{\infty} a_{nk} |\lambda_k \Delta^m x_k - L|^{p_k} = 0, \text{ for some } L\Big\}$$

$$w_\infty[A, \Delta^m, \Lambda, p] = \Big\{x = (x_k) \in w : \sup_n \sum_{k=1}^{\infty} a_{nk} |\lambda_k \Delta^m x_k|^{p_k} < \infty\Big\},$$

If $(x_k) \in w_1[A, \Delta^m, \Lambda, p]$, then we write $x_k \to L(w_1[A, \Delta^m, \Lambda, p])$.

We get the following particular cases of the above sequence spaces by restricting some of the parameters m, p, $A = (a_{nk})$ and $\Lambda = (\lambda_k)$.

When $A = (a_{nk}) = (C, 1)$, Cesàro matrix, we mention the above mentioned spaces by $w_0[\Delta^m, \Lambda, p]$, $w_1[\Delta^m, \Lambda, p]$ and $w_\infty[\Delta^m, \Lambda, p]$. For instance

$$w_1[\Delta^m, \Lambda, p] = \Big\{x = (x_k) \in w : \lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{\infty} a_{nk} |\lambda_k \Delta^m x_k - L|^{p_k} = 0, \text{ for some } L\Big\}.$$

When $m = 0$ and $\Lambda = e = (1, 1, 1, \dots)$, we obtain the sequence spaces $[A, p]_0$, $[A, p]_\infty$ and $[A, p]_1$, introduced and studied by Maddox [10]. If $x \in [A, p]_1$, we say that x is strongly A-summable to L.

When $A = (C, 1)$ i.e. the Cesàro matrix, $m = 0$ and $\Lambda = e$, we obtain the sequence spaces $w_0(p)$, $w_\infty(p)$ and $w_1(p)$, introduced and studied by Maddox [10]. If $x \in [A, p]_1$, we say that x is strongly p-Cesàro summable to L.

Let $p = (p_k)$ be a bounded sequence of strictly positive real numbers. Let $H = \sup p_k$ and $D = \max(1, 2^{H-1})$. Then we have (see for instance Maddox [11]),

$$|a_k + b_k|^{p_k} \leq D(|a_k|^{p_k} + |b_k|^{p_k}). \tag{2}$$

3. Main results

The proof of the following result is obvious.

Theorem 1. *Let $A = (a_{nk})$ be a non-negative matrix and $p - (p_k)$ be a bounded sequence of positive real numbers. Then*

(i) $w_0[A, \Delta^m, \Lambda, p]$, $w_1[A, \Delta^m, \Lambda, p]$ *and* $w_\infty[A, \Delta^m, \Lambda, p]$ *are linear spaces over the field* C.

(ii) $w_1[A, \Delta^m, \Lambda, p] \subset w_\infty[A, \Delta^m, \Lambda, p]$.

Theorem 2. *Let $A = (a_{nk})$ be a non-negative matrix and $p = (p_k)$ be a bounded sequence of positive real numbers. Then the spaces $w_0[A, \Delta^m, \Lambda, p]$ and $w_1[A, \Delta^m, \Lambda, p]$ are complete linear topological spaces, paranormed by*

$$g(x) = \sum_{i=1}^{m} |\lambda_i x_i| + \sup_n \Big\{ \sum_{k=1}^{\infty} a_{nk} |\lambda_k \Delta^m x_k|^{p_k} \Big\}^{\frac{1}{M}},$$

where $M = \max\Big\{1, \sup_k p_k\Big\}$.

Proof. Clearly $g(\theta) = 0$, $g(-x) = g(x)$ and by Minkowski's inequality $g(x+y) \leq g(x) + g(y)$. We now show that the scalar multiplication is continuous. Whenever $\xi \to 0$ and $x \to \theta$, imply $g(\xi x) \to 0$. Also $x \to \theta$, we have $g(\xi x) \to 0$. Now we show that $\xi \to 0$ and x fixed imply $g(\xi x) \to 0$. Without loss of generality let $|\xi| < 1$. Then the required proof follows from the following inequality.

