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# A GEOMETRIC PROCEDURE FOR ROBUST DECOUPLING CONTROL OF CONTACT FORCES IN ROBOTIC MANIPULATION

PAOLO MERCORELLI AND DOMENICO PRATTICHIZZO

This paper deals with the problem of controlling contact forces in robotic manipulators with general kinematics. The main focus is on control of grasping contact forces exerted on the manipulated object. A visco–elastic model for contacts is adopted. The robustness of the decoupling controller with respect to the uncertainties affecting system parameters is investigated. Sufficient conditions for the invariance of decoupling action under perturbations on the contact stiffness and damping parameters are provided. These conditions are meaningful for several classes of manipulation systems with general kinematics.

*Keywords:* geometric approach, robustness, manipulators, elastic contacts
*AMS Subject Classification:* 93D09, 19L64, 70Q05, 14L24,

## 1. INTRODUCTION

This paper deals with the problem of controlling contact forces in robotic manipulators with general kinematics. Manipulation systems with general kinematics, are a class of mechanisms including non–conventional manipulators such as, for example, multiple fingers in a robot hand, multiple arms in cooperating tasks, robots interacting with the object by using their inner links and so on [11, 12]. Contact elasticity cannot be neglected in advanced robotics. This occurs in industrial applications whenever it is necessary to assemble and manipulate non–rigid (rubber or plastic) parts. In medical applications, like micro manipulation of tissues in surgery or in laparoscopy, it could be necessary to squeeze the tissue part of the patient's organ in order to exert a cutting action (see Figure 1).

Moreover, modelling contact elasticity is mandatory not only for deformable manipulated parts but also for soft fingertips as those in [1]. For these reasons a visco–elastic model for contacts is considered in this paper. The main focus is on the control of grasping (or squeezing) contact forces exerted on the object.

Actually, a simultaneous control of contact forces and object motions is needed during manipulation. Furthermore, in many advanced manipulation tasks, the decoupling control of contact forces with respect to the object motion is a basic requirement of the control design. As an example, consider the manipulation system of Figure 1 and model the compliant contacts through visco–elastic lumped parameters as depicted in Figure 2 a). One can image to have to grasp or to squeeze the tissue without moving it in order to have precise incision.

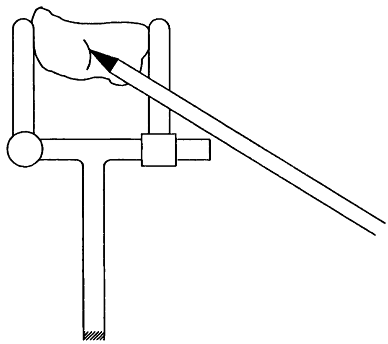

**Fig. 1.** Micro manipulation of internal tissues in surgery or laparoscopy.

Interests in control by using geometric approach are recently increased in theoretical aspects and applications as well, see for instance [3], [7], [10], [13]. In particular on problems like non-interaction and noise localization. A non–decoupling control policy, for instance a force step on the prismatic joint, squeezes the manipulated object but gives rise to an undesired and dangerous transient motion of the object. In [13] geometric control properties and structures are derived for mechanical systems in order to find noise localizing laws. In [6] the decoupling control law is derived through a geometric approach to the problem. The decoupling noninteracting controller is based on the knowledge of the manipulation system dynamics and the visco–elastic contact behaviour whose identification is a difficult task. Thus, a certain degree of uncertainty is always present in the parameters of the visco–elastic contact model. In this paper, robustness of the decoupling controller with respect to the uncertainties affecting system parameters is investigated. In particular, sufficient conditions are provided under which the decoupling control action is invariant with respect to the perturbations on the contact stiffness and damping parameters. These conditions are meaningful for several classes of manipulation systems with general kinematics.

## 2. NOTATION AND DYNAMIC MODEL

This section summarizes notation and some results on the analysis of dynamics for manipulation with general kinematics. The model of the general manipulation system we consider is comprised of a mechanism with an arbitrary number of actuated links and of an object which is in contact, at one or more points, with some of the links.

Let $\mathbf{q} \in I\!\!R^q$ be the vector of generalized coordinate, completely describing the configuration of the manipulation system, let $\tau \in I\!\!R^q$ be the vector of actuated (rotoidal) joint torques and (prismatic) joint forces. Moreover, let $\mathbf{u} \in I\!\!R^d$ be the vector of the generalized coordinates for the object ($d = 3$ for 2D cases while $d = 6$ for 3D cases) and let $\mathbf{w} \in \mathbb{R}^d$ be the vector of external disturbances acting on the object. To clarify the vector notation see Figure 3.

