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Francesco Amato

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STABILITY ANALYSIS AND SYNTHESIS OF SYSTEMS SUBJECT TO NORM BOUNDED, BOUNDED RATE UNCERTAINTIES

FRANCESCO AMATO

In this paper we consider a linear system subject to norm bounded, bounded rate time-varying uncertainties. Necessary and sufficient conditions for quadratic stability and stabilizability of such class of uncertain systems are well known in the literature. Quadratic stability guarantees exponential stability in presence of arbitrary time-varying uncertainties; therefore it becomes a conservative approach when, as it is the case considered in this paper, the uncertainties are slowly-varying in time. The first contribution of this paper is a sufficient condition for the exponential stability of the zero input system; such condition, which takes into account the bound on the rate of variation of the uncertainties, results to be a less conservative analysis tool than the quadratic stability approach. Then the analysis result is used to provide an algorithm for the synthesis of a controller guaranteeing closed loop stability of the uncertain forced system.

1. INTRODUCTION

In the past decade a big effort has been spent to study the stability robustness problem for linear systems subject to time-varying uncertainties taking values in a given compact set.

The classical *quadratic stability* test (see the pioneering work [3] and the tutorial paper [5]) guarantees, if satisfied, exponential stability of the system under consideration for *arbitrary* time variation of the uncertainties within their value set; such test can be reduced to a feasibility problem involvig Linear Matrix Inequalities (LMIs) (see [4]) for systems subject to *norm bounded* uncertainties (that is systems in the form $\dot{x}(t) = (A + F\Delta(t)\,E)\,x(t)$ with $||\Delta(t)|| \leq 1$) and for systems depending affinely on *parametric* uncertainties (that is systems in the form $\dot{x}(t) = A(p(t))\,x(t)$ with $A(\cdot)$ affine).

When, as often it happens in real situations, the rate of variation of the uncertainties is bounded the quadratic stability approach is clearly conservative. In this context and for systems depending on parametric uncertainties, it has been shown (see [2] and the bibliography therein) that the use of parameter dependent Lyapunov functions, which allows to take into account the rate of variation of parameters, attains less conservative results.

The aim of this paper is that of considering the "bounded rate" problem for systems subject to norm bounded uncertainties. As for systems depending on parametric uncertainties, to perform the stability analysis we shall use Lyapunov functions depending on the uncertainty matrix; this will lead in Section 3 to a sufficient condition (Theorem 1) for exponential stability in terms of a feasibility problem involving LMIs.

Then the analysis result contained in Theorem 1 will be used in a synthesis context in Section 4. However the design problem requires the solution of an optimization problem whose constraints are Bilinear Matrix Inequalities (BMIs). BMIs problems are guaranteed to converge but not necessarily to the global optimum (like the LMIs based problems) and are dependent on the initial data; however there exist efficient algorithms which work well in many situations (see [6], [10] and the bibliography therein).

An example to clarify the application and the effectiveness of the proposed technique will be provided at the end of Section 4. Finally some concluding remarks will be given in Section 5.

This work is an extended version of the conference paper [1].

2. PROBLEM STATEMENT

Let us consider the following linear system subject to norm bounded, bounded rate uncertainties

$$\dot{x}(t) = (A + F\Delta(t)\, E)\, x(t) \tag{1}$$

where $A \in \mathbb{R}^{n\times n}$ is strictly Hurwitz, $F \in \mathbb{R}^{n\times m}$, $E \in \mathbb{R}^{r\times n}$ and

$$\Delta(\cdot) \in \mathcal{D} := \left\{\Delta(\cdot) \in \mathcal{C}^1_{m\times r}\,,\ \Delta^T(t)\Delta(t) \leq I\,,\ \dot{\Delta}^T(t)\dot{\Delta}(t) \leq D\,,\ t \in [0,+\infty)\right\}\,. \tag{2}$$

In (2) $\mathcal{C}^1_{m\times r}$ denotes the set of continuously differentiable matrix-valued functions taking values in $\mathbb{R}^{m\times r}$ and D is a given positive semidefinite matrix.

The term "norm bounded" follows from the fact that the uncertainties satisfy the normalized bound (the scaling factors are included in the matrices F and E)

$$\Delta^T(t)\Delta(t) \leq I\,; \tag{3}$$

the term "bounded rate" follows from the condition

$$\dot{\Delta}^T(t)\dot{\Delta}(t) \leq D \tag{4}$$

which represents an upper bound on the rate of variation of $\Delta(\cdot)$.