$$\begin{aligned} g(\xi x) &= \sum_{i=1}^{m} |\xi \lambda_i x_i| + \sup_n \Big\{ \sum_{k=1}^{\infty} a_{nk} |\xi \lambda_k \Delta^m x_k|^{p_k} \Big\}^{\frac{1}{M}}, \\ &\leq |\xi| \sum_{i=1}^{m} |\lambda_i x_i| + \max\{|\xi|, |\xi|^{\frac{H}{M}}\} \sup_n \Big\{ \sum_{k=1}^{\infty} a_{nk} |\lambda_k \Delta^m x_k|^{p_k} \Big\}^{\frac{1}{M}}, \\ &\leq \max\{|\xi|, |\xi|^{\frac{H}{M}}\} \cdot g(x) \to 0, \qquad \text{as} \quad |\xi| \to 0. \end{aligned}$$

Let (x^s) be a Cauchy sequence in $w_0[A, \Delta^m, \Lambda, p]$. Then $g(x^s - x^t) \to 0$, as $s, t \to \infty$. For a given $\epsilon > 0$, let n_0 be such that

$$\sum_{i=1}^{m} |\lambda_i (x_i^s - x_i^t)| + \sup_n \Big\{ \sum_{i=1}^{\infty} a_{nk} |\lambda_k \Delta^m (x_k^s - x_k^t)|^{p_k} \Big\}^{\frac{1}{M}} < \epsilon, \qquad \text{for all} \quad s, t \geq n_0. \tag{3}$$

Hence $\sum_{i=1}^{m} |\lambda_i x_i^s - \lambda_i x_i^t| < \epsilon$, for all $s, t \geq n_0$.

$\Longrightarrow$ $\{\lambda_i x_i^s\}$ is a Cauchy sequence for each $i = 1, 2, \ldots, m$.

$\Longrightarrow$ $\{\lambda_i x_i^s\}$ converges in C for each $i = 1, 2, \ldots, m$.

Let $\lim_{s\to\infty} \lambda_i x_i^s = y_i$, for each $i = 1, 2, \ldots, m$. Let

$$\lim_{s\to\infty} x_i^s = x_i, \text{ say,} \qquad \text{where} \quad x_i = y_i\lambda_i^{-1}, \quad \text{for each } i = 1, 2, \ldots, m. \tag{4}$$

From (3), we have $\sup_n\Big\{\sum_{k=1}^{\infty} a_{nk}|\lambda_k\Delta^m(x_k^s - x_k^t)|^{p_k}\Big\}^{\frac{1}{M}} < \epsilon$, for all $s, t \geq n_0$.

$\implies |\lambda_k\Delta^m x_k^s - \lambda_k\Delta^m x_k^t| < \epsilon$, for all $s, t \geq n_0$, since a_{nk} are strictly positive.

$\implies \{\lambda_k\Delta^m x_k^s\}$ is a Cauchy sequence in C for each $k \in \mathbb{N}$.

Hence $\{\lambda_k\Delta^m x_k^s\}$ converges for each $k \in \mathbb{N}$. Let $\lim_{s\to\infty} \lambda_k\Delta^m x_k^s = z_k$, for each $k \in \mathbb{N}$.

Let

$$\lim_{s\to\infty} \Delta^m x_k^s = y_k = z_k\lambda_k^{-1} \qquad \text{for each} \quad k \in \mathbb{N}. \tag{5}$$

Hence from (1), (4) and (5) it follows that $\lim_{s\to\infty} x_{m+1}^s = x_{m+1}$. Proceeding in this way inductively, we have

$$\lim_{s\to\infty} x_k^s = x_k, \qquad \text{for each} \quad k \in \mathbb{N}.$$

By (3) we have

$$\lim_{t\to\infty}\Big\{\sum_{i=1}^{m}|\lambda_i(x_i^s - x_i^t)| + \sup_n\Big\{\sum_{k=1}^{\infty} a_{nk}|\lambda_k\Delta^m(x_k^s - x_k^t)|^{p_k}\Big\}^{\frac{1}{M}}\Big\} < \epsilon,$$

for all $s \geq n_0$.

$$\implies \Big\{\sum_{i=1}^{m}|\lambda_i(x_i^s - x_i)| + \sup_n\Big\{\sum_{k=1}^{\infty} a_{nk}|\lambda_k\Delta^m(x_k^s - x_k)|^{p_k}\Big\}^{\frac{1}{M}}\Big\} < \epsilon,$$

for all $s \geq n_0$.

$\implies g(x^s - x) < \epsilon, \qquad$ for each $\quad s \geq n_0$.

$\implies (x^s - x) \in w_0[A, \Delta^m, \Lambda, p]$.

