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INPUT–OUTPUT DECOUPLING OF NONLINEAR RECURSIVE SYSTEMS

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The input-output decoupling problem is studied for a class of recursive nonlinear systems (RNSs), i. e. for systems, modelled by higher order nonlinear difference equations, relating the input, the output and a finite number of their time shifts. The solution of the problem via regular static feedback known for discrete-time nonlinear systems in state space form, is extended to RNSs. Necessary and sufficient conditions for local solvability of the problem are proposed. This is the alternative to be used when some nonlinear input-outpt models cannot be realized in the state-space form.

1. INTRODUCTION

When designing controllers for unknown (possibly nonlinear) engineering plants, one starts with experimentally obtained input-output data, and little else. The task is to first obtain a mathematical model of the system, and then design the controller. Nonlinear system identification is in itself an active research field, and the most successful identification techniques are currently based on neural networks (NN) [10, 11] models. The field of nonlinear control system synthesis has seen a lot of progress during the last two decades. However, practically all the previous work in this field is concentrated on nonlinear systems having a state space representation [5, 6, 13]. It is interesting to note that despite the large volume of work going on in both nonlinear system identification and controller design fields, there is serious gap between the two.

On one hand, researchers working on controller design assume that the classical state space model is available. Unfortunately, this is not always the case. On the other hand, neural networks researchers in nonlinear system identification have developed sophisticated methods of accurately estimating NN-based NARMA-type models, but haven't paid attention to the fact that these NN models, in general, are not realizable in the classical state space form [5, 6], and consequently, not useful in controller design using existing state space theory. There are two possible ways to fill the above mentioned gap, either to develop a new class of NN-based models that can be realized in the classical state-space form or develop feedback control laws directly for NARMA-type i/o models. Our goal is to explore the second possibility on the example of the input-output decoupling problem.

To the best of our knowledge, there exist only very few papers [1, 7, 8, 11] dealing with the synthesis problems for nonlinear systems described by recursive i/o equations. This is in remarkable contrast with the wide application of linear input-output difference equations in digital control. The success of the *linear* control motivates us to search for new feedback design methods that can directly applied to nonlinear i/o models.

A system is said to be input-output decoupled if each of its inputs influences one and only of its outputs. In the case the system does not possess the above property, one may try to satisfy this property via feedback compensator. During the last 20 years, a lot of progress has been made in the solution of the input-output decoupling problem (IODP), both via the static and the dynamic feedback (see [5, 6, 13] for an overview). However, all previous work on this subject has concentrated on systems having a state space representation.

The purpose of this paper is to study the IODP for nonlinear system described by a set of higher order difference equations relating the inputs, the outputs, and a finite number of their time shifts:

$$y(t) = F(y(t-1), \dots, y(t-\mu), u(t-1), \dots, u(t-\nu)). \tag{1}$$

Systems of the form (1) are called recursive nonlinear systems (RNS) [4, 10] or alternatively, NARMA-type models [2, 11, 12].

In our study we follow the approach used in the state space formulation, where the IODP is closely related to the right invertibility problem and where the necessary and sufficient condition for local solvability of the IODP via regular static state feedback was nonsingularity of the so-called decoupling matrix. We shall show that the above result has its direct counterpart for the RNS. We concentrate on the local solutions around an equilibrium point of the system. However, the paper does not give any algorithm which explicitly constructs the solution; it only presents necessary and sufficient conditions under which the feedback locally exists and describes how to obtain it. Solution given in the paper relies on the application of the implicit function theorem.

2. DESCRIPTION OF RECURSIVE NONLINEAR SYSTEM

In this section, besides recalling the notion of recursive nonlinear system [1, 4], we establish some notations and introduce some preliminary material.

We denote by $S(R^m)$ the set of all two-sided infinite sequences of the form

$$\{z(t)\} = (\dots, z(-1), z(0), z(1), z(2), \dots)$$

where $z(t) \in R^m$ for all integers t.

Dynamical system. A dynamical system is a map

$$\Sigma : S(R^m) \mapsto S(R^p) : \{u(t)\} \mapsto \{y(t)\}$$

which transforms input sequence $\{u(t)\}$ into output sequence $\{y(t)\}$.

Given two systems $\Sigma_1 : S(R^m) \mapsto S(R^p)$ and $\Sigma_2 : S(R^p) \mapsto S(R^q)$, we denote by $\Sigma_2 \circ \Sigma_1 : S(R^m) \mapsto S(R^q)$ the system represented by the composite map.

