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FURTHER RESULTS ON SLIDING MANIFOLD DESIGN AND OBSERVATION FOR A HEAT EQUATION

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This article presents new extensions regarding a nonlinear control design framework that is suitable for a class of distributed parameter systems with uncertainties (DPS). The control objective is first formulated as a function of the distributed system state. Then, a control is sought such that the set in the state space where this relation is true forms an integral manifold reachable in finite time. The manifold is called a Sliding Manifold. The Sliding Mode controller implements a theoretically infinite gain but with finite control amplitudes serving as an effective tool to suppress the influence of matched disturbances and uncertainties in the system behavior. The theory is developed generically for a finite dimensional Jordan Canonical representation of the DPS. The controller manifold design is described in detail and the observer manifold design can be described in a dual manner. Finally, the control law is expressed in terms of the distributed state. However, in a temperature field control problem motivated by a robotic arc-welding application, the simulations presented are done in the standard manner: a reduced-order finite-dimensional model is used to design the controller which is then implemented on a higher-order, still finite-dimensional (truth) model of the system. An analysis of the potential spillover problem shows the effectiveness of our approach. The article concludes with a brief description of the development of an experimental setup that is underway in our Control Systems Lab.

1. INTRODUCTION

We are developing an implementable theory of stable integral manifold synthesis for control and observation problems for a general type of distributed parameter systems (DPS). The first step is to design a manifold which defines control goals and specifications as a function of the system states. Therefore, when the state is confined to the manifold by a suitably chosen control, the required goals are achieved. This general problem statement is seen to consist of three parts:

1. Manifold Design: the requirements are written as a function of the system states.

2. Control Design: a controller is specified which forces the states to the manifold and maintains them there. At this stage, one assumes the availability of the state vector.

3. State Estimator Design: an observer is specified to implement the controller.

Typical examples of DPS include nonlinear diffusion equations modeling the evolution of the temperature field in arc welding, and Euler–Bernoulli or Timoshenko equations modeling the vibrations of a flexible manipulator. The main difficulties to control these systems are complexity and strong uncertainty. Although one can assume for simulation purposes that the geometry and properties of the heated material are known, or that the manipulator payload is negligible, in practice they may vary in a wide range. For these reasons, the control engineer selects a simplified DPS model together with a suitable set of boundary conditions that describe a closely-related problem. For example, a flexible manipulator with a payload may be modeled by an Euler–Bernoulli beam equation with clamped-free boundary conditions. In arc-welding applications, one may select 2D or 3D heat equations on a plate with Dirichlet-Neumann boundary conditions. After the simplified DPS model is selected, one solves the associated eigenvalue/eigenfunction problem and invokes the *assumed-mode method* [9, 3] to derive a model for the original system. The resulting model comprises a theoretically infinite number of uncoupled differential equations for the so called system modes. In practice one truncates the model for controller design and simulates on a higher dimensional model so that the effect of unmodelled dynamics or spillover effects can be examined. This familiar technique of truncated modal expansions is therefore seen to lead very naturally to a finite dimensional model in the *Jordan Canonical Form.* In our work we allow the state vector of the finite dimensional model to evolve in the complex vector space C^n for convenience. The spectrum of the system matrix may consist of real and complex, simple and repeated eigenvalues, with complex eigenvalues appearing in complex conjugate pairs.

The sliding mode control methodology is well developed for finite dimensional systems (see, for example [4] and references therein) but relatively very little has appeared in the literature for infinite dimensional systems [5, 8]. The appeal of the sliding mode controller is its natural insensitivity to matched disturbances and parameter variations thereby providing robustness to the controller. We are interested in employing this design methodology for DPS as applied to the specific engineering problem of robotic arc-welding [2, 6, 7]. The design idea is based on the following procedure: in order to solve a control problem such as the stabilization of the weld width or heat penetration, we reformulate these objectives as a certain function of the system states which defines a desired manifold [7]. A control is found such that the set in the state space where this relation is true forms a sliding manifold, that is, an integral manifold reachable in finite time. The sliding mode algorithm implements a high (theoretically infinite) gain needed to keep the state on the manifold and as a result, the influence of disturbances and uncertainties in the system behavior is suppressed.

