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WEAK APPROXIMATION BY POSITIVE MAPS ON C*-ALGEBRAS

B. V. LIMAYE—M. N. N. NAMBOODIRI

1. Introduction

Let A and B denote C*-algebras with identities 1_A and 1_B respectively. A *linear map $\phi: A \to B$ is said to be positive if for every $a \in A$, there is some $b \in B$ such that $\phi(a^*a) = b^*b$. For a_1 and a_2 in A, we write $a_1 \leq a_2$ if there is some $a \in A$ such that $a_2 - a_1 = a^*a$. Let

$$\mathbf{P}(A, B)_1 = \{\phi: A \to B: \phi \text{ positive}, \phi(1_A) \leq 1_B\} .$$

If $\phi \in \mathbf{P}(A, B)$ in fact satisfies

$$\phi(a)^*\phi(a) \leq \phi(a^*a)$$

for all $a \in A$, then ϕ is called a *Schwarz map.* A J*-subalgebra (resp., C*--subalgebra) of A is a *subspace A that is closed under the Jordan product $a_1 \circ a_2 = (a_1a_2 + a_2a_1)/2$ (resp., the usual product a_1a_2).

The main Korovkin-type result for weak convergence (which we denote by $\overset{w}{\to}$) given in [8], Theorem 2 can be improved by a minor modification of its proof as follows:

Theorem. *Let $\phi_0, \phi_1, \phi_2, \ldots$ be a sequence in $\mathbf{P}(A, B)_1$. Then*

$$C = \{a \in A: \phi_n(a) \overset{w}{\to} \phi_0(a), \phi_n(a^* \circ a) \overset{w}{\to} \phi_0(a^* \circ a) = \phi_0(a)^* \circ \phi_0(a)\}$$

is a J-subalgebra of A. If each ϕ_n is a Schwarz map, then C is, in fact, a C*-subalgebra of A.*

It is of interest to know when C actually equals A, so that the approximation method (ϕ_n) would work on the entire algebra A. This question is closely related to the uniqueness of the extension of $\phi_0|_C$ to A as a positive map. We give sufficient conditions for this to happen in terms of extreme points of $\mathbf{P}(A, B)_1$ (Theorems 2.2 and 2.5).

As a particular case we consider $A = C(X)$, the set of all complex-valued continuous functions on a compact Hausdorff space X and $B=\mathcal{M}_k$, the set of all $k\times k$ matrices with complex entries (Corollary 2.3). The special case $k=1$ gives the well-known Korovkin-type result for positive functionals on $C(X)$. Also, by taking $A=\beta(H)$, the set of all bounded operators on a complex Hilbert space H, and $B=\mathbb{C}$, the set of all complex numbers, we improve a previous result of the authors about the approximations of a simple eigenvalue of a normal operator on H. It would be interesting to obtain Korovkin-type results for the case $A=\beta(H)$ and $B=\mathcal{M}_k$.

2. Korovkin-type results for weak convergence

If J is a J*-subalgebra of A, then a *C*-homomorphism* $\phi_0: J\to B$ is a *linear map satisfying

$$\phi_0(a^2)=\phi_0(a)^2$$

for all $a\in J$. Clearly, a C*-homomorphism ϕ_0 on A is positive and satisfies $\phi_0(1_A)\leqslant 1_B$, i.e., it belongs to $\mathbf{P}(A,B)_1$.

If $\phi\in\mathbf{P}(A,B)_1$, Kadison has proved in [5] that

$$\phi(a)^2\leqslant\phi(a^2)$$

for all $a\in A$ with $a^*=a$. We begin with a lemma on extreme points of $\mathbf{P}(A,B)_1$.

Lemma 2.1. *Let J be a J^*-subalgebra of A, and $\phi_0: J\to B$ be a C*-homomorphism. Let*

$$Q_0=\{\phi\in\mathbf{P}(A,B)_1:\phi|_J=\phi_0\}\ .$$

Then Q_0 is a convex extremal subset (i.e., a face) of $\mathbf{P}(A,B)_1$, so that the extreme points of Q_0 are precisely those extreme points of $\mathbf{P}(A,B)_1$ which lie in Q_0.