A finite subsequence of the infinite sequence $\{z(t)\}$ between time instances t and $t - \tau$ stacked in the column vector is denoted by

$$Z(t, t-\tau) = (z^T(t), z^T(t-1), \ldots, z^T(t-\tau))^T, \quad \tau \geq 0.$$

If $\tau \leq 0$, it is understood that $Z(t, t-\tau)$ denotes an empty subsequence of $\{z(t)\}$.

If for every input sequence $\{u(t)\}$, the corresponding output sequence $\{y(t)\}$ of the system Σ satisfies the equation

$$y(t) = F(Y(t-1, t-\mu), U(t-1, t-\nu)) \tag{2}$$

where $F : R^{\mu p + \nu m} \mapsto R^p$ is a C^ω map and $1 \leq \mu < \infty$, $1 \leq \nu < \infty$, then the system Σ is said to have a causal finite dimensional realization.

Definition 2.1. (Recursive nonlinear system.) Recursive nonlinear system (RNS) is a system which has a causal finite dimensional realization of the form (2).

Definition 2.2. (Equilibrium point.) The pair of constant values (u^0, y^0) is called the equilibrium point of the recursive nonlinear system (2) if (u^0, y^0) satisfies the equality $y^0 = F(Y^0, U^0)$ where $Y^0 = (y^{0,T}, \ldots, y^{0,T})^T, U^0 = (u^{0,T}, \ldots, u^{0,T})^T$.

From now on, we consider RNS (2) at nonnegative time steps in a finite time interval $0 \leq t \leq t_F$ under the initial conditions

$$x(0) = \begin{bmatrix} Y(-1, -\mu) \\ U(-1, -\nu) \end{bmatrix} = \begin{bmatrix} (y^T(-1) \ldots y^T(-\mu))^T \\ (u^T(-1) \ldots u^T(-\nu))^T \end{bmatrix}.$$

Then the system (2) has as inputs the sequence $\boldsymbol{u} = \{u(t); 0 \leq t \leq t_F\}$.

Throughout the paper we shall adopt a local viewpoint. More precisely, we work around an equilibrium point (u^0, y^0) of the system (2). Let us denote by $\mathcal{U}^0$ (resp. $\bar{U}$) the set of control sequences $\boldsymbol{u} = \{u(t); 0 \leq t \leq t_F\}$ (resp. $U(t-1, t-\nu)$) such that the controls $u(t)$ for every t are sufficiently close to u^0, i.e. that $||u(t) - u^0|| \leq \delta$ for some $\delta > 0$. Analogously, let us denote by $\mathcal{Y}^0$ (resp. $\bar{Y}$) the set of output sequences $\{y(t); 0 \leq t \leq t_F\}$ (resp. $Y(t-1, t-\mu)$) such that the outputs $y(t)$ for every t are sufficiently close to y^0, i.e. that $||y(t) - y^0|| < \epsilon$ for some $\epsilon > 0$. Denote by x^0 a $(\mu p + \nu m)$-dimensional vector $(y^{0,T}, \ldots, y^{0,T}, u^{0,T}, \ldots, u^{0,T})^T$. Finally, let us denote by X^0 the neighbourhod of x^0 such that for every $x \in X^0, ||x - x^0|| < \gamma$ for some $\gamma > 0$.

For difference equation (2) under initial conditions $x(0)$, as long as F is a well-defined function of $R^{\mu p + \nu m}$, there is no problem regarding existence and uniqueness of its solution $y(t; 0 \leq t \leq t_F)$, for arbitrary control sequence $\boldsymbol{u} \in \mathcal{U}^0$ and arbitrary initial condition $x(0) \in X^0$. Such a solution will be denoted as $y(t, x(0), \boldsymbol{u})$ which is a shorthand writing for $y(t, x(0), u(0), \ldots . u(t-1))$.

3. THE DELAY ORDERS WITH RESPECT TO THE CONTROL AND THE CONCEPT OF FORWARD TIME-SHIFT RIGHT INVERTIBILITY

For recursive nonlinear systems, the delay orders $d_i, i = 1, \dots, p$, with respect to the control have been defined [7], one for each output component. These system structural parameters tell us how many inherent delays there are between the ith component y_i of the output and the control, or equivalently, for how many first time instances y_i is completely defined by the initial conditions and which is the first time instant for which the possibility arises to change y_i arbitrarily.