The remainder of the article is organized as follows: Section 2 describes the general model of DPS that is being considered and its finite dimensional model in the Jordan form; Section 3 introduces two transformations that convert the model to the phase-canonic form where the design of the sliding surface becomes very natural and transparent; Section 4 details the design of the controller sliding manifold; Section 5 provides the control law synthesis and stability analysis via a standard Lyapunov argument; Section 6 illustrates the results by simulation, describes the development of an experimental setup, and takes a glimpse at the spillover problem; and Section 7 draws several conclusions and lists topics for further research.

2. DISTRIBUTED PARAMETER SYSTEM MODEL

The distributed parameter systems (DPS) under consideration evolve in a *Banach* space $\mathcal{W}$ with norm $\|\cdot\|$ and governed by partial differential equations (PDE) of the form

$$\frac{\partial Q(t,x)}{\partial t} = \mathcal{A}Q(t,x) + F(t,x) + b(x)\,u(t), \tag{1}$$

where Q is the state, $F(t,x)$ (continuous in t) is a vector of disturbances that, like $b(x)$, belongs to the class $C^1(\Omega)$ of continuously differentiable functions of $x \in \Omega$, where $\Omega \in R^3$ is a spatial region with a smooth boundary $\partial\Omega$. The system is controlled by a scalar control input u. The standard restrictions on $\mathcal{A}$ are that it is a closed, linear, differential operator, that generates a semigroup of strongly continuous bounded operators $e^{\mathcal{A}t}$ defined for $t \geq 0$ [11]. The natural and geometric boundary conditions imposed on (1) render the corresponding boundary value problem well posed with a unique solution for $x \in \Omega$ and $t > 0$.

The operator $\mathcal{A}$ has eigenvalues μ_j and normalized eigenvectors $\phi_j(x)$, $j = 1, 2, \ldots$ ($\|\phi_j\| = 1$) that satisfy the equation

$$\mathcal{A}\phi_j(x) = \mu_j \phi_j(x). \tag{2}$$

Although one is usually concerned with real, self-adjoint, linear operators [12, 10] which are known to have real eigenvalues and eigenvectors, we allow for generality in the system description (1) operators which may have pairs of complex conjugate eigenvalues and eigenvectors as well. Using the standard technique of separation of variables it is possible to write the solution of system (1) in the form of an infinite series

$$Q(t,x) = \sum_{j=1}^{\infty} \phi_j(x)\, q_j(t), \tag{3}$$

where $q_j(t)$ are scalar functions of time known as *modes*. The state vector $q(t) = [q_1\ q_2\ \ldots]$ satisfies the system of ordinary differential equations

$$\dot{q}(t) = Jq(t) + bu + f(t) \tag{4}$$

where the system matrix J is comprised of Jordan blocks on the diagonal,

$$J = \text{blockdiag}\{J_1, J_2, \ldots\}$$

and $b = [b_1, b_2, \ldots]' \in \ell^2$ and $f(t) = [f_1(t), f_2(t), \ldots]' \in \ell^2$ are obtained by expanding $b(x)$ and $f(t,x)$, respectively, in terms of the eigenfunctions ϕ_j. Clearly, a component in b corresponding to a complex eigenvalue λ_j, has a complex conjugate entry corresponding to λ_j^*. Each block J_j can be of the form

$$J_j = \begin{bmatrix} \lambda_j & 1 & 0 & \dots & 0 \\ 0 & \lambda_j & 1 & \dots & 0 \\ \vdots & 0 & \dots & \dots & \vdots \\ \vdots & \vdots & \dots & \lambda_j & 1 \\ 0 & 0 & \dots & 0 & \lambda_j \end{bmatrix} \tag{5}$$

for repeated eigenvalues, or

$$J_j = [\lambda_j] \tag{6}$$

for distinct eigenvalues. For example, the differential operator associated with a cantilever Euler–Bernoulli beam with constant parameters defined by

$$\mathcal{A}w \triangleq -a\frac{\partial^4}{\partial x^4}w$$

is an unbounded, positive, self-adjoint operator with positive eigenvalues μ_j. The resulting modal differential equations are of the form (4) with eigenvalues $\lambda_j = \pm \jmath\sqrt{\mu_j}$. In the case of a slewing beam or flexible robotic manipulator, an additional degree of freedom called "rigid" mode is characterized by the twice repeated eigenvalue $\lambda_1 = 0$.

Finally, in the ideal situation we require the following assumption to hold

$$f(t) \in \text{Range}\{b\} \longrightarrow f(t) = bg(t)$$

which is known as the *Matching Condition* in the Sliding Mode Control literature. It basically restricts the admissible perturbations to those entering the system through the input channels.

