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INFORMATION BOUNDEDNESS PRINCIPLE IN FUZZY INFERENCE PROCESS

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The information boundedness principle requires that the knowledge obtained as a result of an inference process should not have more information than that contained in the consequent of the rule. From this point of view relevancy transformation operators as a generalization of implications are investigated.

1. PRELIMINARIES

The main goal of the fuzzy modeling inference process is to find a value of the output associated with a particular input value. Fuzzy system modeling usually involves the use of a fuzzy rule base consisting in several fuzzy statements. The most widely used type of fuzzy statements are if-then rules with fuzzy predicates (generalized modus ponens).

Suppose that our rules have formulations:

$$\text{If } U \text{ is } A_i \text{ then } V \text{ is } B_i \qquad i = 1, 2, \dots, n$$

where U, V are variables taking the values over spaces X and Y respectively and A_i, B_i are fuzzy sets on X and Y. So, the use of the if-then rule in generalized modus ponens needs three factors: The antecedent A_i, the consequent B_i and the current input (crisp or fuzzy).

The fuzzy system modeling is often based on the use of fuzzy implications or fuzzy conjunctions [2]. The relevancy transformation operator (RET) introduced by Yager [7] is a generalization of both approaches. RET operator can be defined (see below) as a binary operation defined on the unit interval. It is appropriate for obtaining of the effective individual rule output from the rule relevancy and the rule consequent.

Overall fuzzy system output can be obtained by aggregation [3, 7] of individual rule outputs and by a possible defuzzification [7]. In this paper we shall deal with the problem of finding the individual rule output only.

Now we suppose that the relevancy of the individual rule is obtained from the antecedent A_i and the current input. For example, if the input value x is crisp then the relevancy of the rule can be determined by the membership grade of the input value x in the antecedent fuzzy set A_i, i. e.,

$$r_i = A_i(x).$$

If the input value is a fuzzy set C on X then the relevancy r_i of the rule can be determined by

$$r_i = \sup_x \min(C(x), A_i(x)).$$

We also assume that the individual rule output F_i can be obtained from the relevancy r_i of the rule and the rule consequent B_i pointwisely, i. e.,

$$F_i(y) = h(r_i, B_i(y)) \quad y \in Y.$$

Below we shall give a definition of the relevancy transformation operator h [5, 7]. The operator h is closely related to the aggregation operator. If the rule relevancy is zero, the effective output should not influence the aggregation process. Therefore, in this case, we suppose that the rule output should be the element c which is neutral under aggregation operation [3].

Definition 1. Let $c \in [0,1]$ be a given element. A binary operation $h : [0,1]^2 \to [0,1]$ is called a relevancy transformation (RET) operator with respect to the element c if it satisfies the following axioms:

(R1) $h(1,a) = a$ for all $a \in [0,1]$,

(R2) $h(0,a) = c$ for all $a \in [0,1]$,

(R3) $h(r,a_1) \leq h(r,a_2)$ for all $a_1, a_2 \in [0,1]$, such that $a_1 < a_2$ and all $r \in [0,1]$,

(R4) if $a \geq c$, then $h(r_1,a) \leq h(r_2,a)$ for all $r_1, r_2 \in [0,1]$ such that $r_1 < r_2$,

(R5) if $a \leq c$ then $h(r_1,a) \geq h(r_2,a)$ for all $r_1, r_2 \in [0,1]$ such that $r_1 < r_2$.

Note that the axiom (R1) means that the effective rule output is equal to the consequent if the rule relevancy is full. The element c is related to the global aggregation operator, namely it should be its neutral element, i. e.,

$$Agg(a_1, a_2, \ldots, a_i, c, a_{i+1}, \ldots, a_n) = Agg(a_1, a_2, \ldots, a_i, a_{i+1}, \ldots, a_n).$$

The last two conditions are called a consistency in antecedent argument and they imply together with (R2) a necessary condition for h to be a RET operator, namely

$$h(r,c) = c \quad \text{for any} \quad r \in [0,1].$$

In [7] one can find more about the philosophical background of these properties.