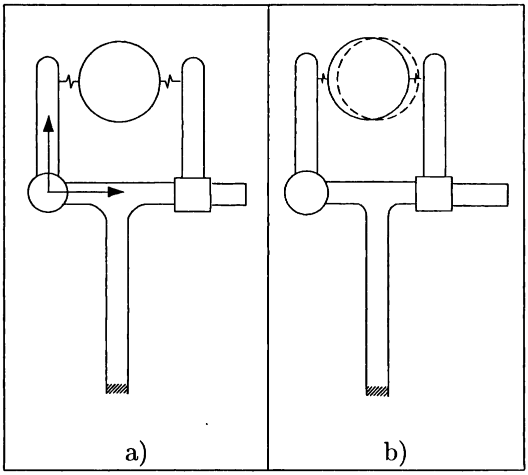


**Fig. 2.** Deformable contacts: squeezing action of the prismatic joint.

The lumped parameter visco–elastic model at the $i$th contact is simply described by introducing contact vectors $\mathbf{c}_i^m$ and $\mathbf{c}_i^o$. The $d$–dimensional contact vector $\mathbf{c}_i^m$ ($\mathbf{c}_i^o$) represents the coordinates of a fixed frame at the contact point thought on the link of the manipulator (on the object). The contact force $\mathbf{f}_i$ and the moment $\mathbf{m}_i$ exchanged at the contact are represented by the vector $\mathbf{t}_i = [\mathbf{f}_i^T, \mathbf{m}_i^T]$ which, according to the visco–elastic model, can be written as

$$\mathbf{t}_i = \mathbf{K}_i\mathbf{H}_i(\mathbf{c}_i^m - \mathbf{c}_i^o) + \mathbf{B}_i\mathbf{H}_i(\dot{\mathbf{c}}_i^m - \dot{\mathbf{c}}_i^o).$$

The parameters indicated with $\mathbf{K}_i$ and $\mathbf{B}_i$ are the stiffness and damping matrices, respectively and $\mathbf{H}_i$ is a constant selection matrix describing several types of contact models. To be more precise, in the three dimension space in presence of hard-contact, matrix $\mathbf{K}_i$ and $\mathbf{B}_i$ are matrix $3 \times 3$. If the contact is a soft one these matrices have dimension $4 \times 3$, see [8]. About matrix $\mathbf{H}_i$, in presence of hard-contact the dimension is $3 \times 6$ and if the contact is soft one the dimension is $4 \times 6$. Notice that the presence of moment $\mathbf{m}_i$ in $\mathbf{t}_i$ depends upon the contact interaction type.

Now let $\mathbf{t} = [\mathbf{f}_1, \cdots, \mathbf{f}_n, \mathbf{m}_1, \cdots, \mathbf{m}_n]$ be the overall contact force vector built by grouping all the vectors $\mathbf{t}_i$ for the $n$ contacts. Accordingly, vector $\mathbf{t}$ is given by $\mathbf{t} = \mathbf{KH}(\mathbf{c}^m - \mathbf{c}^o) + \mathbf{BH}(\dot{\mathbf{c}}^m - \dot{\mathbf{c}}^o)$. The Jacobian $\mathbf{J}$ and grasp matrix $\mathbf{G}$ are defined as usual as $\mathbf{J} = \mathbf{H}\frac{\delta \mathbf{c}^m}{\delta \mathbf{q}}$ and $\mathbf{G}^T = \mathbf{H}\frac{\delta \mathbf{c}^o}{\delta \mathbf{u}}$. Thus the local approximation of the contact force vector $\mathbf{t}$ can be written as

$$\delta\mathbf{t} = \mathbf{K}\left(\mathbf{J}\delta\mathbf{q} - \mathbf{G}^T\delta\mathbf{u}\right) + \mathbf{B}\left(\mathbf{J}\delta\dot{\mathbf{q}} - \mathbf{G}^T\delta\dot{\mathbf{u}}\right). \tag{1}$$

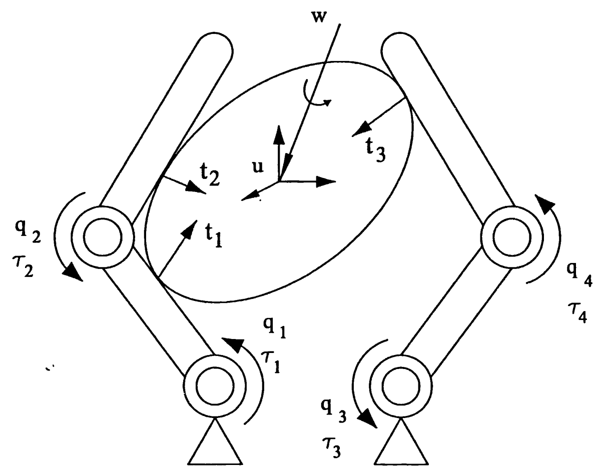


**Fig. 3.** Vector notation for general manipulation system analysis.

The dynamic model considered in the following is the linearization of the nonlinear dynamics of manipulation systems derived in [11, 12]. Consider a reference equilibrium configuration $(\mathbf{q}, \mathbf{u}, \dot{\mathbf{q}}, \dot{\mathbf{u}}, \tau, \mathbf{t}) = (\mathbf{q}_o, \mathbf{u}_o, \mathbf{0}, \mathbf{0}, \tau_o, \mathbf{t}_o)$, such that $\tau_o = \mathbf{J}^T \mathbf{t}_o$ and $\mathbf{w}_o = -\mathbf{G}\mathbf{t}_o$. In the neighborhood of such an equilibrium, linearized dynamics of the manipulation system can be written as