In summary then, we are led to consider a linear, time-invariant, controllable system in the Jordan Canonical form

$$\dot{q} = Jq + b(u + g(t)). \tag{7}$$

Due to physical implementation constraints, in the development that follows we consider the finite dimensional version of (7). Consequently, the model retains N modes, q is of dimension n, and each J_j is of size $n_j \geq 1$ such that $n_1+n_2+\ldots+n_N = n$.

3. PHASE CANONIC VARIABLES

The model derived in the previous section is transformed to phase canonic variables (controllable canonic form) for which the manifold design becomes transparent. To that end, we introduce the following two transformations

$$q = T(\alpha)\tilde{q}$$

$$\tilde{q} = V^{-1}z \tag{8}$$

where V is defined below and

$$T^{-1}(\alpha) = \text{blockdiag}\{T_1^{-1}, T_2^{-1}, \dots, T_N^{-1}\}$$

where $T_i^{-1}(\alpha) \in C^{n_i \times n_i}$ are upper triangular matrices of the form

$$T_i^{-1} = \begin{bmatrix} \alpha_{i1} & \alpha_{i2} & \alpha_{i3} & \alpha_{i4} & \dots & \alpha_{i,n_i} \\ 0 & \alpha_{i1} & \alpha_{i2} & \alpha_{i3} & \dots & \alpha_{i,n_i-1} \\ 0 & 0 & \alpha_{i1} & \alpha_{i2} & \dots & \alpha_{i,n_i-2} \\ \vdots & \dots & \dots & \dots & \dots & \vdots \\ 0 & 0 & \dots & \dots & 0 & \alpha_{i1} \end{bmatrix}$$

for blocks corresponding to repeated eigenvalues, or

$$T_i^{-1} = [\alpha_{i1}]$$

for blocks corresponding to distinct eigenvalues.

The new system of equations are found to be

$$\dot{z} = VJV^{-1}z + VT^{-1}bu = Az + e(u + g(t)) \tag{9}$$

where the pair $\{A, e\}$ is in the standard Phase Canonic (controllable) form, that is

$$A = \begin{bmatrix} 0 & 1 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 1 \\ -h_0 & -h_1 & \dots & -h_{n-1} \end{bmatrix}; \qquad e = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}$$

and $\det(\lambda I - A) = \lambda^n + h_{n-1}\lambda^{n-1} + \dots + h_0$. The transformation V in (8) is a Vandermonde matrix of eigenvectors and generalized eigenvectors of A.

Such a construction can be done by selecting the vector α in the transformation $T(\alpha)$ as shown next. The vector b is partitioned as

$$b = [b_1', \ b_2', \ \dots \ , b_N']'$$

where

$$b_k = [b_{k1}, \ b_{k2}, \ b_{k3}, \ \dots, \ b_{k,n_k}]'.$$

Let

$$B = \text{blockdiag}\{B_1, B_2, \dots, B_N\}$$

with each block B_i given by

$$B_i = \begin{bmatrix} b_{i1} & b_{i2} & \dots & b_{i,n_i} \\ b_{i2} & bi3 & \dots & 0 \\ \vdots & \dots & \dots & \vdots \\ b_{i,n_i} & 0 & \dots & 0 \end{bmatrix}$$

Then,

$$\alpha = B^{-1}V^{-1}e \tag{10}$$

The following proposition answers the question of invertibility of matrix B.

Proposition 1. If the pair $\{J, b\}$ is controllable, then matrix B is invertible.

Proof. The necessary and sufficient conditions for the pair $\{J, b\}$ to be controllable are that

- any two blocks J_i and J_j, $i \neq j$, have distinct eigenvalues;
- none of the entries b_{il} in b corresponding to the *last* row of each Jordan block be zero.