Definition 1. [3] System (1) is said to be quadratically stable if there exists a positive definite symmetric matrix P such that for all Δ with $\Delta^T\Delta \leq I$

$$(A + F\Delta E)^T P + P(A + F\Delta E) < 0. \tag{5}$$

It is readily seen that quadratic stability guarantees exponential stability of system (1) for arbitrary time-varying uncertainties satisfying the bound (3). The following result is well known in the literature (see [5] and the bibliography therein).

Theorem 1. System (1) is quadratically stable *if and only if* there exists a symmetric matrix P which satisfies

$$P > 0 \tag{6a}$$

$$\begin{pmatrix} A^T P + PA + E^T E & PF \\ F^T P & -I \end{pmatrix} < 0. \tag{6b}$$

Therefore the quadratic stability test can be reduced to a feasibility problem involving the Linear Matrix Inequalities (LMIs) constraints (6); such feasibility problem can be easily solved by one of the algorithms proposed in [4].

The simplicity of the quadratic stability test, rendered this approach quite popular in the control community in the last years. At the same time it is clear that such approach *is conservative* when the rate of variation of the uncertainties is bounded as in the case of this paper.

The first goal of this paper is stated in the following problem

Problem 1. (Analysis) Find a sufficient condition for the exponential stability of system (1) for all $\Delta(\cdot) \in \mathcal{D}$

Now consider the forced system

$$\dot{x}(t) = (A + F\Delta(t)\,E)\,x(t) + Bu(t)\,. \tag{7}$$

Problem 2. (Synthesis) Find a state feedback controller

$$u = Kx \tag{8}$$

such that the closed loop system (7) – (8) is exponentially stable for all $\Delta(\cdot) \in \mathcal{D}$.

3. MAIN RESULT: ANALYSIS

The stability analysis will be performed with the aid of parameter dependent Lyapunov functions in the form

$$v(x, \Delta) = x^T P(\Delta)\,x \tag{9}$$

where $P(\cdot)$ is required to be a continuously differentiable matrix-valued function.

We consider the following structure for the matrix-valued function $P(\cdot)$ in (9)

$$P(\Delta) = P_0 + N\Delta E. \tag{10}$$

In (10) $P_0 \in \mathbb{R}^{n\times n}$ and $N \in \mathbb{R}^{n\times m}$ are left free for optimization purposes. P_0 is required to be a symmetric matrix; note, however, that $P(\Delta)$ is not symmetric. Moreover note that, in some sense, the Lyapunov function (9) – (10) extends the structure of the Lur'è–Postnikov Lyapunov function (see [9]) to the full matrix case.

The following theorem is the first main result of the paper.

Theorem 2. System (1) is exponentially stable for all $\Delta(\cdot) \in \mathcal{D}$ if there exist

i) A symmetric matrix $P_0 \in \mathbb{R}^{n\times n}$;

ii) A matrix $N \in \mathbb{R}^{n\times m}$;

iii) Nonnegative scalars τ_0, τ_1, τ_2, τ_3, τ_4

such that

$$\begin{pmatrix} P_0 - \tau_0 E^T E & \frac{1}{2}N \\ \frac{1}{2}N^T & \tau_0 I \end{pmatrix} > 0 \tag{11a}$$

$$\begin{pmatrix} L_0 + \tau_1 E^T E + \tau_2 A^T E^T E A + \tau_4 E^T D E & \frac{1}{2}A^T N + P_0 F & \frac{1}{2}N & \frac{1}{2}N & \frac{1}{2}N \\ \frac{1}{2}N^T A + F^T P_0 & L_1 - \tau_1 I + \tau_3 F^T E^T E F & O & O & O \\ \frac{1}{2}N^T & O & -\tau_2 I & O & O \\ \frac{1}{2}N^T & O & O & -\tau_3 I & O \\ \frac{1}{2}N^T & O & O & O & -\tau_4 I \end{pmatrix} < 0 \tag{11b}$$

where

$$L_0 = A^T P_0 + P_0 A \tag{12a}$$

$$L_1 = \frac{1}{2}F^T N + \frac{1}{2}N^T F. \tag{12b}$$

Proof. First we note that system (1) can be rewritten as follows

$$\dot{x} = Ax + Fu \tag{13a}$$

$$y = Ex \tag{13b}$$

$$u = \Delta y. \tag{13c}$$

Positive definiteness of v. Now we show that (11a) implies that for all Δ with $\|\Delta\| \le 1$

$$v(x,\Delta) = x^T (P_0 + N\Delta E)\, x > 0. \tag{14}$$

We have, using the fact that $u = \Delta E x$,

$$v(x,\Delta) = x^T P_0 x + x^T N u \tag{15}$$

therefore $v(x,\Delta) > 0$ for all Δ with $\|\Delta\| \le 1$ if

$$\begin{pmatrix} x^T & u^T \end{pmatrix} \begin{pmatrix} P_0 & \frac{1}{2}N \\ \frac{1}{2}N^T & O \end{pmatrix} \begin{pmatrix} x \\ u \end{pmatrix} > 0\,, \quad \text{for all } x \neq 0 \text{ and } u = \Delta E x\,,\ \|\Delta\| \le 1. \tag{16}$$