Example 1. (i) Let T be a t-norm [5]. Put $h(r, a) = T(r, a)$ where $r, a \in [0, 1]$. Then h is a RET operator with respect to the element $c = 0$.

(ii) Let S be a t-conorm [5]. Put $h(r, a) = S(1 - r, a)$ where $r, a \in [0, 1]$. Then h is a RET operator with respect to the element $c = 1$. Note that in this case h is called S-implicator.

(iii) Let $c \in [0, 1]$ be a given element. Define

$$h(r, a) = ra + (1 - r)c$$

Then h is a RET operator with respect to the element c. We shall call it Product RET (PRET) operator.

We have described how to obtain an individual rule output using a RET operator. We shall concentrate on the problem whether this process is always meaningful. Namely, we shall investigate whether the RET operator fulfills the natural requirement of any inference process: Knowledge obtained as a results of this process should not have more information than that contained in the consequent of the rule [6]. This principle is usually called the information boundedness principle (IBP).

The next example shows a fuzzy modeling inference process with one simple rule only and a RET operator such that IBP is evidently not fulfilled.

Example 2. Let the rule has the form:

If U is A then V is B

where variables U and V take values over space of real numbers and fuzzy sets A and B are triangular fuzzy numbers given by expressions

$$A(x) = \max(0, 1 - |x|), \quad B(y) = \max(0, 1 - |y|), \quad x, y \in \mathbb{R}.$$

It means that this rule says: if U is "around zero", then V is "around zero". Let h be a RET operator with respect to $c = 0$ given by

$$h(r, a) = \begin{cases} 1 & \text{if } r > 0 \text{ and } a = 1 \\ a & \text{if } r = 1 \\ 0 & \text{elsewhere.} \end{cases}$$

Then for any relevance $r \in]0, 1[$ we obtain the current output

$$F(y) = \begin{cases} 1 & \text{if } y = 0 \\ 0 & \text{elsewhere.} \end{cases}$$

It means that F is a singleton and the rule gives the certainty that V is zero. Of course, our rule says only that V is "around zero".

Many information measures have been proposed attached to fuzzy sets or bodies of evidence (Shannon's entropy, fuzziness, measure of imprecision etc. [2]). Yager [6] introduced a concept of specificity of fuzzy sets, measuring to what extent a fuzzy set restricts a small number of values for a variable. Dubois and Prade [1] extended this concept for bodies of evidence. They utilized the fact that bodies of evidence with the nested basic probability assignment can be characterized by a possibility distributions. Such possibility distribution can be viewed as the membership function of some fuzzy set. From such point of view the maximal specificity measure relates to the complete knowledge which can be represented by singleton and minimal specificity measure relates to complete ignorance which can be represented by constant fuzzy set equal to one.

Since considered mentioned membership functions are always normal (at least one element is fully possible), the older definitions of specificity measure are convenient for normal fuzzy sets only. In [6] Yager introduced definition of specificity measure which did not require normal fuzzy sets.

Now we formulate the information boundedness principle (IBP) related to a given RET operator h and a given specificity measure Sp.

(IBP) Let $F_1(y) = h(r_1, B(y))$, $F_2(y) = h(r_2, B(y))$ be individual outputs of the same rule with the consequent B and r_1, r_2 are the levels of relevancy such that $1 \geq r_1 > r_2 \geq 0$. Then

$$Sp(F_1) \geq Sp(F_2).$$

Putting $r_1 = 1$, $r_2 = r$ and using (R1) we obtain a weaker form of IBP.

(IBP*) Let $F(y) = h(r, B(y))$ be an individual output of the rule. Then

$$Sp(F) \leq Sp(B).$$

In the next sections we shall try to reformulate the definition of specificity measure [6] for fuzzy sets on a finite universe and then classify RET operators with respect to various specificity measures.