Since $w_0[A, \Delta^m, \Lambda, p]$ is a linear space, so we have

$$x = x^s - (x^s - x) \in w_0[A, \Delta^m, \Lambda, p].$$

This complete the proof. □

THEOREM 3. *Let $A = (a_{nk})$ be a non-negative regular matrix, $0 < p_k \leq q_k$ and $(\frac{q_k}{p_k})$ be bounded. Then $w_1[A, \Delta^m, \Lambda, q] \subseteq w_1[A, \Delta^m, \Lambda, p]$.*

Proof. Let $x \in w_1[A, \Delta^m, \Lambda, q]$. Define

$$u_k = \begin{cases} y_k, & \text{for } y_k \geq 1, \\ 0, & \text{for } y_k < 1 \end{cases}$$

and

$$v_k = \begin{cases} 0, & \text{for } y_k \geq 1 \\ y_k, & \text{for } y_k < 1, \end{cases}$$

where $y_k = |\lambda_k \Delta^m x_k - L|^{p_k}$.

Therefore $y_k = u_k + v_k$ and $y_k^{t_k} = u_k^{t_k} + v_k^{t_k}$, where $t_k = \frac{q_k}{p_k}$.

Now it follows that $u_k^{t_k} \leq u_k \leq y_k$ and $v_k^{t_k} \leq v_k^{\zeta}$ for $0 < \zeta \leq t_k \leq 1$.

Following Maddox [10], we have the following inequality

$$\sum_{k=1}^{\infty} a_{nk} y_k^{t_k} \leq \sum_{k=1}^{\infty} a_{nk} u_k + \left(\sum_{k=1}^{\infty} a_{nk} v_k \right)^{\zeta} \|A\|^{1-\zeta}.$$

Hence we have $x \in w_1[A, \Delta^m, \Lambda, q]$. □

THEOREM 4. *Let A be a non-negative regular matrix and $p = (p_k)$ be such that $0 < h = \inf p_k \leq p_k \leq H = \sup p_k$. Then*

$$X(\Delta^m, \Lambda) \subset w_\infty[A, \Delta^m, \Lambda, p], \qquad \textit{for} \quad X = \ell_\infty, c, c_0,$$

where $X(\Delta^m, \Lambda) = \{x = (x_k) : \ (\lambda_k \Delta^m x_k) \in X\}$.

Proof. Let $x \in \ell_\infty(\Delta^m, \Lambda)$. Then there exists $K > 0$, such that $|\lambda_k \Delta^m x_k| \leq K$, for all $k \in \mathbb{N}$. We have

$$\sum_{k=1}^{\infty} a_{nk} |\lambda_k \Delta^m x_k|^{p_k} \leq \max\{K^h, K^H\} \sum_{k=1}^{\infty} a_{nk} < \infty.$$

Hence $\ell_\infty(\Delta^m, \Lambda) \subset w_\infty[A, \Delta^m, \Lambda, p]$. The other cases can be established similarly. □

THEOREM 5.

(i) *Let $0 < \inf p_k \leq p_k \leq 1$. Then $w_1[A, \Delta^m, \Lambda, p] \subset w_1[A, \Delta^m, \Lambda]$.*

(ii) *Let $1 \leq p_k \leq \sup p_k < \infty$. Then $w_1[A, \Delta^m, \Lambda] \subset w_1[A, \Delta^m, \Lambda, p]$.*

(iii) *Let $m_1 \leq m_2$. Then $w_1[A, \Delta^{m_2}, \Lambda, p] \subset w_1[A, \Delta^{m_1}, \Lambda, p]$.*

Proof.

(i) Let $0 < \inf p_k \leq p_k \leq 1$ and $(x_k) \in w_1[A, \Delta^m, \Lambda, p]$. Then there exists L such that

$$\sup_n \sum_{k=1}^{\infty} a_{nk} |\lambda_k \Delta^m x_k - L| \leq \sup_n \sum_{k=1}^{\infty} a_{nk} |\lambda_k \Delta^m x_k - L|^{p_k}.$$

Hence $(x_k) \in w_1[A, \Delta^{m_1}, \Lambda]$.