A RNS (2) with delay orders $d_i, i = 1, \dots, p$ admits a representation of the form [7]

$$\begin{array}{rcl} y_1(t+d_1) & = & F_1^{d_1}(Y(t-1,t-\mu), U(t-1,t-\nu), u(t)) \\ & \vdots & \\ y_p(t+d_p) & = & F_p^{d_p}(Y(t-1,t-\mu), U(t-1,t-\nu), u(t)) \end{array} \tag{3}$$

or in the vector form

$$\begin{bmatrix} y_1(t+d_1) \\ \vdots \\ y_p(t+d_p) \end{bmatrix} = A(x(t), u(t)). \tag{4}$$

where

$$x(t) = \begin{bmatrix} Y(t-1,t-\mu) \\ U(t-1,t-\nu) \end{bmatrix}.$$

Definition 3.1. ($(d_1, \dots, d_p)$-FTS right invertibility.) The RNS (2) is called locally $(d_1, \dots, d_p)$-forward time shift right invertible in a neighbourhood of its equilibrium point (u^0, y^0) if there exist sets $\mathcal{U}^0$, $\mathcal{Y}^0$ and X^0 such that given $x(0) \in X^0$, we are able to find for any sequence $\{y_{\text{ref}}(t); 0 \le t \le t_F\} \in \mathcal{Y}^0$ a control sequence $\{u_{\text{ref}}(t); 0 \le t \le t_F\} \in \mathcal{U}^0$ (not necessarily unique) yielding

$$y_i(t, x(0), u_{\text{ref}}(0), \dots, u_{\text{ref}}(t)) = y_{\text{ref},i}(t),$$

$$d_i \le t \le t_F, i = 1, \dots, p.$$

Denote by $\mathcal{Y}_i^0$ the set of sequences $\{y_i(t); 0 \le t \le t_F\} \in \mathcal{Y}_i^0$.

Then the above definition says that for the ith output component it is possible to reproduce locally all sequences $\mathbf{y}_{\text{ref},i}$ from $\mathcal{Y}_i^0$, beginning from time instant d_i. But $(d_1, \dots, d_p)$-FTS right invertibility does not allow us to reproduce the first d_i terms in the arbitrary sequence $\{y_{\text{ref},i}(t); 0 \le t \le t_F\} \in \mathcal{Y}_i^0$.

Consider the RNS (2) with delay orders $d_i < \infty$, $i = 1, \dots, p$, i.e. the system, described by equations (3). We introduce the so-called decoupling matrix $K(x, u)$ for the system (2) in the following way

$$K(x, u) = \frac{\partial}{\partial u} \begin{bmatrix} F_1^{d_1}(x, u) \\ \dots \\ F_p^{d_p}(x, u) \end{bmatrix}.$$

From the definition of the d_i's the rows of the matrix $K(x,u)$ are nonzero vector functions around (u^0, y^0). It is obvious that the rank of $K(x,u)$ is, in general, input and output dependent. However, we shall assume that $K(x,u)$ has a constant rank around (u^0, y^0). This assumption is formalized in the notion of regularity of an equilibrium point.

Definition 3.2. (Regularity of an equilibrium point.) We call the equilibrium point (u^0, y^0) of the system (2) regular with respect to $(d_1, \ldots, d_p)$-FTS right invertibility, if the rank of the decoupling matrix $K(x,u)$ of the system (2) is constant around (u^0, y^0).

Theorem 3.3. [7] Assume that for the system (2) $d_i < \infty\,,\ i = 1, \ldots, p$. Then the RNS (2) is locally $(d_1, \ldots, d_p)$-forward time-shift right invertible around a regular equilibrium point (u^0, y^0) if and only if rank $K(x^0, u^0) = p$.