2. SPECIFICITY MEASURE

In the next sections we shall consider fuzzy subsets of a finite universe X with the cardinality n. Then each fuzzy set is represented by an n-tuple $(a_1, a_2, \dots, a_n) \in [0,1]^n$. Now we shall define a specificity as a mapping from the system of all fuzzy sets on X to the unit interval. Our definition is reformulation of the definition published in [6].

Definition 2. A mapping $Sp\colon [0,1]^n \to [0,1]$ is called a specificity measure (specificity for short) if it satisfies the following axioms:

(S1) For any permutation $(p_1, p_2, \dots, p_n)$ of $(1, 2, \dots, n)$ is

$$Sp(a_1, a_2, \dots, a_n) = Sp(a_{p_1}, a_{p_2}, \dots, a_{p_n}).$$

(S2) If $1 \geq b_1 > b_2 \geq a_2 \geq a_3 \geq \cdots \geq a_n \geq 0$ then

$$Sp(b_1, a_2, a_3, \ldots, a_n) > Sp(b_2, a_2, a_3, \ldots, a_n).$$

(S3) If $1 \geq a_1 \geq a_2 \geq \cdots \geq a_n \geq 0$, $a_1 \geq b_2 \geq b_3 \geq \cdots \geq b_n \geq 0$ and $a_i \geq b_i$ for each $i = 2, 3, \ldots, n$, then

$$Sp(a_1, a_2, \ldots, a_n) \leq Sp(a_1, b_2, \ldots, b_n).$$

(S4) $Sp(a_1, a_2, \ldots, a_n) = 1$ if and only if exists the unique i such that $a_i = 1$ and $a_j = 0$ for each $j \neq i$.

(S5) $Sp(0, 0, \ldots, 0) = 0$.

Axiom (S1) means that the specificity is invariant with respect to order of membership degrees, the property of symmetry. Axioms (S2) and (S3) mean that specificity is increasing in the greatest membership value but nonincreasing in the others. Axiom (S4) requires that only singletons have maximal specificity. The axiom (S5) says that the specificity of the empty set is equal to zero.

In [6] Yager permits that axiom (S5) may be too strong. On the other hand, if specificity represents how much the greatest membership degree protrudes the others, this claim should be accepted, if not enforced. Moreover, from our viewpoint, this definition of specificty should be weak. Therefore we introduce two additional properties.

Definition 3. We say that a specificity measure Sp is grounded if

(GS) $Sp(b, b, \ldots, b) = 0$ for all $b \in [0, 1]$,
and we say that a specificity measure is shift invariant if

(SIS) $Sp(a_1, a_2, \ldots, a_n) = Sp(a_1 + b, a_2 + b, \ldots, a_n + b)$
for all $b \in [-\min_i(a_i), 1 - \max_i(a_i)]$.

Note that axiom (GS) implies axiom (S5) and axiom (SIS) with (S5) together imply (GS).

Remark 1. If Sp_1 and Sp_2 are two different specificity measures, then any convex combination, i.e., the function $\alpha Sp_1 + (1 - \alpha) Sp_2$ where $\alpha \in [0, 1]$, is a specificity measure. Moreover if both specificities are grounded (shift invariant), then their convex combination is also a grounded (shift invariant) specificity.

Example 3. In this example we consider fuzzy sets over 2-membered universe only, that is $n = 2$.

(i) Put $Sp(x,y) = \max(x,y) - \min(x,y)$. Then Sp is a shift invariant and grounded specificity.

(ii) Specificity $Sp(x,y) = \max(x,y) - 0.5\min(x,y)$ is not grounded and thus not shift invariant.

(iii) Put

$$Sp(x,y) = \begin{cases} 2.4\max(x,y) - 2.4\min(x,y) & \text{if } \min(x,y) \geq 1.5\max(x,y) - 0.5 \\ 0.3\max(x,y) - \min(x,y) + 0.7 & \text{elsewhere.} \end{cases}$$

Then Sp is a grounded specificity but not shift invariant.