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}_\tau \tau' + \mathbf{B}_w \mathbf{w}', \tag{2}$$

where state, input and disturbance vectors are defined as the departures from the reference equilibrium configuration:

$$\begin{aligned} \mathbf{x} &= \left[(\mathbf{q} - \mathbf{q}_o)^T \ (\mathbf{u} - \mathbf{u}_o)^T \ \dot{\mathbf{q}}^T \ \dot{\mathbf{u}}^T\right]^T, \\ \tau' &= \tau - \mathbf{J}^T \mathbf{t}_o, \quad \mathbf{w}' = \mathbf{w} + \mathbf{G}\mathbf{t}_o \end{aligned}$$

and

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ \mathbf{L}_k & \mathbf{L}_b \end{bmatrix}; \quad \mathbf{B}_\tau = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{M}_h^{-1} \\ \mathbf{0} \end{bmatrix}; \quad \mathbf{B}_w = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{M}_o^{-1} \end{bmatrix},$$

where $\mathbf{M}_h$ and $\mathbf{M}_o$ are the inertia matrices of the manipulator and the object, respectively. To simplify notation we will henceforth omit the prime in $\tau'$ and $\mathbf{w}'$.

By neglecting terms due to gravity variations, rolling phenomena at the contacts and local variations of the Jacobian and grasp matrices and under the hypothesis that stiffness and damping matrices are proportional $\mathbf{B} \propto \mathbf{K}$ [9], simple expressions are obtained for $\mathbf{L}_k$ and $\mathbf{L}_b$

$$\mathbf{L}_k = -\mathbf{M}^{-1}\mathbf{P}_k; \qquad \mathbf{L}_b = -\mathbf{M}^{-1}\mathbf{P}_b$$

where

$$\mathbf{M} = \mathrm{diag}(\mathbf{M}_h, \mathbf{M}_o);$$
$$\mathbf{P}_k = \mathbf{S}^T\mathbf{K}\mathbf{S};$$
$$\mathbf{P}_b = \mathbf{S}^T\mathbf{B}\mathbf{S};$$
$$\mathbf{S} = [\mathbf{J} \ \ -\mathbf{G}^T].$$

The remainder of this section provides some results obtained in [11] on the control of internal forces, a problem of paramount importance in robotic manipulation.

Let us define $\mathbf{t}'$ as the first order approximation of departures of contact force vector $\mathbf{t}$ from the reference equilibrium $\mathbf{t}_o$. According to equation (1), $\mathbf{t}'$ (henceforth $\mathbf{t}$) can be regarded as an output of the linearized model (2), $\mathbf{t} = \mathbf{C}_t\mathbf{x}$ where

$$\mathbf{C}_t = \left[\mathbf{KJ} \ \ -\mathbf{KG}^T \ \mathbf{BJ} \ \ -\mathbf{BG}^T\right].$$

When manipulation systems with general kinematics are taken into account [4] not all the internal forces are controllable. In [6] the *reachable internal forces subspace* $\mathcal{R}_{ti,\tau}$ for dynamic system (2) is analyzed and the internal force output $\mathbf{e}_{ti}$ is defined as the projection of the force vector $\mathbf{t}$ onto the subspace $\mathcal{R}_{ti,\tau}$

$$\begin{aligned} &\mathbf{e}_{ti} = \mathbf{E}_{ti}\mathbf{x} && \text{where} \\ &\mathbf{E}_{ti} = [\mathbf{Q}(\mathbf{K}) \ \mathbf{0} \ \mathbf{Q}(\mathbf{K}) \ \mathbf{0}] && \text{and} \\ &\mathbf{Q}(\mathbf{K}) = (\mathbf{I} - \mathbf{KG}^T(\mathbf{GKG}^T)^{-1}\mathbf{G})\mathbf{KJ}. \end{aligned} \tag{3}$$

## 3. INTERNAL FORCES DECOUPLING

As pointed out in the introduction, in many advanced robotics tasks the visco–elasticity at the contacts cannot be neglected and the decoupling control of the internal forces from the object motions is needed. Rigid–body kinematics [5, 12] is motions of the object and manipulator which do not involve visco–elastic deformations. For this reason they are regarded as the low–energy motions of the whole system. Rigid–body kinematics represent the easiest way to move the manipulated object and therefore are of particular interest in contrôlling manipulation. In [5] *coordinated* rigid–body motions of the mechanisms are defined as motions of the manipulator $\delta\mathbf{q}$ and of the object $\delta\mathbf{u}$ such that

$$\begin{bmatrix} \delta\mathbf{q} \\ \delta\mathbf{u} \end{bmatrix} \in \mathrm{im} \begin{bmatrix} \mathbf{\Gamma}_{qc} \\ \mathbf{\Gamma}_{uc} \end{bmatrix}$$

where $\mathbf{J\Gamma}_{qc} = \mathbf{G}^T\mathbf{\Gamma}_{uc}$.

Thus rigid–body object motions are those in the column space of $\mathbf{\Gamma}_{uc}$ and the output $\mathbf{e}_{uc}$ is defined as the projection of object displacements $\mathbf{u}$ onto the column space of $\mathbf{\Gamma}_{uc}$

$$\begin{aligned} &\mathbf{e}_{uc} = \mathbf{E}_{uc}\mathbf{x} && \text{where} \\ &\mathbf{E}_{uc} = \mathbf{\Gamma}_{uc}^P[\mathbf{0} \ \mathbf{I} \ \mathbf{0} \ \mathbf{0}] && \text{and} \\ &\mathbf{\Gamma}_{uc}^P = \mathbf{\Gamma}_{uc}(\mathbf{\Gamma}_{uc}^T\mathbf{\Gamma}_{uc})^{-1}\mathbf{\Gamma}_{uc}^T. \end{aligned} \tag{4}$$

In this paper we adopt the notion of *internal force decoupling control* that is formalized in the following definition.