4. THE FORMULATION AND THE SOLUTION OF THE INPUT–OUTPUT DECOUPLING PROBLEM (IODP) VIA STATIC FEEDBACK

The system with $m = p$ is said to be locally input-output (I/O) decoupled, if the decoupling matrix $K(x,u)$ is diagonal nonsingular matrix for all (x,u) in a neighbourhood of an equilibrium point (u^0, y^0) and if $(\partial y_i(k)/\partial u_j(0)) = 0$ for $j \neq i$ and $k \geq d_i + 1$. In the case the system does not possess the above property one may try to satisfy this property via feedback compensator. In this paper we are looking for a static feedback C, with a new m-dimensional control v, described by equations of the form

$$u(t) = \alpha(x(t), v(t)) = \alpha(Y(t-1, t-\mu), U(t-1, t-\nu), v(t)) \tag{5}$$

defined locally around (to be found) a point (x^0, v^0, u^0) such that $u^0 = \alpha(x^0, v^0)$.

We call the compensator C described by equation (5) regular, if the matrix $\partial\alpha(x,v)/\partial v$ is nonsingular around a point (x^0, v^0, u^0).

The closed-loop system (2), (5), initialized at x_0, that is the system

$$\begin{bmatrix} y_1(t+d_1) \\ \vdots \\ y_p(t+d_p) \end{bmatrix} = A(x(t), \alpha(x(t), v(t))) \tag{6}$$

is denoted by $S \circ C$.

Definition 4.1. (Local static input-output decoupling problem.) Given the system (2) around a regular equilibrium point (x^0, u^0), find if possible, a regular static feedback C, defined by equations of the form (5) together with a point (x^0, v^0, u^0) and neighbourhoods $O = X^0 \times V^0$ of (x^0, v^0) in $X \times V$ and U^0 of u^0 in U, being the domain and the range of C so that the closed-loop system $S \circ C$, described by (2), (5) is locally input-output decoupled on $O \times U^0$, for all $0 \leq t \leq t_F$.

The following theorem holds.

Theorem 4.2. The system (2) with $m = p$ is locally around a regular equilibrium point (x^0, u^0) I/O decouplable by regular static feedback of the form (5), if and only if the system (2) is locally $(d_1, \ldots, d_p)$-forward time-shift right invertible around (u^0, y^0).

Proof. *Sufficiency.* Suppose the system (2) is locally $(d_1, \ldots, d_p)$-forward time-shift right invertible around the regular equilibrium point (u^0, y^0). Then the decoupling matrix $K(x, u)$ has rank p around the point (x^0, u^0). Consider the equation

$$\begin{bmatrix} v_1(t) \\ \vdots \\ v_p(t) \end{bmatrix} = A(x(t), u(t)). \tag{7}$$

Observe, that the Jacobian matrix of the right hand side of (7) with respect to $u(t)$ equals $K(x(t), u(t))$. So we may apply the Implicit Function Theorem yielding locally $u(t)$ as an analutic function of $x(t)$ and $v(t)$, i. e.

$$u(t) = \alpha(x(t), v(t)) \tag{8}$$

and which is such that

$$\begin{bmatrix} v_1(t) \\ \vdots \\ v_p(t) \end{bmatrix} = A(x(t), \alpha(x(t), v(t))). \tag{9}$$

Now we apply the local feedback (8) to (2) yielding the feedback modified system

$$\begin{aligned} y_1(t + d_1) &= v_1(t) \\ &\vdots \\ y_p(t + d_p) &= v_p(t). \end{aligned} \tag{10}$$

Necessity. Suppose the I/O decoupling problem for system (2) is locally solvable in a neighbourhood of the regular equilibrium point (x^0, u^0). It means that there exists a regular feedback

$$u(t) = \alpha(x(t), v(t)) \tag{11}$$

defined around the points (x^0, v^0) and u^0 (with v^0 as the solution of the equation $u^0 = \alpha(x^0, v^0)$) such that the feedback modified system (6) is locally I/O decoupled. This implies that the feedback modified system (6) has a diagonal nonsingular decoupling matrix $\tilde{K}(x, v)$.

But we have also that (see Lemma 4.3 below)

$$\tilde{K}(x, v) = K(x, u)\Big|_{u=\alpha(x,v)} \frac{\partial \alpha(x, v)}{\partial v}. \tag{12}$$

Since the matrix $\partial\alpha/\partial v$ is nonsingular by regularity of the feedback, and each row of $K(x, u)$ is nonzero by construction, (12) implies that $K(x, u)$ is nonsingular. □

Lemma 4.3. Consider the system (2) around an equilibrium point (x^0, u^0), and let $u = \alpha(x, v)$ be an arbitrary analytic state feedback defined around the points (x^0, v^0) and u^0. Then around (x^0, u^0, v^0) we have that

$$\tilde{K}(x, v) = K(x, u)\Big|_{u=\alpha(x,v)} \frac{\partial}{\partial v}\alpha(x, v) \tag{13}$$

where $\tilde{K}(x, v)$ is the decoupling matrix of the feedback modified system (6).