We have that (16) is satisfied if and only if

$$\begin{pmatrix} x^T & u^T \end{pmatrix} \begin{pmatrix} P_0 & \frac{1}{2}N \\ \frac{1}{2}N^T & O \end{pmatrix} \begin{pmatrix} x \\ u \end{pmatrix} > 0 \tag{17}$$

for all pair (x, u) satisfying

$$\begin{pmatrix} x^T & u^T \end{pmatrix} \begin{pmatrix} E^T E & O \\ 0 & -I \end{pmatrix} \begin{pmatrix} x \\ u \end{pmatrix} \geq 0. \tag{18}$$

In [4], p. 24 it is shown that (17) and (18) are equivalent to the existence of a nonnegative scalar τ_0 satisfying condition (11a).

Negative definiteness of $\dot{v}$. We have that

$$\begin{aligned} \dot{v} &= \dot{x}^T (P_0 + N\Delta E)\, x + x^T (P_0 + N\Delta E)\dot{x} + x^T N\dot{\Delta} E x \\ &= (Ax + Fu)^T (P_0 x + Nu) + (x^T P_0 + x^T N\Delta E)(Ax + Fu) + x^T N\dot{\Delta} E x \\ &= x^T \left(A^T P_0 + P_0 A\right) x + x^T P_0 F u + u^T F^T P_0 x + x^T A^T N u + u^T F^T N u \\ &\qquad + x^T N v + x^T N w + x^T N z \end{aligned} \tag{19}$$

where we have used the fact that $u = \Delta E x$ and let

$$v = \Delta E A x \tag{20a}$$

$$w = \Delta E F u \tag{20b}$$

$$z = \dot{\Delta} E x. \tag{20c}$$

Therefore we have that $\dot{v} < 0$ if

$$\begin{pmatrix} x \\ u \\ v \\ w \\ z \end{pmatrix}^T \begin{pmatrix} A^T P_0 + P_0 A & \frac{1}{2} A^T N + P_0 F & \frac{1}{2} N & \frac{1}{2} N & \frac{1}{2} N \\ \frac{1}{2} N^T A + F^T P_0 & \frac{1}{2} F^T N + \frac{1}{2} N^T F & O & O & O \\ \frac{1}{2} N^T & O & O & O & O \\ \frac{1}{2} N^T & O & O & O & O \\ \frac{1}{2} N^T & O & O & O & O \end{pmatrix} \begin{pmatrix} x \\ u \\ v \\ w \\ z \end{pmatrix} < 0\,, \tag{21}$$

for all $x \neq 0$ and $u = \Delta E x$, $v = \Delta E A x$, $w = \Delta E F u$ with $\|\Delta\| \leq 1$ and $v = \dot{\Delta} E x$ with $\dot{\Delta}^T \dot{\Delta} \leq D$.

Again we have that $\dot{v}$ is negative definite for all $\Delta(\cdot) \in \mathcal{D}$ if (21) holds for all five-tuple (x, u, v, w, z) satisfying

$$\begin{pmatrix} x^T & u^T & v^T & w^T & z^T \end{pmatrix} \begin{pmatrix} -E^T E & O & O & O & O \\ O & I & O & O & O \\ O & O & O & O & O \\ O & O & O & O & O \\ O & O & O & O & O \end{pmatrix} \begin{pmatrix} x \\ u \\ v \\ w \\ z \end{pmatrix} \leq 0 \tag{22a}$$

$$\begin{pmatrix} x^T & u^T & v^T & w^T & z^T \end{pmatrix} \begin{pmatrix} -A^T E^T E A & O & O & O & O \\ O & O & O & O & O \\ O & O & I & O & O \\ O & O & O & O & O \\ O & O & O & O & O \end{pmatrix} \begin{pmatrix} x \\ u \\ v \\ w \\ z \end{pmatrix} \leq 0 \tag{22b}$$