**Definition 1.** Consider the couple $(\mathbf{A}, \mathbf{B}_\tau)$ in (2). The control law $\tau = \mathbf{F}\mathbf{x} + \mathbf{U}\tau_{\text{ref}}$ is an *internal force control decoupled from object motion* if the state feedback and the input selection matrices are such that

$$\text{a)} \quad \min \mathcal{I}(\mathbf{A} + \mathbf{B}_\tau \mathbf{F}, \mathbf{B}_\tau \mathbf{U}) \subseteq \ker \mathbf{E}_{uc};$$

$$\text{b)} \quad \text{im}\, \mathbf{E}_{ti} = \mathbf{E}_{ti} \min \mathcal{I}(\mathbf{A} + \mathbf{B}_\tau \mathbf{F}, \mathbf{B}_\tau \mathbf{U})$$

where $\min \mathcal{I}(A, B)$ is the minimal subspace $A$–invariant containing the column space of $B$.

Observe that the decoupling control of internal forces does not affect the rigid–body object motion (claim a) and preserves the reachability of internal forces (claim b).

In [6] it has been proven that for general manipulation systems with $\ker(\mathbf{G}^T) \neq \{0\}$, the problem of finding a decoupling internal force control has always a solution and a decoupling feedback control law is proposed. The choice of matrices $\mathbf{F}$ and $\mathbf{U}$ is based on the geometric concept of *controlled invariant* [2].

## 4. ROBUST DECOUPLING CONTROL OF CONTACT FORCES

The control law in [6] is model based and an accurate identification procedure of the model's parameters is needed. While techniques estimating the dynamics parameters of the object and of the manipulator are well established, the identification of the visco–elastic contact matrices $\mathbf{K}$ and $\mathbf{B}$ still remains a hard task and consequently a certain degree of uncertainty is present in the system model. This section is devoted to the analysis and design of a robust decoupling controller for manipulation systems with structured (visco–elastic) uncertainties. We assume that a structured uncertainty is present in the visco elastic contact behaviour. In particular we assume that the estimated stiffness and damping matrices have the following structures $\mathbf{K}_s = k_s \mathbf{Z}$ and $\mathbf{B}_s = b_s \mathbf{Z}$ respectively where matrix $\mathbf{Z}$ represents the a priori knowledge on the visco–elastic behaviour while $k_s$ and $b_s$ (real positive values) are the estimated stiffness and damping parameters. We assume that the measured value $k_s$ and $b_s$ are corrupted by errors $\Delta k$ and $\Delta b$ described in a set membership context, $\Delta k \in [\underline{\Delta k}, \overline{\Delta k}]$ and $\Delta b \in [\underline{\Delta b}, \overline{\Delta b}]$, thus it holds

$$\begin{aligned} \mathbf{K} &= (k_s + \Delta k)\mathbf{Z}; \\ \mathbf{B} &= (b_s + \Delta b)\mathbf{Z}. \end{aligned} \tag{5}$$

The uncertainties on stiffness and damping matrices reflect on the linearized dynamics (2) and on the output matrix (3) which becomes uncertain,

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{A}(\Delta k, \Delta b)\mathbf{x} + \mathbf{B}_\tau \tau; \\ \mathbf{e}_{ti} = \mathbf{E}_{ti}(\Delta k, \Delta b)\mathbf{x}; \\ \mathbf{e}_{uc} = \mathbf{E}_{uc}\mathbf{x} \end{cases} \tag{6}$$

where

$$\begin{aligned}\mathbf{A}(\Delta k, \Delta b) &= \mathbf{A}_s + \Delta k \mathbf{A}_{ke} + \Delta b \mathbf{A}_{be};\\ \mathbf{E}_{ti}(\Delta k, \Delta b) &= \mathbf{E}_s + \Delta k \mathbf{E}_{ke} + \Delta b \mathbf{E}_{be}\end{aligned} \tag{7}$$

being $\mathbf{A}_s$ and $\mathbf{E}_s$ the state and the force output matrices of Section 2 are calculated with nominal values $\mathbf{K} = k_s\mathbf{Z}$ and $\mathbf{B} = b_s\mathbf{Z}$, while

$$\mathbf{A}_{ke} = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{S}^T\mathbf{Z}\mathbf{S} & \mathbf{0} \end{bmatrix}; \qquad \mathbf{A}_{be} = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{S}^T\mathbf{Z}\mathbf{S} \end{bmatrix};$$

$$\mathbf{E}_{ke} = [\mathbf{Q}(\mathbf{Z})\ \mathbf{0}\ \mathbf{0}\ \mathbf{0}]; \qquad \mathbf{E}_{be} = [\mathbf{0}\ \mathbf{0}\ \mathbf{Q}(\mathbf{Z})\ \mathbf{0}],$$

specify the structure of the system uncertainty.

After having characterized the structured uncertainties affecting the manipulation dynamics, let us formalize the concept of *robust internal force decoupling controller* [3] by extending the requirements on decoupling and reachability of Definition 1.