The class of all specificities is very large, so it is reasonable to search some interesting sub-classes. One of them is the class of so called linear specificities [6].

Definition 4. Let $1 \geq w_2 \geq w_3 \geq \cdots \geq w_n \geq 0$ be given constants such that $\sum_{i=2}^{n} w_i = 1$. If $1 \geq a_1 \geq a_2 \geq \cdots \geq a_n \geq 0$, then the linear specificity is defined as:

$$Sp(a_1, a_2, \ldots, a_n) = a_1 - \sum_{i=2}^{n} w_i a_i.$$

Note that every linear specificity is grounded and shift invariant. For $n = 2$, any linear specificity is equivalent to the specificity from Example 3 (i).

Example 4. Let $1 \geq a_1 \geq a_2 \geq \cdots \geq a_n \geq 0$.

(i) Specificity defined as

$$\underline{Sp}(a_1, a_2, \ldots, a_n) = a_1 - a_2$$

is a linear specificity with weight constants $w_2 = 1$ and $w_3 = w_4 = \cdots = w_n = 0$.

(ii) Specificity defined by the expression

$$\overline{Sp}(a_1, a_2, \ldots, a_n) = a_1 - \frac{1}{n-1}\sum_{i=2}^{n} a_i$$

is also a linear specificty with constants $w_2 = w_3 = \cdots = w_n = \frac{1}{n-1}$. Yager [6] showed that any linear specificity Sp satisfies $\overline{Sp} \geq Sp \geq \underline{Sp}$.

3. INFORMATION BOUNDEDNESS PRINCIPLE FOR RELEVANCY TRANSFORMATION OPERATORS

In this section we are looking for RET operators which fullfil IBP with respect to some specificities. The direct consequence of the result reached in [6] is that any S-implicator satisfies IBP with respect to a linear specificity if and only if the S-implicator is 2-increasing. This property plays an important role in the theory of copulas [4].

Definition 5. We say that a mapping $F\colon \mathbb{R}^2 \to \mathbb{R}$ is 2-increasing if

$$F(x_1, y_1) - F(x_1, y_2) - F(x_2, y_1) + F(x_2, y_2) \geq 0$$

for any $x_1 \geq x_2$ and $y_1 \geq y_2$.

Main result of this article is a generalization of mentioned theorem for S-implicators. First we prove a Lemma.

Lemma 1. Let $Sp : [0,1]^n \to [0,1]$ be a shift invariant specificity. There exist a strictly increasing mapping $g : [0,1] \to [0,1]$ such that $g(0) = 0$, $g(1) = 1$ and

$$g(a_1 - a_n) \geq Sp(a_1, a_2, \dots, a_n) \geq g(a_1 - a_2)$$

for any $1 \geq a_1 \geq a_2 \geq \cdots \geq a_n \geq 0$.

Proof. For simplicity consider $1 \geq a_1 \geq a_2 \geq \cdots \geq a_n \geq 0$. The situation is the same for another order of arguments due to axiom (S1).

Define $g(x) = Sp(x, 0, 0, \dots, 0)$ for any $x \in [0,1]$. According to axiom (S2) of specificity, mapping $g(x)$ is strictly increasing. From axioms (S4) and (S5) follows $g(0) = Sp(0, 0, \dots, 0) = 0$ and $g(1) = Sp(1, 0, \dots, 0) = 1$ respectively.