Proof. The set Q_0 is clearly convex. Let $\phi_1,\phi_2\in P(A,B)_1$, and $\phi=(\phi_1+\phi_2)/2$ belong to Q_0. We show that $\phi_1,\phi_2\in Q_0$.

Let $a\in J$ with $a^*=a$. By Kadison's inequality, we have

$$\phi_1(a)^2\leqslant\phi_1(a^2)\ \text{ and }\ \phi_2(a)^2\leqslant\phi_2(a^2)\ .$$

Since J is a J*-subalgebra, we see that $a^2\in J$. Also, $\phi|_J=\phi_0$, which is a C*-homomorphism. Hence

$$\phi(a^2)=\phi_0(a^2)=\phi_0(a)^2=\phi(a)^2\ .$$

Now,

$$[\phi_1(a)-\phi_2(a)]^2=\phi_1(a)^2+\phi_2(a)^2-2\phi_1(a)\circ\phi_2(a)=$$
$$=2\phi_1(a)^2+2\phi_2(a)^2-[\phi_1(a)^2+\phi_2(a)^2+2\phi_1(a)\circ\phi_2(a)]\leqslant$$

$$\leq 4\left[\frac{\phi_1(a^2)+\phi_2(a^2)}{2}\right]-4\left[\frac{\phi_1(a)+\phi_2(a)}{2}\right]^2=4[\phi(a)^2-\phi(a)^2]=0\ .$$

Thus, $\phi_1(a)=\phi_2(a)$ for all $a\in J$ with $a^*=a$. Since J is *-closed, we have $\phi_1|_J=\phi_2|_J$. Hence $\phi|_J=\phi_1|_J=\phi_0|_J$, i.e., $\phi\in Q_0$. We have thus shown that the set Q_0 is extremal. The final statement about extreme points now follows immediately.

Theorem 2.2. *Let ϕ_0: $A\to\beta(H)$ be a C^*-homomorphism, and let (ϕ_n), $n=1, 2, \ldots$, be a sequence in $\mathbf{P}(A, \beta(H))_1$.*
Assume that if ϕ is an extreme point of $\mathbf{P}(A, \beta(H))_1$ and $\phi\neq\phi_0$, then there is some $a_0\in A$ such that $\phi(a_0)\neq\phi_0(a_0)$,

$$\phi_n(a_0)\overset{w}{\to}\phi_0(a_0)\ \text{and}\ \phi_n(a_0^*\circ a_0)\overset{w}{\to}\phi_0(a_0^*\circ a_0)\ .$$

Then $\phi_n(a)\overset{w}{\to}\phi_0(a)$ for all $a\in A$.

Proof. Let

$$C=\{a\in A\colon \phi_n(a)\overset{w}{\to}\phi_0(a),\ \phi_n(a^*\circ a)\overset{w}{\to}\phi_0(a^*\circ a)\}\ .$$

By the theorem quoted in the Introduction (Cf. Theorem 2 of [8]), C is a J*-subalgebra of A. We claim that $\phi_0|_C$ extends to a unique element of $\mathbf{P}(A, \beta(H))_1$, namely ϕ_0 itself.

Let

$$Q_0=\{\phi\in\mathbf{P}(A, \beta(H))_1\colon \phi|_C=\phi_0|_C\}\ .$$

Since $\mathbf{P}(A, \beta(H))_1$ is compact in the weak operator topology (p. 974 of [4]), we see that the closed convex subset Q_0 is also compact. If Q_0 contains more than one element, then by the Krein—Millman theorem it must contain an extreme point ϕ which does not equal ϕ_0. However, by Lemma 1.1 ϕ is an extreme point of $\mathbf{P}(A, \beta(H))_1$, and by our hypothesis there is a_0 in C such that $\phi(a_0)\neq\phi_0(a_0)$. But $\phi\in Q_0$, so that $\phi|_C=\phi_0|_C$. This contradiction shows that Q_0 is a singleton set, and our claim is justified.