**Definition 2.** Consider the class of linear systems

$$\left(\mathbf{A}(\Delta k, \Delta b), \mathbf{B}_\tau, \begin{bmatrix} \mathbf{E}_{ti}(\Delta k, \Delta b) \\ \mathbf{E}_{uc} \end{bmatrix}\right) \qquad \forall(\Delta k, \Delta b).$$

The control law $\tau = \mathbf{F}\mathbf{x} + \mathbf{U}\tau_{\text{ref}}$ is a *decoupling control of internal forces robust with respect to the visco–elastic uncertainties* if

a) $((\mathbf{A}(\Delta k, \Delta b) + \mathbf{B}_\tau\mathbf{F}), \mathbf{B}_\tau\mathbf{U}) \subseteq \ker \mathbf{E}_{uc}$;

b) $\operatorname{im} \mathbf{E}_{ti}(\Delta k, \Delta b) = \mathbf{E}_{ti}(\Delta k, \Delta b) \min \mathcal{I}\ ((\mathbf{A}(\Delta k, \Delta b) + \mathbf{B}_\tau\mathbf{F}), \mathbf{B}_\tau\mathbf{U})$.

## 5. ROBUST CONTROLLER DESIGN

The design of the robust decoupling controller is based on the concepts of "generalized controlled invariant subspace" and "parameter invariant control" introduced in [3].

**Definition 3.** Consider the set of dynamic systems $(\mathbf{A}(\Delta k, \Delta b), \mathbf{B}_\tau)$ for all $(\Delta k, \Delta b)$, a subspace $\mathcal{V}$ is a *generalized controlled invariant* iff there exists a constant matrix $\mathbf{F}$ such that

$$(\mathbf{A}(\Delta k, \Delta b) + \mathbf{B}_\tau\mathbf{F})\mathcal{V} \subseteq \mathcal{V} \quad \forall(\Delta k, \Delta b).$$

By means of the above mentioned definition, it is an easy matter to extend the generalization of controlled invariants to the self–bounded controlled invariants. Then it is possible to define

$$\mathcal{V}^*(\mathbf{A}(\Delta k, \Delta b), \mathbf{B}_\tau, \ker(\mathbf{E}_{uc})), \tag{8}$$

the maximal generalized $(\mathbf{A}(\Delta k, \Delta b), \mathbf{B}_\tau)$–controlled invariant contained in the null space of the (object motion) output matrix $\mathbf{E}_{uc}$.

The following algorithm [3] allows one to evaluate the subspace $\mathcal{V}^*$ whose computation is the first step of the synthesis of the robust decoupling controller.

**Algorithm for the computation of $\mathcal{V}^*$**

$\mathcal{V}_0 = \mathsf{ker}(\mathbf{E}_{uc})$;

$\mathcal{V}_{k+1} = \mathcal{V}_k \cap \mathbf{A}_s^{-1}(\mathsf{im}\,\mathbf{B}_\tau, \mathcal{V}_k) \cap \mathbf{A}_{ke}^{-1}\mathcal{V}_k \cap \mathbf{A}_{be}^{-1}\mathcal{V}_k$;

if $\mathcal{V}_n = \mathcal{V}_{n-1}$, then $\mathcal{V}_n = \mathcal{V}^*(\mathbf{A}(\cdot), \mathbf{B}_\tau, \mathsf{ker}(\mathbf{E}_{uc}))$.

**Remark 1.** It can be shown that the state feedback matrix $\mathbf{F}$, which makes the subspace $\mathcal{V}^*$ invariant with respect to $(\mathbf{A}_s+\mathbf{B}_\tau\mathbf{F})$, fulfills the condition of Definition 3 as well. Recall that $\mathbf{A}_s$ is equal to $\mathbf{A}(\Delta k, \Delta b)$ for $(\Delta k, \Delta b) = (0,0)$ therefore the subspace $\mathcal{V}^*$ is controlled invariant with respect to the pair $(\mathbf{A}_s, \mathbf{B}_\tau)$.

Let us define the input selection matrix $\mathbf{U}$ as

$$\mathsf{im}(\mathbf{B}_\tau\mathbf{U}) = \mathcal{V}^* \cap \mathsf{im}\mathbf{B}_\tau. \tag{9}$$

Then the following proposition, whose proof comes easily from Remark 1, shows which are the state feedback matrices $\mathbf{F}$ and the input selectors $\mathbf{U}$ decoupling internal forces from object motions, notwithstanding the visco–elastic uncertainties.

**Proposition 1.** The decoupling condition a) of Definition 2 is satisfied iff the maximal controlled invariant $\mathcal{V}^*$ is not empty and the input selection matrix $\mathbf{U}$ is not null.

The proof of this proposition is straightforward. □

It must be stressed that this proposition provides a necessary condition for the existence of a robust decoupling controller. Recall that in order to obtain a robust decoupling controller both conditions a) and b) must be fulfilled. Necessary and sufficient conditions for the second requirement of Definition 2 and for the existence of the robust decoupling controller are provided in Propositions 2, 3 and 4.