It can be easily seen that

$$\det(B) = \prod_i (b_{il})^{n_i}.$$

Thus, controllability ensures solvability of (10). □

The following example should clarify the construction of $T(\alpha)$, V, and the calculation of α. Let

$$J = \begin{bmatrix} \lambda_1 & 0 & 0 & 0 & 0 \\ 0 & \lambda_2 & 0 & 0 & 0 \\ 0 & 0 & \lambda_3 & 1 & 0 \\ 0 & 0 & 0 & \lambda_3 & 1 \\ 0 & 0 & 0 & 0 & \lambda_3 \end{bmatrix}, \quad b = \begin{bmatrix} b_{11} \\ b_{21} \\ b_{31} \\ b_{32} \\ b_{33} \end{bmatrix}. \tag{11}$$

Then, the transformation T is given by

$$T^{-1} = \begin{bmatrix} \alpha_{11} & 0 & 0 & 0 & 0 \\ 0 & \alpha_{21} & 0 & 0 & 0 \\ 0 & 0 & \alpha_{31} & \alpha_{32} & \alpha_{33} \\ 0 & 0 & 0 & \alpha_{31} & \alpha_{32} \\ 0 & 0 & 0 & 0 & \alpha_{31} \end{bmatrix}$$

and the matrix B is given by

$$B = \begin{bmatrix} b_{11} & 0 & 0 & 0 & 0 \\ 0 & b_{21} & 0 & 0 & 0 \\ 0 & 0 & b_{31} & b_{32} & b_{33} \\ 0 & 0 & b_{32} & b_{33} & 0 \\ 0 & 0 & b_{33} & 0 & 0 \end{bmatrix}.$$

The Vandermonde matrix of eigenvectors for the companion form corresponding to matrix J in (11) is given by

$$V = [\, v_1 \; v_2 \; \tilde{v}_1 \; \tilde{v}_2 \; \tilde{v}_3 \,] = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ \lambda_1 & \lambda_2 & \lambda_3 & 1 & 0 \\ \lambda_1^2 & \lambda_2^2 & \lambda_3^2 & 2\lambda_3 & 1 \\ \lambda_1^3 & \lambda_2^3 & \lambda_3^3 & 3\lambda_3^2 & 3\lambda_3 \\ \lambda_1^4 & \lambda_2^4 & \lambda_3^4 & 4\lambda_3^3 & 6\lambda_3^2 \end{bmatrix}.$$

It can be verified that $VJV^{-1} = A$ with A in phase canonic (companion) form.

4. CONTROL MANIFOLD SYNTHESIS

The first step in the control design is to consider a linear, time-varying manifold in the state space of system (9) defined by

$$S = \gamma_1 z_1 + \ldots + \gamma_{n-1} z_{n-1} + z_n - w(t) = 0. \tag{12}$$

We now prove the main result of this article:

Theorem 1. Denote by $P(\lambda)$ a desired, Hurwitz, closed-loop characteristic polynomial written as follows:

$$P(\lambda) = \lambda^{n-1} + \gamma_{n-1}\lambda^{n-2} + \ldots + \gamma_2\lambda + \gamma_1. \tag{13}$$

Consider λ_j an eigenvalue of J and compute the polynomials $\tilde{P}_k$, $k = 2, 3, \ldots, n_j$

$$\tilde{P}_k(\lambda) = \frac{1}{k-1}\frac{\mathrm{d}}{\mathrm{d}\lambda}\tilde{P}_{k-1}(\lambda) = \frac{1}{(k-1)!}\frac{\mathrm{d}^{k-1}}{\mathrm{d}\lambda^{k-1}}\tilde{P}_1(\lambda),$$

$$\tilde{P}_1(\lambda) = P(\lambda).$$

If the control u in system (7) is such that the state q belongs to the time-varying manifold

$$S = \{q \in C^n : Kq(t) - w(t) = 0\} \tag{14}$$

for $t \geq t_1$, then the coefficient vector K is given by

$$K = \left[\underbrace{\tilde{P}_1(\lambda_1)\ \tilde{P}_2(\lambda_1)\ldots}_{n_1}\ \underbrace{\tilde{P}_1(\lambda_2)\ \tilde{P}_2(\lambda_2)\ldots}_{n_2}\ldots\right] T^{-1}. \tag{15}$$

Proof. Since the control u is such that $S = 0$ for $t \geq t_1$, then, from (12) we have

$$z_n = -\gamma_1 z_1 - \ldots - \gamma_{n-1} z_{n-1} + w(t), \tag{16}$$

and the dynamics of the closed loop system restricted to S are described by

$$\begin{aligned} \dot{z}_1 &= z_2 \\ \vdots\ &\ \ \vdots \\ \dot{z}_{n-1} &= -\gamma_1 z_1 - \ldots - \gamma_{n-1} z_{n-1} + w(t). \end{aligned} \tag{17}$$

which indicates that $\gamma_1, \ldots, \gamma_{n-1}$ are the coefficients of the characteristic polynomial (13) for the free motion of the system constrained within the manifold.