Now, let ψ be any cluster point of the sequence (ϕ_n) in $\mathbf{P}(A, \beta(H))_1$, and let (ϕ_α) be a subnet of (ϕ_n) which converges to ψ. Since $\lim\phi_n(a)$ exists for all $a\in C$, we have

$$\psi(a)=\lim\phi_\alpha(a)=\lim\phi_n(a)=\phi_0(a)\ ,$$

i.e., $\psi|_C=\phi_0|_C$. But $\phi_0|_C$ extends to a unique element of $\mathbf{P}(A, \beta(H))_1$ so that $\psi=\phi_0$. Thus, every cluster point of (ϕ_n) in $\mathbf{P}(A, \beta(H))_1$ concides with ϕ_0. This shows that $\phi_n\to\phi_0$ in the weak operator topology, or $\phi_n(a)\overset{w}{\to}\phi_0(a)$ for all $a\in A$.

The usefulness of the above result depends on the specific knowledge of the extreme points of $\mathbf{P}(A, \beta(H))_1$. We now consider such a situation.

Corollary 2.3. *Let $x_1, \ldots, x_m$ be distinct points in a compact Hausdorff space X and let $P_1, \ldots, P_m$ be mutually orthogonal non-zero self-adjoint projections in $\mathcal{M}_k$, the set of all $k \times k$ complex matrices. For $f \in C(X)$, let*

$$\phi_0(t) = f(x_1)P_1 + \ldots + f(x_m)P_m .$$

For a sequence (ϕ_n) in $\mathbf{P}(C(X), \mathcal{M}_k)_1$, let

$$C = \{f \in C(X) \colon \varphi_n(f) \to \phi_0(f),\ \phi_n(|f|^2) \to \phi_0(|f|^2)\} .$$

If C contains the constant function 1 and separates each x_j $(1 \leq j \leq m)$ from every other point of X, then $\phi_n(f) \to \phi_0(f)$ for all $f \in C(X)$.

Note. Since $\mathcal{M}_k$ is finite dimensional, the weak convergence $(\xrightarrow{w})$ is equivalent to the norm convergence $(\to)$.

Proof. Let $A = C(X)$ and $H = \mathbb{C}^k$ so that $\beta(H) = \mathcal{M}_k$. It is clear that ϕ_0 is *linear, and for all $f \in C(X)$,

$$\begin{aligned}\phi_0(f^2) &= f^2(x_1)P_1 + \ldots + f^2(x_m)P_m \\ &= [f(x_1)P_1 + \ldots + f(x_m)P_m]^2 \\ &= [\phi_0(f)]^2 ,\end{aligned}$$

since $P_j^* = P_j = P_j^2$ and $P_iP_j = 0$ for $i \neq j$, $1 \leq i, j \leq m$. Thus, ϕ_0 is a C*-homomorphism.

Let ϕ be an extreme point of $\mathbf{P}(C(X), \mathcal{M}_k)_1$ such that $\phi|_C = \phi_0|_C$. In order to apply Theorem 2.2, we show that $\phi = \phi_0$.

Let $\phi_0(1) = P_0$. Since $1 \in C$, we see that ϕ is an extreme point of

$$\{\psi \colon C(X) \to \mathcal{M}_k \colon \psi \text{ positive and } \psi(1) = P_0\} .$$

Now, the algebra $C(X)$ is commutative and hence every positive map on $C(X)$ is completely positive ([10], 3.9 of Ch. IV). It then follows by Theorem 1.4.10 of [1] that

$$\phi(f) = f(y_1)Q_1 + \ldots + f(y_p)Q_p ,$$

for all $f \in C(X)$, where $y_1, \ldots, y_p$ are distinct points of X and $Q_1, \ldots, Q_p$ are positive matrices in $\mathcal{M}_k$ satisfying $Q_1 + \ldots + Q_p = P_0$.

Since C separates each x_j from every other point of X, and since C is an algebra containing 1, it follows that there are $f_1, \ldots, f_m$ in C with

$$f_j(x_j) = 1,\ f_j(x_i) = 0 \text{ for } i \neq j,\ 1 \leq i, j \leq m .$$

First we show that each $x_j \in \{y_1, \ldots, y_p\}$. For otherwise, we can find $f_0 \in C$ such that $f_0(x_j)=1$ and $f_0(y_i)=0$ for all $1 \leqslant i \leqslant p$. Then $f_0 f_j \in C$ and

$$\phi_0(f_0 f_j) = P_j = 0 = \phi(f_0 f_j) ,$$

which is a contradiction to $\phi|_C = \phi_0|_C$. Thus, each x_j equals some y_i. Hence $m \leqslant p$. By renumbering the y_i's and the corresponding Q_i's, we may assume that $y_1 = x_1, \ldots, y_m = x_m$. Then for all $f \in C(X)$,

$$\phi(f) = f(x_1)Q_1 + \ldots + f(x_m)Q_m + f(y_{m+1})Q_{m+1} + \ldots + f(y_p)Q_p .$$