**Proposition 2.** A necessary condition for claim b) in Definition 2 to hold is

$$\mathbf{E}_{ti}\mathcal{V}^* = \mathsf{im}\mathbf{E}_{ti}.$$

Proof. The proposition is simply proven by observing that for any choice of $\mathbf{F}$ and $\mathbf{U}$, the minimal invariant min $\mathcal{I}$ $((\mathbf{A}(\Delta k, \Delta b) + \mathbf{B}_\tau\mathbf{F}), \mathbf{B}_\tau\mathbf{U})$ is a subspace of $\mathcal{V}^*$ for all $(\Delta k, \Delta b)$. Notice that the existence of non–empty $\mathcal{V}^*$ is necessary for the fulfillment of the decoupling condition a). □

**Proposition 3.** Choose matrices $\mathbf{F}$ and $\mathbf{U}$ according to Remark 1 and Proposition 1. Conditions

$$\dim(\mathcal{V}^*) = \operatorname{rank}(\mathbf{B}_\tau \mathbf{U}); \\ \mathbf{E}_{ti}\mathcal{V}^* = \operatorname{im}\mathbf{E}_{ti}$$

are sufficient for claim b) in Definition 2 to hold.

Proof. Simply observe that under these conditions $\min \mathcal{I}((\mathbf{A}(\Delta k, \Delta b) + \mathbf{B}_\tau \mathbf{F}), \mathbf{B}_\tau \mathbf{U}$ is equal to $\mathcal{V}^*$ which does not depend on corrupting errors $(\Delta k, \Delta b)$. □

**Proposition 4.** Choose matrices $\mathbf{F}$ and $\mathbf{U}$ according to Remark 1 and Proposition 1. Condition

$$\operatorname{rank}(\mathbf{Q}(\mathbf{Z})) = \operatorname{rank}(\mathbf{E}_{ti}\mathbf{B}_\tau \mathbf{U})$$

is sufficient for claim b) in Definition 2 to hold.

Proof. Being $\mathbf{K} \propto \mathbf{Z}$, from the definition of matrix $\mathbf{Q}(\cdot)$ in (3), it ensures that the column spaces of $\mathbf{E}_{ti}$, $\mathbf{E}_s$, $\mathbf{E}_{ke}$, $\mathbf{E}_{be}$ coincide with the column space of $\mathbf{Q}(\mathbf{Z})$. Thus $\operatorname{rank}(\mathbf{Q}(\mathbf{Z})) = \operatorname{rank}(\mathbf{E}_{ti}\mathbf{B}_\tau \mathbf{U})$ implies that $\operatorname{im}(\mathbf{E}_{ti}) = \operatorname{im}(\mathbf{E}_{ti}\mathbf{B}_\tau \mathbf{U})$ and the proof ends. □

Before presenting the basic procedure one wants to do some considerations. If the subspace $\mathcal{V}^*$ is also a robust conditioned invariant subspace with respect to the couple $(\mathbf{A}(\Delta k, \Delta b), \ker(\mathbf{E}_{uc}))$, then it is possible to extend these results to the output feedback. Both in application cases and in theoretical ones the decoupling from the outputs are an interesting problem. In [7] and [10] constructive conditions for disturbance decoupling with algebraic output feedback are presented but without considering their robustness. One has to remark that $\mathbf{E}_{uc}$ is the measured output while $\mathbf{E}_{ti}$ is the controlled output[1]. One remarks that the robust conditioned invariant subspace $\mathcal{V}^*$ is also a robust conditioned invariant subspace iff

$$\mathbf{A}(\Delta k, \Delta b)(\mathcal{V}^* \cap \ker(\mathbf{E}_{uc})) \subseteq \mathcal{V}^* \quad \forall (\Delta k, \Delta b),$$

see [2].

If the subspace $\mathcal{V}^*$, as calculated above, is also robust conditioned invariant subspace then it is possible to find a constant matrix $\mathbf{K}$ such that

$$(\mathbf{A}(\Delta k, \Delta b) + \mathbf{B}_\tau \mathbf{K}\mathbf{E}_{uc})\mathcal{V}^* \subseteq \mathcal{V}^*.$$

This allows us to extend Remark 1, Proposition 3 and 4 to matrix $\mathbf{K}$ so that one can obtain the robust output feedback decoupling. About the calculation of matrix $\mathbf{K}$ one can observe that all our systems must be *no left invertible* because of (9) and because of the Proposition 1 which yields

$$\mathcal{V}^* \cap \operatorname{im}\mathbf{B}_\tau \neq 0.$$

[1]Recall that our task is to squeeze the object or to stretch it with a desired force without moving it.

In fact, one recalls that a system is called *left invertible* if and only if

$$\mathcal{V}^* \cap \mathrm{im}\mathbf{B}_\tau = 0,$$

where $\mathcal{V}^*$ is defined as in (8). In case of *no left invertibility* and stable internal unassignable eigenvalues of $\mathcal{V}^*$[2], then the calculation of the matrix $\mathbf{K}$ becomes immediate from the calculation of the matrix $\mathbf{F}$. In particular, choosing $\mathbf{F}$ such that:

$$\ker\mathbf{E}_{uc} \subseteq \ker\mathbf{F},$$

then

$$\mathbf{K} = \mathbf{F}\mathbf{E}_{uc}^{\dagger},$$

where the matrix $\mathbf{E}_{uc}^{\dagger}$ is the pseudo inverse matrix of $\mathbf{E}_{uc}$, $\mathbf{E}_{uc}^{\dagger} = \mathbf{E}_{uc}^{T}(\mathbf{E}_{uc}\mathbf{E}_{uc}^{T})^{-1}$. For further detail on the calculation of the matrix $\mathbf{K}$ and $\mathbf{F}$ see [7] and [13].