(ii) Let $1 \leq p_k \leq \sup p_k < \infty$, for all $k \in \mathbb{N}$ and $(x_k) \in w_1[A, \Delta^m, \Lambda]$. Then for each $0 < \epsilon < 1$, there exists a positive integer J such that $\sum\limits_{k=1}^{\infty} a_{nk}|\lambda_k \Delta^m x_k - L| < \epsilon < 1$, for all $n > J$. This implies that

$$\sum_{k=1}^{\infty} a_{nk}|\lambda_k \Delta^m x_k - L|^{p_k} \leq \sum_{k=1}^{\infty} a_{nk}|\lambda_k \Delta^m x_k - L| < \epsilon, \qquad \text{for all} \quad n > J$$

Hence $(x_k) \in w_1[A, \Delta^{m_1}, \Lambda, p]$.

(iii) The Proof is a routine verification. □

4. Statistical convergence

A complex number sequence $x = (x_k)$ is said to be statistically convergent to the number L if for every $\epsilon > 0$,

$$\lim_{n\to\infty} \frac{1}{n}\big|\{k \leq n : \ |x_k - L| \geq \epsilon\}\big| = 0,$$

where the vertical bars indicate the number of elements in the enclosed set. In this case we write stat-lim $x_k = L$.

The idea of statistical convergence for sequence of real numbers was studied by Fast [4] and Schoenberg [16] at the initial stage. Later on it was studied from sequence space point of view and linked with summability methods by Šalát [15], Fridy [5], Fridy and Orhan [6], Tripathy [18] and many others.

A complex number sequence $x = (x_k)$ is said to be Δ^m_Λ*-statistically convergent* to the number L if for every $\epsilon > 0$, and fixed $m \in \mathbb{N}$,

$$\lim_{n\to\infty} \frac{1}{n}\big|\{k \leq n : \ |\lambda_k \Delta^m x_k - L| \geq \epsilon\}\big| = 0,$$

in this case we write Δ^m_Λ-stat-lim $x_k = L$ and by $S(\Delta^m_\Lambda)$ we denote the class of all Δ^m_Λ-statistically convergent sequences.

When $m = 0$ and $\Lambda = e$, the space $S(\Delta^m_\Lambda)$ represents the ordinary statistical convergence.

When $\Lambda = e$, the space $S(\Delta^m_\Lambda)$, becomes the generalized difference statistically convergent sequence space defined and studied by Et and Nuray [3].

Now, we shall give some inclusion relations between $w_1[\Delta^m, \Lambda, p]$-convergence and Δ^m_Λ-statistical convergence.

Theorem 6.

(i) $x_k \to L(w_1[\Delta^m, \Lambda, p])$, $0 < p < \infty$ *implies* $x_k \to L[S(\Delta^m_\Lambda)]$.

(ii) $(x_k) \in \ell_\infty(\Delta^m, \Lambda)$ *and* $x_k \to L[S(\Delta^m_\Lambda)]$ *imply* $x_k \to L(w_1[\Delta^m, \Lambda, p])$, $0 < p < \infty$.

(iii) $S(\Delta^m_\Lambda) \cap \ell_\infty(\Delta^m, \Lambda) = w_1[\Delta^m, \Lambda, p] \cap \ell_\infty(\Delta^m, \Lambda)$.

Proof.

(i) Let $x_k \to L(w_1[\Delta^m, \Lambda, p])$, $0 < p < \infty$ and $\epsilon > 0$ be given, we can write

$$\frac{1}{n}\sum_{k=1}^{n} |\lambda_k \Delta^m x_k - L|^p = \frac{1}{n} \sum_{\substack{k \le n, \\ |\lambda_k \Delta^m x_k - L| \ge \epsilon}} |\lambda_k \Delta^m x_k - L|^p + \frac{1}{n} \sum_{\substack{k < n, \\ |\lambda_k \Delta^m x_k \quad L| \quad \epsilon}} |\lambda_k \Delta^m x_k - L|^p$$

$$\ge \frac{1}{n} \left|\left\{k \le n : \ |\lambda_k \Delta^m x_k - L| \ge \epsilon\right\}\right| \epsilon^p.$$

Hence $x_k \to L[S(\Delta^m_\Lambda)]$.