Were $p > m$, then we could find $g_0 \in C$ such that $g_0(x_j)=0$ for all $1 \leqslant j \leqslant m$ and $g_0(y_i)=1$ for all $m+1 \leqslant i \leqslant p$. Then

$$Q_{m+1} + \ldots + Q_p = \phi(g_0) = \phi_0(g_0) = 0 .$$

Since $Q_i \geqslant 0$, we see thatk$Q_{m+1} = \ldots = Q_p = 0$. Thus, for all $f \in C(X)$,

$$\phi(f) = f(x_1)Q_1 + \ldots + f(x_m)Q_m .$$

But for $1 \leqslant j \leqslant m$,

$$Q_j = \phi(f_j) = \phi_0(f_j) = P_j .$$

Hence $\phi = \phi_0$. Now Theorem 2.2 applies and we obtain the desired result.

Remark 2.4. Often one can choose a finite number of functions $f_1, \ldots, f_p$ in $C(X)$ which separate any two distinct points of X. Also, we can easily see, as in Corollary 4 of [8], that the conditions $\phi_n(f_j) \to \phi_0(f)$ for $j = 1, \ldots, p$ and $\phi_n\left(\sum_{j=1}^{p} |f_j|^2\right) \to \phi_0\left(\sum_{j=1}^{p} |f_j|^2\right)$ imply $\phi_n(|f_j|^2) \to \phi_0(|f_j|^2)$ for each j. Then, the result in Theorem 2.3 says that $\phi_n(f) \to \phi_0(f)$ for all $f \in C(X)$, provided

$$\phi_n(1) \to \phi_0(1) ,$$
$$\phi_n(f_j) \to \phi_0(f_j), \quad j = 1, \ldots, p, \quad \text{and}$$
$$\phi_n\left(\sum_{j=1}^{p} |f_j|^2\right) \to \phi_0\left(\sum_{j=1}^{p} |f_j|^2\right) .$$

For example, if X is a compact subset of the Euclidean space $\mathbf{R}^p$, then we can take f_j to be the jth co-ordinate function, $j = 1, \ldots, p$. If X denotes the p-dimensional torus $\{(e^{i\theta_1}, \ldots, e^{i\theta_p}) : 0 \leqslant \theta_j \leqslant 2\pi,\ j = 1, \ldots, p\}$, then we can let $f_j((e^{i\theta_1}, \ldots, e^{i\theta_p})) = e^{i\theta_j}$. Since in this case, $|f_j|^2 = 1$ for $1 \leqslant j \leqslant p$, we need the convergence of (ϕ_n) only on $1, f_1, \ldots, f_p$. These results generalize earlier results proved for the case $\mathcal{M}_1 = \mathbb{C}$, i.e., for positive functionals on $C(X)$. (See Corollaries 2.5 and 2.6 of [9].)

When the map ϕ_0 that is being approximated is not a C*-homomorphism, the following version of Theorem 2.2 is useful.

Theorem 2.5. *Let $\phi_0, \phi_1, \phi_2, \ldots$ be a sequence in $\mathbf{P}(A, \beta(H))_1$ (resp., a sequence of Schwarz maps from A to $\beta(H)$), and let $E \subset A$ be such that for every $a \in E$,*

$$\phi_n(a) \xrightarrow{w} \phi_0(a) \text{ and } \phi_n(a^* \circ a) \rightarrow \phi_0(a^* \circ a) = \phi_0(a)^* \circ \phi_0(a) .$$

Assume that if ϕ *is an extreme point of* $\mathbf{P}(A, \beta(H))_1$ *a* $\phi \neq \phi_0$*, then there is* a_0 *in the J*-subalgebra (resp., the C*-subalgebra) generated by* E *in* A *such that* $\phi(a_0) \neq \phi_0(a_0)$.

Then $\phi_n(a) \xrightarrow{w} \phi_0(a)$ *for all* $a \in A$.