Since we suppose $1 \geq a_1 \geq a_2 \geq \cdots \geq a_n \geq 0$, from the axiom (S3) we obtain inequality.

$$Sp(a_1, a_n, \dots, a_n) \geq Sp(a_1, a_2, \dots, a_n) \geq Sp(a_1, a_2, \dots, a_2).$$

Let us subtract constant a_n from each argument of the left specificity and a_2 from each argument of the right one. Due to Sp is shift invariant, such change has no effect to its value and following inequality hold

$$Sp(a_1 - a_n, 0, \dots, 0) \geq Sp(a_1, a_2, \dots, a_n) \geq Sp(a_1 - a_2, 0, \dots, 0).$$

Rewriting expression for definition of g we obtain

$$g(a_1 - a_n) \geq Sp(a_1, a_2, \dots, a_n) \geq g(a_1 - a_2)$$

which is our claim. □

Theorem 1. Let $Sp : [0,1]^n \to [0,1]$ be a shift invariant specificity and let h be a RET operator. Following statements are equivalent:

(a) h satisfies IBP with respect to Sp,

(b) h is 2-increasing.

Proof. For simplicity consider $1 \geq a_1 \geq a_2 \geq \cdots \geq a_n \geq 0$. The situation is the same for another order of arguments due to axiom (S1).

(a) $\Rightarrow$ (b) RET operator h satisfies IBP with respect to Sp, therefore

$$Sp(h(r_1,a_1),h(r_1,a_2),\dots h(r_1,a_n)) \geq Sp(h(r_2,a_1),h(r_2,a_2),\dots h(r_2,a_n))$$

for any $r_1 > r_2$. Putting $a_1 > a_2 = a_3 = \cdots = a_n$ we obtain

$$Sp(h(r_1,a_1),h(r_1,a_2),\dots h(r_1,a_2)) \geq Sp(h(r_2,a_1),h(r_2,a_2),\dots h(r_2,a_2)).$$

According to Lemma 1 there exist strictly increasing mapping $g : [0,1] \to [0,1]$ such that $g(0) = 0$ and $g(1) = 1$ and

$$\begin{aligned} g(h(r_1,a_1) - h(r_1,a_2)) &\geq Sp(h(r_1,a_1),h(r_1,a_2),\dots h(r_1,a_2)) \\ \geq \quad Sp(h(r_2,a_1),h(r_2,a_2),\dots h(r_2,a_2)) &\geq g(h(r_2,a_1) - h(r_2,a_2)) \end{aligned}$$

or simply

$$g(h(r_1,a_1) - h(r_1,a_2)) \geq g(h(r_2,a_1) - h(r_2,a_2)).$$

Because of g is strictly increasing, for RET operator h follows that

$$h(r_1,a_1) - h(r_1,a_2) \geq h(r_2,a_1) - h(r_2,a_2)$$

or

$$h(r_1,a_1) - h(r_1,a_2) - h(r_2,a_1) + h(r_2,a_2) \geq 0$$

for any $a_1 > a_2$ and $r_1 > r_2$ and thus h is 2-increasing.

(b) $\Rightarrow$ (a) Let h be 2-increasing RET operator and let $1 \geq r_1 > r_2 \geq 0$. Consider expression

$$Sp(h(r_2,a_1),h(r_2,a_2),\dots h(r_2,a_n)).$$

First we will show that $h(r_1,a_1) - h(r_2,a_1)$ is a proper "shift" for this expression in the sense of Definition 3 (SIS), i.e., following inequalities holds

$$1 - h(r_2,a_1) \geq h(r_1,a_1) - h(r_2,a_1) \geq -h(r_2,a_n)$$

for any $r_1 > r_2$ and $a_1 \geq a_n$. First inequality follows from $1 \geq h(r_1,a_1)$ which is a general property of RET operators. The second inequality follows from 2-increasingness of h, we have

$$h(r_1,a_1) - h(r_1,a_n) - h(r_2,a_1) + h(r_2,a_n) \geq 0$$

or

$$h(r_1, a_1) - h(r_2, a_1) + h(r_2, a_n) \geq h(r_1, a_n) \geq 0$$

which implies

$$h(r_1, a_1) - h(r_1, a_n) \geq -h(r_2, a_n).$$

Let us add $h(r_1, a_1) - h(r_2, a_1)$ to each argument of specificity in first expression. Due to specificity Sp is shift invariant, the expression above is equal to