Proof. The proof is straightforward. The ith row of the decoupling matrix $K(x, u)$ is determined as

$$\frac{\partial}{\partial u} F_i^{d_i}(x, u).$$

On the other hand, by (6) the ith row of $\tilde{K}(x, v)$ is determined as

$$\frac{\partial}{\partial v} F_i^{d_i}(x, \alpha(x, v)) = \frac{\partial}{\partial u} F_i^{d_i}(x, u)\Big|_{u=\alpha(x,v)} \frac{\partial}{\partial v}\alpha(x, v).$$

These two expressions yield (13). □

Remark. We should like to stress that the assumption of regularity of the equilibrium point (u^0, y^0) in Theorem 4.2 is extremely vital. If the point (u^0, y^0) is not regular, that is around the point (u^0, y^0) the rank of the decoupling matrix $K(x, u)$ is not necessarily constant, then (like in the case of the state space representation) the condition $K(x, u) = p$ is not necessary for local I/O decoupling.

5. EXAMPLES

Example 1. Consider the system

$$\begin{aligned} y_1(t) &= y_1(t-3)\, y_1(t-2)\, u_1(t-3) + y_1^2(t-3)\, u_1(t-1) \\ y_2(t) &= u_2(t-1) + u_1(t-1)\, y_1(t-3). \end{aligned}$$

The delay orders $d_1 = d_2 = 1$, since

$$\begin{aligned} y_1(t+1) &= y_1(t-2)\, y_1(t-1)\, u_1(t-2) + y_1^2(t-2)\, u_1(t) \\ y_2(t+1) &= u_2(t) + u_1(t)\, y_1(t-2). \end{aligned}$$

and the IODP is solvable around an equilibrium point (u^0, y^0) such that $y_1^0 \neq 0$. In order to obtain the equations of the compensator (see the proof of the theorem), we have to solve the system of equations

$$\begin{aligned} v_1(t) &= y_1(t-2)\, y_1(t-1)\, u_1(t-2) + y_1^2(t-2)\, u_1(t) \\ v_2(t) &= u_2(t) + u_1(t)\, y_1(t-2). \end{aligned}$$

for $u(t)$:

$$\begin{aligned} u_1(t) &= [v_1(t) - y_1(t-2)\, y_1(t-1)\, u_1(t-2)]/y_1^2(t-2) \\ u_2(t) &= v_2(t) - [v_1(t) - y_1(t-2)\, y_1(t-1)\, u_1(t-2)]/y_1(t-2). \end{aligned}$$

Applying this compensator we get

$$\begin{aligned} y_1(t+1) &= v_1(t) \\ y_2(t+1) &= v_2(t). \end{aligned}$$

We conjecture (though we are not able to give a proof at moment since the the necessary and sufficient realizability conditions are still missing for multi-input multi-output nonlinear systems), that the system is not realizable in the classical state space form.

6. CONCLUSIONS

The local input-output decoupling problem is studied for a class of recursive nonlinear systems (RNSs), i. e. for systems, modelled by higher order nonlinear difference equations relating the input, the output and a finite number of their time shifts. The solution of the problem via the regular static feedback known for the discrete-time nonlinear systems in the state space form, is extended to the RNSs. The necessary and sufficient conditions for local solvability of the problem are proposed. This is the alternative to be used when some nonlinear input-output models cannot be realized in state-space form.

Of course, it would be extremely interesting to relate the concepts and the results obtained in this paper to the known concepts and results of nonlinear systems in the state space form, by first realizing the RNS in a state space representation. Unfortunately, the restrictive integrability conditions imply that RNSs which can be transformed into the classical state space form exhibit a very special structure. In general, there are structural obstructions for obtaining Kalmanian realizations. Since linear recursive systems always admit classical state space realizations, these obstructions are typical nonlinear phenomena. Since the RNSs, in general, can not be realized by the standard state equations [3, 9], this comparison is not an easy task and can be done only for a subclass of realizable systems. As the necessary and sufficient realizability conditions for multi-input multi-output systems are still missing, we leave this topic for the future research.

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