Now, let us return to the original state variable q. The function S can be written as

$$S = [\gamma_1, \ldots, \gamma_{n-1}, 1]\begin{bmatrix} z_1 \\ z_2 \\ \cdot \\ \vdots \\ \cdot \\ z_n \end{bmatrix} - w(t) = \gamma' z - w(t). \tag{18}$$

or equivalently,

$$S = \gamma' V T^{-1} q - w(t) \tag{19}$$

from which the result follows as we observe that the product $\gamma' V$ can be written as shown in (15). □

Remark. We show next that (15) is equivalent to Ackermann's Pole Placement formula. For brevity we show this for a second order Jordan system. Ackermann's formula [1] for a gain vector K in the full-state feedback law $u = Kq$ for system (7) can be written as follows

$$K = e' \mathcal{C}^{-1}(J, b)\, P_{\mathrm{des}}(J) \tag{20}$$

where $\mathcal{C}(J, b)$ is the Controllability matrix of the pair (J, b), and P_{des} denotes the desired closed-loop characteristic polynomial. Using the notation in this paper, it is straightforward to verify that

$$\begin{aligned} K &= e'(V')^{-1}(B')^{-1} P_{\mathrm{des}}(J) = \alpha' P_{\mathrm{des}}(J) \\ &= [\alpha_1 \ \alpha_2] \begin{bmatrix} \tilde{P}_1(\lambda_1) & \tilde{P}_2(\lambda_1) \\ 0 & \tilde{P}_1(\lambda_1) \end{bmatrix} = [\tilde{P}_1(\lambda_1) \quad \tilde{P}_2(\lambda_1)]\, T^{-1}. \end{aligned}$$

5. CONTROL DESIGN

Let us assume that the control u in (1) is such that it steers and keeps the state Q on the manifold

$$\begin{aligned} S(t, Q) &= \int_\Omega \sigma(x) Q(t, x)\, \mathrm{d}x - w(t) \\ &= \langle \sigma(x), Q(t, x) \rangle_{L^2} - w(t) = 0. \end{aligned} \tag{21}$$

Then

$$S = \sum_{j=1}^{N} q_j(t)\, \langle \sigma(x), \phi_j(x) \rangle_{L^2} - w(t). \tag{22}$$

Comparing (22), (14), and (15) we obtain that, to guarantee the desired dynamics,

$$\int_\Omega \sigma(x) \phi_j(x)\, \mathrm{d}x = K_j \quad \Longrightarrow \quad \sigma(x) = \sum_{j=1}^{N} K_j \phi_j(x) \tag{23}$$

that is, the function $\sigma(x)$ is easily resolved by invoking the orthogonality property of the eigenfunctions $\phi_j(x)$.

The general expression for the control law can be written as

$$u = -\langle \sigma(x), b(x) \rangle_{L^2}^{-1} \langle \mathcal{A}\sigma(x), Q(t, x) \rangle_{L^2} - g(t) + v(S), \tag{24}$$

where $v(S)$ is any function continuous or discontinuous such that $Sv(S) < 0$. In this case, differentiating (21) and using (1) and (24) we obtain

$$\dot{S} = -v(S) \tag{25}$$

which guarantees convergence $S \to 0$. If $g(t)$ cannot be measured as it usually happens in practice, then let $v(S) = -M sign(S)$. In this case the control law

$$u = -\langle\sigma(x), b(x)\rangle_{L^2}^{-1}\langle\mathcal{A}\sigma(x), Q(t,x)\rangle_{L^2} - M\, sign(S), \tag{26}$$

results in $\dot{S} = -M\, sign(S) + g(t)$ and convergence is guaranteed if $|g| < M$, that is, if the control magnitude M dominates the bound on the disturbance vector g.

5.1. Observer design

The problem of designing an observer can be treated as the dual of the controller design. Due to space limitations however, the details are not included in this paper.

6. A TEMPERATURE FIELD CONTROL PROBLEM

Within the broad class of systems discussed in this article, we consider a *parabolic* partial differential equation in two spatial dimensions modeling the heat diffusion in a plate. The experimental setup we are developing is illustrated in Figure 1.