$$Sp(h(r_2, a_1) + h(r_1, a_1) - h(r_2, a_1), h(r_2, a_2) + h(r_1, a_1) - h(r_2, a_1), \ldots$$

$$\ldots, h(r_2, a_n) + h(r_1, a_1) - h(r_2, a_1)).$$

Since considered $1 \geq a_1 \geq a_2 \geq \cdots \geq a_n \geq 0$ and due to RET operator h has 2-increasing property follows

$$h(r_2, a_i) + h(r_1, a_1) - h(r_2, a_1) \geq h(r_1, a_i)$$

for any $i = 1, 2, \ldots n$. Moreover the first argument is equal to $h(r_1, a_1)$. Applying axiom (S3) of specificity measure we have that former expression is less or equal than expression

$$Sp(h(r_1, a_1), h(r_1, a_2), \ldots h(r_1, a_n))$$

so that the following inequality holds

$$Sp(h(r_1, a_1), h(r_1, a_2), \ldots h(r_1, a_n)) \geq Sp(h(r_2, a_1), h(r_2, a_2), \ldots, h(r_2, a_n))$$

which means that h satisfies IBP property. □

Example 5. Let $c \in [0, 1]$ be a given element.

(i) It can be easily shown that PRET operator from Example 1 (iii) is 2-increasing.

(ii) Let $b \in [0, 1]$ be a given element. Put

$$h(r, a) = \begin{cases} c & \text{if } r < b \\ ra + (1 - r)\, c & \text{elsewhere} \end{cases} \qquad r, a \in [0, 1]$$

then h is a 2-increasing RET operator.

(iii) Let $b \in (0, 1]$ be a given element. Define

$$h(r, a) = \begin{cases} c & \text{if } r < b \\ a & \text{elsewhere} \end{cases} \qquad r,\, a \in [0, 1]$$

then h is a 2-increasing RET operator.

Next we show that some 2-increasing RET operators can be easily generated. First, recall the definition of a copula [4, 5].

Definition 6. A two-dimensional copula (copula for short) is a mapping $C : [0,1]^2 \to [0,1]$, such that

(C1) C is a 2-increasing function,

(C2) $C(x,0) = C(0,x) = 0$ for all $x \in [0,1]$

(C3) $C(x,1) = C(1,x) = x$ for all $x \in [0,1]$.

The next theorem gives a possibility to generate RET operators by some copulas [5].

Theorem 2. Let $c \in [0,1]$ be a given element. If C is a copula which for all $r \in [0,1]$ satisfies the property $C(r,c)+C(1-r,c) = c$, then the function $h\colon [0,1]^2 \to [0,1]$ defined as

$$h(r,a) = C(r,a) + C(1-r,c)$$

is a RET operator with respect to the element c.

Now we shall show that RET operators generated by a copula are 2-increasing.

Theorem 3. Let h be a RET operator generated by a copula C in the sense of Theorem 2. Then h is 2-increasing.

P r o o f. By the assumption h is defined by the expression

$$h(r,a) = C(r,a) + C(1-r,c)$$

where C is a copula, and thus 2-increasing function. So for any $a_1 \geq a_2$ and for any $r_1 \geq r_2$, both from the unit interval, is

$$C(r_1,a_1) - C(r_2,a_1) - C(r_1,a_2) + C(r_2,a_2) \geq 0.$$

Extending this inequality by two zeros we obtain

$$C(r_1,a_1) + C(1-r_1,c) - C(r_2,a_1) - C(1-r_2,c)$$

$$-C(r_1,a_2) - C(1-r_1,c) + C(r_2,a_2) + C(1-r_2,c) \geq 0$$

which is equivalent to

$$h(r_1,a_1) - h(r_1,a_2) - h(r_2,a_1) + h(r_2,a_2) \geq 0$$

which is our claim. □

In the next example we show a 2-increasing RET operator and a nonlinear specificity such that IBP is not satisfied.

