Book Reviews

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Dvurečenskij, A. — Pulmannová, S.:
NEW TRENDS IN QUANTUM STRUCTURES.
Kluwer Academic Publishers, Dordrecht; Ister Science, Bratislava 2000, xvi + 541 pp.
ISBN 0-7923-6471-6; ISBN 80-88683-23-8

The present book is devoted to the latest trends and results concerning different types of quantum structures, that means an important interdisciplinary area which joins mathematics, logic and fuzzy reasonings with mathematical foundations of quantum mechanics. Moreover, it shows various possibilities and directions of their applications.

As is known, G. Birkhoff and J. von Neumann in 1936 showed that the set of assertions of quantum mechanics (in contrast to the classical case) has algebraic properties different from Boolean algebra. An intensive study of algebras of quantum theories began mainly in the sixties and there are numerous monographs reflecting their development.

Recently, in the nineties, new important types of quantum structures, such as difference posets (D-posets), weak orthoalgebras and effect algebras (which all also include, among others, the space of effect operators of Hilbert spaces and MV-algebras), have been introduced and turbulently developed. The authors of the book and their school have played an outstanding role in this development. The methods of the research which are systematically respected and used in the book present a very effective and strong combination of algebraic and fuzzy set approaches and ideas.

The text of the book is divided into seven chapters.

In the *first Chapter*, D-posets and effect algebras as partial algebraic structures are introduced and studied, a categorical equivalence between the both classes is described and relations of these structures with partially ordered Abelian groups are shown. In particular, some important basic classes of effect algebras (orthoalgebras, orthomodular posets and lattices) are investigated and the notions of compatibility, observables and commeasurability are introduced.

The *second Chapter* contains some basic facts on MV-algebras and introduces QMV-algebras which are a common quantum abstraction of MV-algebras and orthomodular lattices and also include the standard Hilbert space effect algebras. A hierarchy of ideals and congruences is described there.

In the *third Chapter*, partial Abelian monoids as a generalization of effect algebras are introduced and their congruences and ideals are studied. An application to dimension theory is given and a connection with the Grothendick groups of AF C^*-algebras is shown.

The *fourth Chapter* is devoted to tensor products of D-posets and effect algebras. Relations between D-test spaces and tensor products of D-posets are described and partition logics and their connections with automata are studied.

In the fifth Chapter, the authors deal with BCK-algebras. They describe properties of important commutative BCK-algebras with the relative cancellation property and present a categorical equivalence between commutative BCK-algebras and a category of Abelian ℓ-groups which generalizes Mundici's equivalence between MV-algebras and unital Abelian ℓ-groups.

The *sixth Chapter* is devoted to applications of BCK-algebras. Semisimple and simple BCK-algebras are presented and measures on them are described and Dedekind complete BCK-algebras are studied. Pseudo MV-algebras (a non-commutative generalization of MV-algebras) are presented.

The *seventh Chapter* deals with Loomis-Sikorski type representation theorems for σ-complete MV-algebras and BCK-algebras. Weakly divisible MV-algebras are characterized by the set of continuous fuzzy sets on compact Hausdorff spaces and, furthermore, MV-observables are characterized.

All structures studied in the book are illustrated by means of numerous appropriate examples and exercises, which, moreover, often lead to important applications.

The book successfully gathers together results in theories that have been recently very intensively developed and applied by many specialists, among them the authors have a significant position. It is addressed to mathematicians, physicists, logicians, philosophers as well as to specialists in quantum computing and in fuzzy sets as a welcome addition to the literature on quantum structures.

Jiří Rachůnek, Olomouc