Proof. By the theorem quoted in the Introduction, the set

$$\{a \in A: \phi_n(a) \xrightarrow{w} \phi_0(a), \phi_n(a^* \circ a) \xrightarrow{w} \phi_0(a^* \circ a) = \phi_0(a)^* \circ \phi_0(a)\}$$

is a J*-subalgebra and it contains E. Hence it contains the J*-subalgebra J_E generated by E in A. Thus, for every $a_0 \in J_E$, we have

$$\phi_n(a_0) \xrightarrow{w} \phi_0(a_0), \phi_n(a_0^* \circ a_0) \xrightarrow{w} \phi_0(a_0^* \circ a_0) = \phi_0(a_0)^* \circ \phi_0(a_0) .$$

Then the proof of Theorem 2.2 holds verbatim if we replace C by J_E throughout. In case each ϕ_n in a Schwarz map, we merely have to replace C by the C*-subalgebra C_E generated by E in A.

Remark 2.6. If either A is commutative, or if $\beta(H)$ is commutative (i.e., H is of dimension 1), then every $\phi \in \mathbf{P}(A, \beta(H))_1$ is, in fact, a Schwarz map ([10], 3.5, 3.9 and 3.8 of Ch. IV). When $H = \mathbb{C}$, we obtain the following result from Theorem 2.5:

Let $\phi_0, \phi_1, \phi_2, \ldots$ be positive functionals on a C*-algebra A with $\phi_n(1_A) \leq 1$. Let $E \subset A$ be such that for very $a \in E$,

$$\lim \phi_n(a) = \phi_0(a)$$

and

$$\lim \phi_n(a^* \circ a) = \phi_0(a^* \circ a) = |\phi_0(a)|^2 .$$

If the C*-algebra C_E generated by E in A separates ϕ_0 from every other extreme point of $\mathbf{P}(A, \mathbb{C})_1$, then $\phi_n(a) \rightarrow \phi_0(a)$ for all $a \in A$.

This result improves upon Theorem 1.2 of [7] for a C*-algebra A with identity, because the earlier result assumed in addition that $\phi_0|_{C_E}$ was an extreme point of the set of all positive functionals of norm ≤ 1 on C_E, and it required

$$\lim \phi_n(a^*a) = \phi_0(a^*a) = |\phi_0(a)|^2 = \phi_0(aa^*) = \lim \phi_n(aa^*) .$$

Various concrete cases of this result about positive functionals are given in [7]. We choose to improve one of them.

Corollary 2.7. *Let* $T_0 \in \beta(H)$ *be normal and* λ_0 *be a simple eigenvalue of* T_0 *with* *a corresponding unit eigenvector x_0. Let (ϕ_n) be a sequence of positive functionals on $\beta(H)$ such that*

$$\phi_n(I) \to 1,$$
$$\phi_n(T_0) \to \lambda_0, \text{ and}$$
$$\phi_n(T_0^* T_0) \to |\lambda_0|^2.$$

Then $\phi_n(T) \to \langle Tx_0, x_0 \rangle$ for all $T \in \beta(H)$.

Proof. Let $A = \beta(H)$ and $\phi_0(T) = \langle Tx_0, x_0 \rangle$ for $T \in \beta(H)$. On replacing ϕ_n by $\phi_n / \phi_n(I)$, we can assume without loss of generality that $\phi_n(I) = 1$. Let $E = \{1, T_0\}$. Since $T_0^* T_0 = T_0 T_0^*$, we see that $\phi_n(T_0^* \circ T_0) \to \phi_0(T_0^* \circ T_0) = \|T_0 x_0\|^2 = |\lambda_0|^2 = |\phi_0(T_0)|^2$.

Let $\sigma(T_0)$ denote the spectrum of the normal operator T_0, and μ_0 denote the corresponding spectral measure. If f_0 is the characteristic function of the set $\{\lambda_0\}$, then f_0 is continuous on $\sigma(T_0)$, since λ_0 is an isolated point of $\sigma(T_0)$. Hence f_0 is a uniform limit of polynomials in z and $\bar{z}$ on $\sigma(T_0)$. The spectral mapping theorem shows, in turn, that

$$f_0(T_0) = \int_{\sigma(T_0)} f_0(z) \, d\mu_0(z) = \mu_0(\{\lambda_0\})$$

is a limit in $\beta(H)$ of polynomials in T and T^*. Thus, $f_0(T_0) \in C_E$, the C*-subalgebra generated by E in $\beta(H)$. But $f_0(T_0)$ is an orthogonal projection and its range is the eigenspace corresponding to λ_0, which is one dimensional. Thus, $f_0(T_0)x = \langle x, x_0 \rangle x_0$ for all $x \in H$.