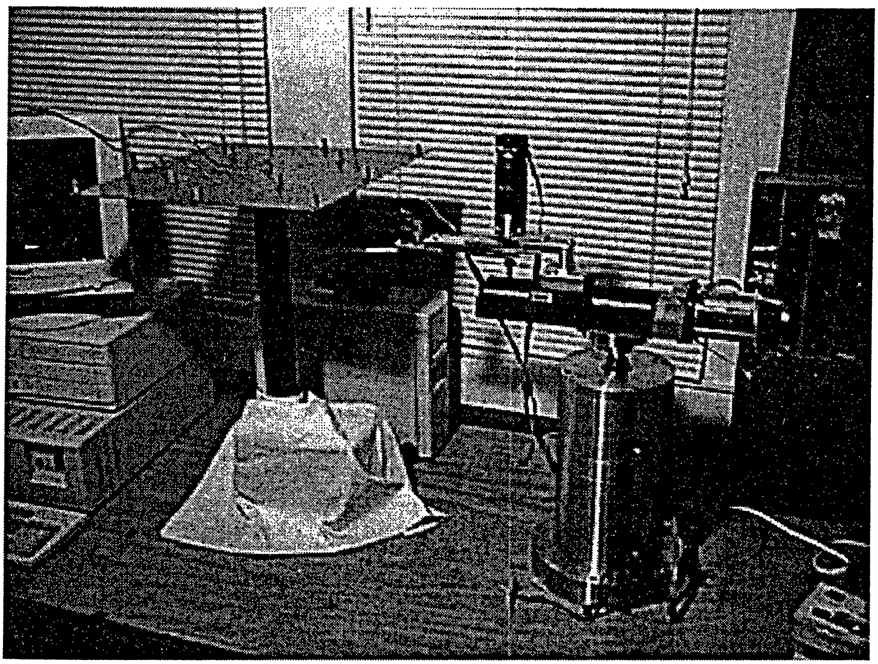

Fig. 1. Experimental testbed.

It consists of a thin square aluminum plate instrumented with 13 thermocouples for temperature measurements. The planar manipulator holds a heating element with its end-effector that is used as the heat input. The motivation is a robotic arc-welding application. The controller is synthesized using the SIMULINK environment and implemented in a DSP board by dSPACE. In this article we only report on simulation results.

The operator $\mathcal{A}$ is the *Laplacian* defined in $L^2(\Omega)$ which, together with Dirichlet-Neumann boundary conditions, is known to be a positive self-adjoint operator with positive (real) eigenvalues λ_j, $j = 1, 2, \ldots$ and normalized eigenfunctions $\phi_j(x)$, $j = 1, 2, \ldots$ forming an orthonormal basis for the Hilbert space $L^2(\Omega)$.

Figure 2 is a Simulink diagram of a controller/observer simulation for a finite dimensional model of a heat equation. The simulation model contains 20 modes (see block DPS with NMODES) with eigenvalues ranging from $\lambda_1 = -0.1$ to $\lambda_{20} = -10$. These numerical values were chosen to facilitate the simulation runs. The controller/observer is designed from a model that includes only the first 10 modes. The output of the simulation model contains the influence of all 20 modes and is used to generate the observation error $y - \hat{y}$ that drives the observer signum block.

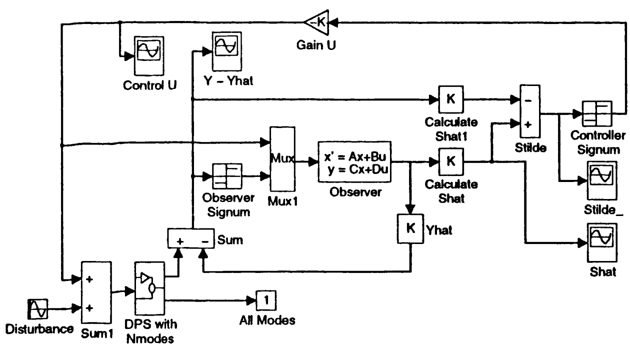

Fig. 2. Simulink diagram of a heat equation with sliding mode controller and observer.

Figures 3 through 6 illustrate a typical simulation run. The plots clearly show that sliding occurs after a finite convergence time and the 20 mode-model remains stable.

6.1. A glimpse at the residual modes

The simulations presented in this section could present a potentially destabilizing effect known as *spillover*. The effect is basically caused by the unmodelled dynamics that are present in the simulation model and that were not included in the controller design. In what follows, we provide a preliminary result along the lines of spillover analysis.