A procedure for designing the robust decoupling controller of internal forces for a given manipulation system is reported in the sequel. The procedure is based on propositions and remarks of this section.

**Procedure.**

*Step 1.* Compute $\mathcal{V}^*$.

*Step 2.* If $\mathcal{V}^* \neq \{\mathbf{0}\}$, choose $\mathbf{F}$ and $\mathbf{U}$ according to Remark 1 and Proposition 1. If $\mathbf{U} \neq \mathbf{0}$, Proposition 1 holds and claims a) of Definition 2 is satisfied, otherwise the robust decoupling controller does not exist and the procedure ends.

*Step 3.* Check the sufficient conditions of Proposition 2. If they are satisfied stop.

*Step 4.* Check the sufficient conditions of Proposition 3. If these are satisfied stop.

*Step 5.* If Step 3 and 4 fail, check the necessary condition of Proposition 4.

*Step 6.* If the necessary condition of Proposition 1 is satisfied, check condition b) of Definition 2.

**Remark 2.** Steps 1 and 2 refer to the decoupling property (claim a) of the robust controller. Steps 3 to 6 check that $\mathbf{F}$ and $\mathbf{U}$ fulfill the reachability condition (claim b).

Notice that if the procedure does not end at Step 3 or 4, a different choice of the state–feedback matrix $\mathbf{F}$ and of the input selection matrix may be needed in order to prove the robustness of the proposed control law.

[2]This condition is mostly guaranteed in industrial manipulators (no anthropomorphic m-niplators).

## 6. AN APPLICATION EXAMPLE

In this section numerical results are reported for the gripper described in Figure 2 a). This system is a planar device with 2 degrees of freedom, a prismatic and a rotoidal joint. Joint variables are positive when links move left. In the reference frame, the contacts are $\mathbf{c}_1 = (0,1)$, $\mathbf{c}_2 = (1,1)$ and the object center of mass is $c_b = (0.5,1)$. As already explained $\mathbf{J} = \mathbf{H}\frac{\delta \mathbf{c}^m}{\delta \mathbf{q}}$ and $\mathbf{G}^T = \mathbf{H}\frac{\delta \mathbf{c}^o}{\delta \mathbf{u}}$, in the presented case the matrix $\mathbf{H}$ is an identity matrix. The inertia matrices of the object and manipulator are assumed to be normalized to the identity matrix. The contact behavior is assumed isotropic at the contact points. Given $\mathbf{q} = [q_1, q_2]^T$ like the vector of generalized coordinate, being in general $\mathbf{c}_1^m = (\cos q_1, 1 - \sin q_1)$, $\mathbf{c}_2^m = (1 - q_2, 1)$, the jacobian matrix and its linearization around the point $q_1 = \frac{\pi}{2}$ assume the following values

$$\mathbf{J} = \begin{bmatrix} -\sin q_1 & 0 \\ -\cos q_1 & 0 \\ 0 & -1 \\ 0 & 0 \end{bmatrix}^T ; \qquad \mathbf{J}_l = \begin{bmatrix} -1 & 0 \\ 0 & 0 \\ 0 & -1 \\ 0 & 0 \end{bmatrix}^T .$$

About the grasp matrix, once assumed $\mathbf{u} = [x, y, \theta]^T$ to be the vector of the generalized coordinates for the object then the contact points could be represented as follows $\mathbf{c}_1^o = (x + \cos\theta, 1 + y + 0.5\,\mathrm{sen}\,\theta)$, $\mathbf{c}_2^o = (1 + x - 0.5\cos\theta, 1 + y - 0.5\,\mathrm{sen}\,\theta)$. The grasp matrix and its linearization around $\theta = 0$ have the following form:

$$\mathbf{G} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0.5\sin\theta & -0.5\cos\theta & 0.5\sin\theta & -0.5\cos\theta \end{bmatrix} ; \qquad \mathbf{G}_l = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & -0.5 & 0 & 0.5 \end{bmatrix} .$$

It ensues [9] that matrix $\mathbf{Z}$, the a priori knowledge on the visco–elastic behavior, is equal to the identity $\mathbf{Z} = \mathbf{I}$. Assume that the measured stiffness and damping are $k_s = 1$ $b_s = 2$ and that the corrupting errors belong to the sets $\Delta k \in [-0.1, 0.1]$ and $\Delta b \in [-0.2, 0.2]$. The nominal and the uncertain matrices of the internal force output in (7) assume the values

$$\mathbf{E}_s = [0.71,\ -0.41,\ 0,\ 0,\ 0,\ 1.42,\ -1.42,\ 0,\ 0,\ 0];$$

$$\mathbf{E}_{ke} = [9.71,\ -5.71,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0];$$