Let ϕ be an extreme point of $\mathbf{P}(\beta(H), \mathbb{C})_1$ and $\phi \neq \phi_0$. Then by Theorem 2.5.2 of [5], either $\phi(T) = 0$ for all compact $T \in \beta(H)$, or $\phi(T) = \langle Tx_1, x_1 \rangle$ for some $x_1 \in H$ with $\|x_1\| = 1$ and all $T \in \beta(H)$. In the former case, $\phi(f_0(T_0)) = 0$ since $f_0(T_0)$ is compact, while $\phi_0(f_0(T_0)) = \langle f_0(T_0)x_0, x_0 \rangle = \langle x_0, x_0 \rangle = 1 \neq 0$. In the latter case, $\phi(f_0(T_0)) = \phi_0(f_0(T_0))$ implies $\langle f_0(T_0)x_1, x_1 \rangle = 1$ so that x_1 is in the range of the projection $f_0(T_0)$, i.e., x_1 and x_0 are scalar multiples of each other. But then $\phi = \phi_0$, which is not the case. Thus, we see that the element $f_0(T_0)$ in C_E separates ϕ from ϕ_0. By the result in Remark 2.6, we see that $\phi_n(T) \to \phi_0(T) = \langle Tx_0, x_0 \rangle$ for all $T \in \beta(H)$.

Remark 2.8. The above result is better than Corollary 3.2 of [7] since the earlier result required in addition that the operator T_0 be compact and that λ_0 satisfy $|\lambda_0| = \|T_0\|$.

In order to apply this result to specific situations, we must have examples of operators which have simple eigenvalues. In this connection the following results are known:

1. Let an $n \times n$ non-singular normal matrix A_0 be such that all its minors have non-negative determinants and the elements just above and justs below the

principal diagonal are non-zero. Then all the eigenvalues of A_0 are simple (Chapter II, Theorem 6 of Sec. 6 and Theorem 10 of Sec. 7 in [2]).

2. Let $k(s, t)$ be a continuous real-valued function for $(s, t) \in [a, b] \times [a, b]$. For $f \in L^2([a, b]) = H$, let

$$T_0(f)(s) = \int_a^b k(s, t) f(t)\, \mathrm{d}t, \quad s \in [a, b]$$

be a normal operator in $\beta(H)$. If $k(s, t) > 0$ for $a \leqslant s, t \leqslant b$, and if for $a < s_1 < \ldots < s_n < b$, $a < t_1 < \ldots < t_n < b$, the determinant of the matrix $(k(s_i, t_j))$ is non-singular, then all the eigenvalues of T_0 are simple (Chapter IV, Sec. 2, pp. 239 and 240 of [2]).

Addendum: Question similar to the ones consider in this note, but for completely positive linear maps on $\beta(H)$ are consider in the Weak Korovkin approximation by completely positive linear maps on $\beta(H)'$ by the authors. This paper is to appear in the Journal of Approximation Theory.

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СЛАБАЯ АППРОКСИМАЦИЯ ПОЛОЖИТЕЛЬНЫХ ОТОБАЖЕНИЙ C^*-АЛГЕБР

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Резюме

Пусть A — C*-алгебра с единицей 1_A и $\beta(H)$ — множество всех ограниченных операторов в пространстве Гильберта H. Пусть $\phi_n: A \to \beta(H)$, $n = 0, 1, 2, \ldots$, последовательность положительных отображений, для которых $\phi_n (1_A) \leq I$ и $\phi_n(a) \to \phi_0(a)$ слабо для a, принадлежащих некоторому подмножеству A. В терминах экстремальных точек положительных отображений приводятся достаточные условия для слабой сходимости $\phi_n(a) \to \phi_0(a)$ для всех $a \in A$.

Улучшается результат автора о приближении простого собственного значения нормального оператора.