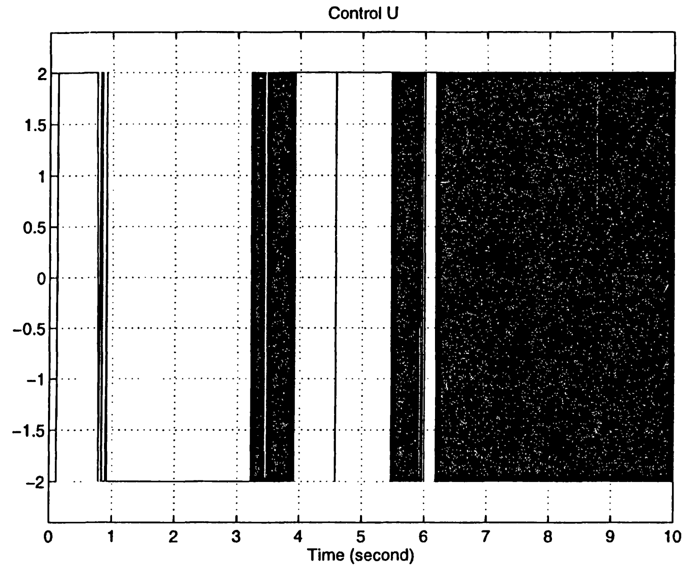

Fig. 3. Control U.

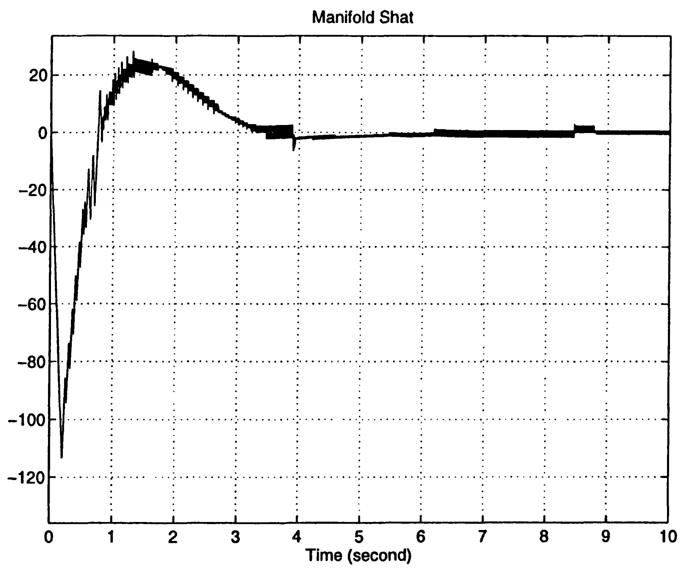

Fig. 4. Plot of Manifold $\hat{S}$ Showing Convergence and sliding.

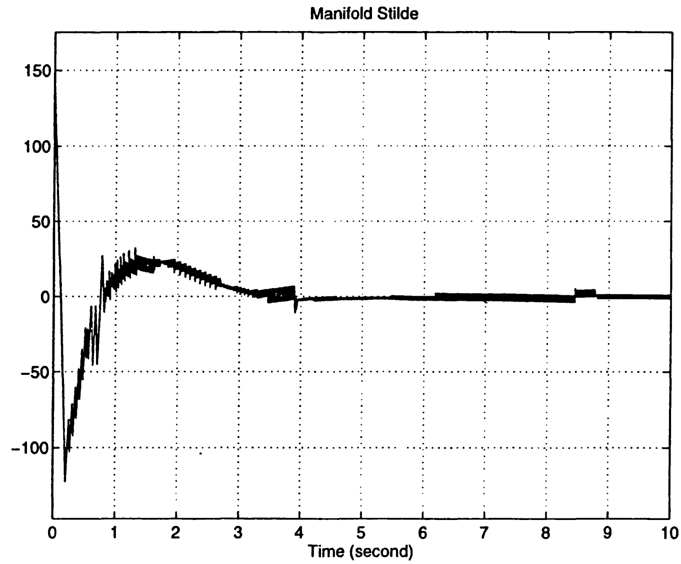

Fig. 5. Plot of Manifold $\tilde{S}$ Showing Convergence and Sliding.