$$\mathbf{E}_{be} = [0,\ 0,\ 0,\ 0,\ 0,\ 0.81,\ -0.71,\ 0,\ 0,\ 0];$$

while the object motion output matrix $\mathbf{E}_{uc}$ is

$$\mathbf{E}_{uc} = [0,\ 0,\ -0.58,\ 1,\ 0,\ 8,\ 0,\ 4,\ 0,\ 0].$$

The algorithm for the computation of $\mathcal{V}^*$ stops for $n = 4$ and $\mathcal{V}^* \neq \{0\}$. Matrix $\mathbf{F}$ is chosen according to Remark 1

$$\mathbf{F} = \begin{bmatrix} -14 & -13 & -16 & 0 & 0 & -2.5 & -1.5 & -7 & 0 & 0 \\ -13 & -14 & -16 & 0 & 0 & -1.5 & -2.5 & -3 & 0 & 3 \end{bmatrix}$$

and from (9) the input selection matrix is obtained as

$$\mathbf{U} = \begin{bmatrix} -0.7071 \\ 0.7071 \end{bmatrix}.$$

It ensues that, notwithstanding the parameter uncertainties, the control law $\mathbf{u} = \mathbf{F}\mathbf{x} + \mathbf{U}\tau_{\mathrm{ref}}$ is such that the trajectories of the systems lie on the null space of the object motion output matrix. As regards the complete reachability of internal forces (claim b), being 6 the dimension of $\mathcal{V}^*$ and 1 the rank of $\mathbf{B}_\tau\mathbf{U}$ (Step 3), Proposition 3 does not hold. At Step 4 the computation of $\mathbf{E}_{ti}\mathbf{B}_\tau\mathbf{U}$ is needed. In this case we obtain that $\mathbf{E}_{ti}\mathbf{B}_\tau\mathbf{U} = -2 - \Delta b$ whose rank is unitary for all corrupting errors $\Delta b \in [-0.2, 0.2]$. The procedure ends by observing that the rank of $\mathbf{Q}(\mathbf{Z}) = [0.71, \ -0.71]$ is unitary as well. Hence, the chosen state feedback matrix $\mathbf{F}$ and the input selection matrix $\mathbf{U}$ synthesize a force control law which is robust with respect to the uncertainties affecting system visco–elastic parameters, preserves the complete controllability of internal contact forces and does not interact with the manipulated object motion. An interesting feature of the robustness property of the system is that the same control law can be used to grasp and manipulate different objects, provided that all of them are characterized by the same visco–elastic matrix $\mathbf{Z}$, the a priori knowledge on visco–elastic behaviour.

## 7. CONCLUSIONS

The control of internal forces was the focus of this paper. Special attention was paid to the noninteraction between the contact forces and the object motion control, a fundamental requirement in advanced robotics.

Since in advanced manipulation tasks the visco–elastic behaviour at the mechanical contacts cannot be neglected, a lumped parameter model of the visco–elastic behavior was taken into account.

The robustness of the decoupling control with respect to the uncertainties in the contact model was investigated. Sufficient conditions for the invariance of the decoupling action under perturbations on the contact stiffness and damping parameters were provided.

These conditions are meaningful for several classes of manipulation systems with general kinematics.

An example was reported to show applications of the obtained results.

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## REFERENCES

[1] P. N. Akella and M. R. Cutkosky: Contact transition control with semiactive soft fingertips. Trans. Robotic and Automation *11* (1995), 6, 859–867.

[2] G. Basile and G. Marro: Controlled and Conditioned Invariants in Linear System Theory. Prentice Hall, Englewood Cliffs, N.J. 1992.

[3] S. P. Bhattacharyya: Generalized controllability, controlled invariant subspace and parameter invariant control. SIAM J. Algebraic Discrete Methods *4* (1983), 4, 529–533.

[4] A. Bicchi: Force distribution in multiple whole-limb manipulation. In: ICRA, 1993.

[5] A. Bicchi, C. Melchiorri. and D. Balluchi: On the mobility and manipulability of general multiple limb robots. IEEE Trans. Automat. Control *11* (1995), 2, 215–228.

[6] A. Bicchi D. Prattichizzo, P. Mercorelli, and A. Vicino: Noninteracting force/motion control in general manipulation systems. In: Proc. 35th IEEE Conference on Decision Control, CDC'96, Kobe 1996.

[7] G. Marro and F. Barbagli: The algebraic output feedback in the light of dual-lattice structures. Kybernetika *35* (1999), 6, 693–706.

[8] M. T. Mason and J. K. Salisbury: Robot Hands and the Mechanics of Manipulation. MIT Press, Cambridge, MA 1985.

[9] L. Meirovitch: Analytical Methods in Vibrations. Macmillan, New York 1967.

[10] F. Barbagli, G. Marro, P. Mercorelli, and D. Prattichizzo: Some results on output algebraic feedback with applications to mechanical systems. In: Proc. 37th IEEE Conference on Decision Control, CDC'98, Tampa 1998.

[11] D. Prattichizzo: Structural Properties and Control of Robotics Manipulation. Ph.D. Thesis, University of Pisa, 1995.

[12] D. Prattichizzo and A. Bicchi: Dynamic analysis of mobility and graspability of general manipulation systems. Trans. Robotic and Automation *14* (1998), 2, 251–218.

[13] D. Prattichizzo and P. Mercorelli: On some geometric control properties of active suspension systems. Kybernetika *36* (2000), 5, 549–570.



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