Example 6. Let Sp be the specificity from Example 3 (iii). As h take PRET operator with respect to $c = 0$, $h(r, a) = ra$ introduced in Example 1 (iii). According to Example 5 (i) such h is 2-increasing. Put $a_1 = 1$, $a_2 = 0.5$, $r_1 = 1$ and $r_2 = 0.5$. We obtain

$$Sp(h(r_1, a_1), h(r_1, a_2)) = Sp(1, 0.5) = 0.5$$

and

$$Sp(h(r_2, a_1), h(r_2, a_2)) = Sp(0.5, 0.25) = 0.6.$$

Due to $r_1 > r_2$, IBP is violated.

On the other hand, there exist RET operators which satisfy IBP with respect to any specificity. Such RET operator is for example the function h from Example 5 (iii) if $c = 0$.

For simplicity we shall restrict our considerations on the PRET operators from Example 1 (iii). From the Theorem 1 follows, that any PRET operator satisfies IBP with respect to any shift invariant specificity. The next theorem is a necessary condition for specificity if any PRET satisfies IBP with respect to this specificity.

Theorem 4. If any product RET operator satisfies IBP with respect to some specificity Sp, then Sp is grounded.

Proof. Take two PRET operators with respect to $c = 0$ and $c = 1$ respectively, $h_1(r, a) = ra$ and $h_2(r, a) = ra + 1 - r$. Because h_1 satisfies IBP with respect to Sp, for any $r_1 > r_2$ we have

$$Sp(h_1(r_1, 1), h_1(r_1, 1), \dots, h_1(r_1, 1)) \geq Sp(h_1(r_2, 1), h_1(r_2, 1), \dots, h_1(r_2, 1)).$$

Due to $h_1(r_1, 1) = r_1$ and $h_1(r_2, 1) = r_2$, the last inequality can be rewriten to

$$Sp(r_1, r_1, \dots, r_1) \geq Sp(r_2, r_2, \dots, r_2)$$

so $Sp(x, x, \dots, x)$ is nondecreasing function of variable x.

Similary, using $h_2(1 - r_1, 0) = r_1$ and $h_2(1 - r_2, 0) = r_2$, it can be shown, that $Sp(x, x, \dots, x)$ is nonincreasing function of x. Consequently $Sp(x, x, \dots, x)$ is a constant function. Using axiom (S5) we obtain $Sp(x, x, \dots, x) = 0$ for all $x \in [0, 1]$. $\square$

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REFERENCES

[1] D. Dubois and H. Prade: Properties of measures of information in evidence and possibility theories. Fuzzy Sets and Systems *100* (1999), Supplement, 35–49.

[2] G. J. Klir and Y. Bo: Fuzzy Sets and Fuzzy Logic. Theory and Applications. Prentice–Hall, Englewood Cliffs, N. J. 1995.

[3] A. Kolesárová and E. E. Kerre: Computational rule of inference based on triangular norms. In: Fuzzy If–Then Rules in Computational Inteligence. Theory and Applications (Da Ruan and E. E. Kerre, eds.), Kluwer Academic Publishers, Dordrecht 2000, pp. 61–80.

[4] R. B. Nelsen: An Introduction to Copulas. Lecture Notes in Statistics, Springer, Berlin 1999.

[5] M. Šabo, A. Kolesárová, and Š. Varga: RET operators generated by triangular norms and copulas. Internat. J. Uncertainty and Knowledge–Based Systems *9* (2001), 2, 169–181.

[6] R. R. Yager: Global requirements for implication operators in fuzzy modus ponens. Fuzzy Sets and Systems *106* (1999), 3–10.

[7] R. R. Yager: Uninorms in fuzzy modeling. Fuzzy Sets and Systems *122* (2001), 167–175.

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