$$
\begin{pmatrix} x^T & u^T & v^T & w^T & z^T \end{pmatrix}
\begin{pmatrix}
O & O & O & O & O \\
O & -F^TE^TEF & O & O & O \\
O & O & O & O & O \\
O & O & O & I & O \\
O & O & O & O & O
\end{pmatrix}
\begin{pmatrix} x \\ u \\ v \\ w \\ z \end{pmatrix} \leq 0 \quad (22c)
$$

$$
\begin{pmatrix} x^T & u^T & v^T & w^T & z^T \end{pmatrix}
\begin{pmatrix}
-E^TDE & O & O & O & O \\
O & O & O & O & O \\
O & O & O & O & O \\
O & O & O & O & O \\
O & O & O & O & I
\end{pmatrix}
\begin{pmatrix} x \\ u \\ v \\ w \\ z \end{pmatrix} \leq 0. \quad (22d)
$$

Again in [4] it is shown that (21) and (22) are implied by the existence of nonnegative scalars τ_1, τ_2, τ_3 and τ_4 satisfying condition (11b). □

By virtue of Theorem 2, Problem 1 is solvable if the following *LMIs based* [4] feasibility problem admits a solution.

Problem 3. Find a symmetric matrix $P_0 \in \mathbb{R}^{n\times n}$, a matrix $N \in \mathbb{R}^{n\times m}$ and nonnegative scalars τ_0, τ_1, τ_2, τ_3 and τ_4 which satisfy (11).

Remark 1. (Recover of the Quadratic Stability Approach) Note that when $\|D\| \to \infty$, the application of Theorem 2 will automatically find, if existing, an *uncertainty independent* Lyapunov function with negative definite derivative. Indeed letting $N = 0$, τ_0, τ_2, τ_3, τ_4 sufficiently small and $\tau_1 = 1$ we obtain that conditions (11a) and (11b) are equivalent to the existence of a positive definite P_0 satisfying the condition

$$
\begin{pmatrix} A^TP_0 + P_0A + E^TE & P_0F \\ F^TP_0 & -I \end{pmatrix} < 0 \tag{23}
$$

which is necessary and sufficient for quadratic stability for $\|\Delta\| \leq 1$ of system (1) (see Theorem 1).

Therefore in the limit case of arbitrary time-varying uncertainties Theorem 2 recovers the quadratic stability approach; conversely, for slowly-varying uncertainties, the approach of this paper, by optimizing over the matrix N, can lead to less conservative results than quadratic stability based methods.

Remark 2. (Robust stability for time-invariant uncertainties) When $D = O$ Theorem 2 allows us to study robust stability versus *time-invariant* uncertainties which is still an open problem in the field of stability analysis of uncertain systems subject to norm bounded uncertainties. In this case condition (11b) relaxes as follows

$$
\begin{pmatrix}
L_0 + \tau_1E^TE + \tau_2A^TE^TEA & \frac{1}{2}A^TN + P_0F & \frac{1}{2}N & \frac{1}{2}N \\
\frac{1}{2}N^TA + F^TP_0 & L_1 - \tau_1I + \tau_3F^TE^TEF & O & O \\
\frac{1}{2}N^T & O & -\tau_2I & O \\
\frac{1}{2}N^T & O & O & -\tau_3I
\end{pmatrix} < 0.
$$

4. THE SYNTHESIS PROBLEM

From Theorem 2 we can derive the following result.

Theorem 3. Problem 2 is solvable if there exist

i) A symmetric matrix $P_0 \in \mathbb{R}^{n \times n}$;

ii) Matrices $N \in \mathbb{R}^{n \times m}$ and $K \in \mathbb{R}^{m \times n}$;

iii) Nonnegative scalars τ_0, τ_1, τ_2, τ_3, τ_4

such that

$$\begin{pmatrix} P_0 - \tau_0 E^T E & \frac{1}{2}N \\ \frac{1}{2}N^T & \tau_0 I \end{pmatrix} > 0 \tag{24a}$$

$$\begin{pmatrix} L_{0CL} + \tau_1 E^T E + \tau_4 E^T D E & A_{CL}^T E^T & \frac{1}{2}A_{CL}^T N + P_0 F & \frac{1}{2}N & \frac{1}{2}N & \frac{1}{2}N \\ E A_{CL} & -\tau_2 I & O & O & O & O \\ \frac{1}{2}N^T A_{CL} + F^T P_0 & O & L_1 - \tau_1 I + \tau_3 F^T E^T E F & O & O & O \\ \frac{1}{2}N^T & O & O & -\tau_2 I & O & O \\ \frac{1}{2}N^T & O & O & O & -\tau_3 I & O \\ \frac{1}{2}N^T & O & O & O & O & -\tau_4 I \end{pmatrix} < 0 \tag{24b}$$

where

$$\begin{aligned} L_{0CL} &= A_{CL}^T P_0 + P_0 A_{CL} && (25a) \\ A_{CL} &= A + BK. && (25b) \end{aligned}$$