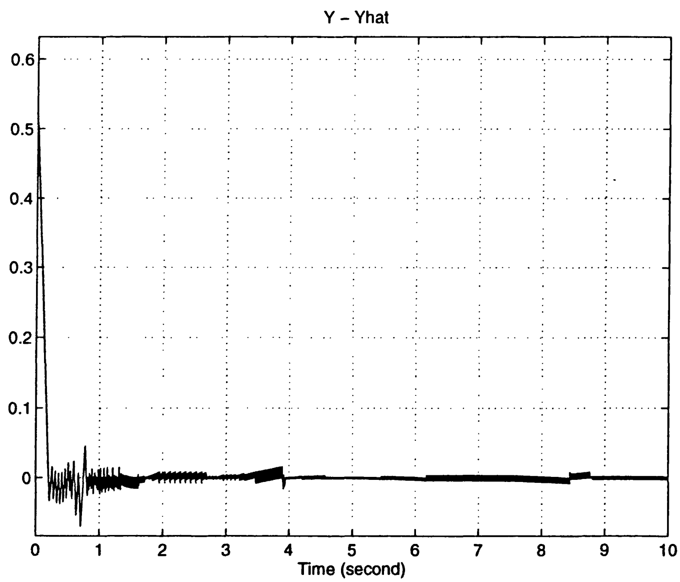

Fig. 6. Plot of $y - \hat{y}$ showing stability.

Consider the following infinite dimensional model

$$\begin{aligned} \dot{X} &= \begin{bmatrix} \Lambda_m & 0 \\ 0 & \Lambda_r \end{bmatrix} \begin{bmatrix} X_m \\ X_r \end{bmatrix} + \begin{bmatrix} b_m \\ b_r \end{bmatrix} (u(t) + g(t)) \\ Y &= [C_m \ C_r] X \end{aligned} \tag{27}$$

where the subscript m denotes modeled and the subscript r denotes residual. Therefore, $X_m \in R^m$ and in practice, one would consider $X_r \in R^r$ where $r \gg m$.

The design of a sliding mode controller/observer pair discussed in the simulations section leads to the following sliding surface dynamics

$$\begin{bmatrix} \dot{\tilde{S}} \\ \dot{\sigma} \end{bmatrix} = \begin{bmatrix} \gamma' b_m + (\gamma' L)(C_m L)^{-1} C_r b_r & 0 \\ C_r b_r & -C_m L \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} + \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} \tag{28}$$

where $\tilde{S}$ is the controller sliding function, σ is the observer sliding function, the respective control/observer inputs are u and v, the observer gain matrix is L, and F_1 and F_2 are disturbance functions.

In order to design a sliding mode controller of the form $u = M\, sign(\tilde{S})$ in the first equation of (28), it is necessary that the *sign* of the coefficient of u be known. Upon examining (28), we establish the following fairly conservative bound that makes the coefficient of u positive. The bound also provides a guideline in the selection of L and of the relative sizes of m and r, that is, the modeled and residual subsystem sizes:

Proposition 2. If

$$||C_r b_r|| < ||(\gamma' L)(C_m L)^{-1}||$$

then the coefficient of u in (28) is positive. Moreover, the observer gain L may be selected in accordance with the following mini-max problem:

$$\min \max \lambda[\ L(C_m L)^{-1} (C_m L)^{-'} L'\ ]$$

where $\lambda(\cdot)$ is an eigenvalue of the indicated matrix.

We observe that the above proposition also gives an indication of how to tackle the sensor placement problem. This direction is currently being investigated.

7. CONCLUSIONS

We have investigated the problems of control and observation designs for a class of distributed parameter systems. In particular, we write the system in the Jordan canonical form and develop a formula for the sliding manifolds. The main result states that the manifolds can be synthesized in terms of the desired closed-loop characteristic polynomial evaluated at the known open-loop eigenvalues. This result was initially reported at the 1997 American Control Conference by Drakunov et al [6] for the special case of a diagonal system matrix. The recent work in 1998 by

Ackermann and Utkin [1] makes a connection between the manifold design problem and Ackermann's formula for eigenvalue placement. The connection between our result and Ackermann's formula is also shown. Simulations are included for a heat equation and a brief analysis of the spillover problem is presented. Further research efforts are underway to (1) obtain experimental results on a square plate temperature control problem; (2) to tackle the sensor placement problem using Proposition 2; and (3) to apply the results of this paper to an arc-welding problem for weld-quality control.

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