(ii) Suppose that $(x_k) \in \ell_\infty(\Delta^m, \Lambda)$ and $(x_k) \in [S(\Delta^m_\Lambda)]$. Let $B = |\lambda_k \Delta^m x_k + |L|$ and $\epsilon > 0$ be given, let $n_0(\epsilon)$ be such that

$$\frac{1}{n}\left|\left\{k \le n : \ |\lambda_k \Delta^m x_k - L| \ge (\tfrac{\epsilon}{2})^{\frac{1}{p}}\right\}\right| < \tfrac{\epsilon}{2B^p},$$

for all $n > n_0(\epsilon)$, let $L_n = \left\{k \le n : \ |\lambda_k \Delta^m x_k - L| \ge \left(\frac{\epsilon}{2}\right)^{\frac{1}{p}}\right\}$. Now for all $n > n_0(\epsilon)$, we have

$$\frac{1}{n}\sum_{k=1}^{n} |\lambda_k \Delta^m x_k - L|^p = \frac{1}{n} \sum_{k \in L_n} |\lambda_k \Delta^m x_k - L|^p + \frac{1}{n} \sum_{k \notin L_n} |\lambda_k \Delta^m x_k - L\ ^p$$

$$< \frac{1}{n}\left(\frac{n\epsilon}{2B^p}\right) B^p + \frac{1}{n} n \frac{\epsilon}{2} = \epsilon.$$

Hence $x_k \to L(w_1[\Delta^m, \Lambda, p])$.

(iii) Follows from (i) and (ii). □

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REFERENCES

[1] ET, M.—COLAK, R.: *On generalized difference sequence spaces*, Soochow J. Math. **21** (1995), 377–386.

[2] ET, M.—ESI, A.: *On Köthe-Toeplıtz duals of generalized difference sequence spaces*, Bull. Malays. Math. Sci. Soc. (2) **23** (2000), 25 32.

[3] ET, M.—NURAY, F.: *Δ^m-Statistical convergence*, Indian J. Pure Appl. Math. **32** (2001), 961 969.

[4] FAST, H.: *Sur la convergence statistique*, Colloq. Math. **2** (1951), 241–244.

[5] FRIDY, J. A.: *On statistical convergence*, Analysis (Munich) **5** (1985), 301–313.

[6] FRIDY, J. A.—ORHAN, C.: *Lacunary statistical summability*, J. Math. Anal. Appl. **173** (1993), 497–504.

[7] GOES, G.—GOES, S.: *Sequences of bounded variation and sequences of Fourier coefficients*, Math. Z. **118** (1970), 93–102.

[8] KIZMAZ, H: *On certain sequence spaces*, Canad. Math. Bull. **24** (1981), 169–176.

[9] LASCARIDES, C. G.: *A study of certain sequence spaces of Maddox and generalization of a theorem of Iyer*, Pacific J. Math. **38** (1971), 487–500.

[10] MADDOX, I. J.: *Spaces of strongly summable sequences*, Quart. J. Math. Oxford Ser. (2) **18** (1967), 345 355.

[11] MADDOX, I. J.: *Elements of Functional Analysis*, Cambridge University Press, Cambridge, 1970.

[12] NAKANO, H.: *Modular sequence spaces*, Proc. Japan Acad. Ser. A Math. Sci. **27** (1951), 508 512.

[13] NANDA, S.: *Strongly almost summable and strongly almost convergent sequences*, Acta Math. Hungar. **49** (1987), 71–76.

[14] RATH, D.—TRIPATHY, B. C.: *Matrix maps on sequence spaces associated with sets of integers*, Indian J. Pure Appl. Math. **27** 1996), 197–206.

[15] ŠALÁT, T.: *On statistically convergent sequences of real numbers*, Math. Slovaca **30** (1980), 139 150.

[16] SCHOENBERG, I. J.: *The integrability of certain functions and related summability methods*, Amer. Math. Monthly **66** (1959), 361–375.

[17] SIMONS, S.: *The spaces $\ell(p_\nu)$ and $m(p_\nu)$*, Proc. London Math. Soc. **15** (1965), 422 436.

[18] TRIPATHY, B. C.: *Matrix trasformation between some classes of sequences*, J. Math. Anal. Appl. **206** (1997), 448–450.

[19] TRIPATHY, B. C.: *On a class of difference sequences related to the p-normed space ℓ^p*, Demonstratio Math. **36** (2003), 867–872.

[20] TRIPATHY, B. C.: *Generalized difference paranormed statistically convergent sequences defined by Orlicz functions in a locally convex space*, Soochow J. Math. **30** (2004), 431 446.

[21] TRIPATHY, B. C.—SEN, M.: *Vector valued paranormed bounded and null sequence spaces associated with multiplier sequences*, Soochow J. Math. **29** (2003), 313 326.

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