Proof. From Theorem 2 we have that the closed loop system (7) – (8) is exponentially stable if there exists a symmetric matrix $P_0 \in \mathbb{R}^{n \times n}$, a matrix $N \in \mathbb{R}^{n \times m}$ and nonnegative scalars τ_0, τ_1, τ_2, τ_3, τ_4 such that

$$\begin{pmatrix} P_0 - \tau_0 E^T E & \frac{1}{2}N \\ \frac{1}{2}N^T & \tau_0 I \end{pmatrix} > 0 \tag{26a}$$

$$\begin{pmatrix} L_{0CL} + \tau_1 E^T E + \tau_2 A^T E^T E A + \tau_4 E^T D E & \frac{1}{2}A_{CL}^T N + P_0 F & \frac{1}{2}N & \frac{1}{2}N & \frac{1}{2}N \\ \frac{1}{2}N^T A_{CL} + F^T P_0 & L_1 - \tau_1 I + \tau_3 F^T E^T E F & O & O & O \\ \frac{1}{2}N^T & O & -\tau_2 I & O & O \\ \frac{1}{2}N^T & O & O & -\tau_3 I & O \\ \frac{1}{2}N^T & O & O & O & -\tau_4 I \end{pmatrix} < 0 \tag{26b}$$

By applying the Schur Complements Lemma [4] to the $1-1$ block in (26b) the proof follows. □

By virtue of Theorem 3, Problem 2 is solvable if the following feasibility problem admits a solution.

Problem 4. Find a symmetric matrix $P_0 \in \mathbb{R}^{n\times n}$, matrices $N \in \mathbb{R}^{n\times m}$, $K \in \mathbb{R}^{m\times n}$ and nonnegative scalars τ_0, τ_1, τ_2, τ_3 and τ_4 which satisfy (24).

Unfortunately, Problem 4 is not based on LMIs constraints because of the product between the optimization variables. Indeed condition (24b) is a Bilinear Matrix Inequality [10]. To solve this problem we propose a procedure which consists in alternating the optimization over P_0 and N, with fixed K, with the optimization over K, with fixed P_0 and N (see also [6]). In this way each optimization becomes an LMI problem; this procedure is guaranteed to converge but not necessarily to the global minimum and the solution is dependent on the initial data. With this procedure we solved the following example.

Example 1. Consider system (7) with

$$A = \begin{pmatrix} 0 & 1 \\ -5 & -2 \end{pmatrix}; \; B = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \; F = \begin{pmatrix} 0.8 & 0 \\ 0 & 0.8 \end{pmatrix} \; E = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \; D = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \tag{27}$$

We solved Problem 4 by using the LMI toolbox; a stabilizing K is given by

$$K = (\,-35.46 \quad -22.74\,). \tag{28}$$

5. CONCLUDING REMARKS

In this paper we have considered an unforced linear system subject to norm bounded, bounded rate time-varying uncertainties. For such a system we have provided a sufficient condition for exponential stability in terms of a feasibility problem based on LMIs constraints. Our approach, which makes use of uncertainty dependent Lyapunov functions, has been shown to be less conservative than the classical quadratic stability approach.

Then the analysis result has been used in a synthesis context. Unfortunately the design problem cannot be formulated as an LMIs feasibility problem; therefore we propose a procedure based on the alternation of analysis and synthesis LMIs optimization problems.

Concerning future work, we note that by using Lyapunov functions in the form (9) we have two degrees of freedom in the optimization process, namely P_0 and N. Better results could be obtained by considering more general classes of uncertainty dependent Lyapunov functions; however such Lyapunov functions should possess a structure leading to convex optimization problems. For instance with

$$v(x, \Delta) = x^T (P_0 + N\Delta G)\, x \tag{29}$$

we introduce a new degree of freedom, namely the matrix $G \in \mathbb{R}^{r\times n}$; unfortunately the derivative of the Lyapunov function (29) along the solutions of system (1) does not lead, as in Theorem 2, to an LMIs optimization problem. It seems to the author that the problem of finding an uncertainty dependent Lyapunov function more general than (9)–(10) and leading to a convex feasibility problem is not a trivial one.

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Dr. Francesco Amato, Associate Professor of System Theory, Dipartimento di Informatica e Sistemistica, Università degli Studi di Napoli "Federico II", via Claudio 21, 80125 Napoli. Italy.
e-mail: framato@unina.it