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ROBUST OBSERVER DESIGN FOR TIME-DELAY SYSTEMS: A RICCATI EQUATION APPROACH

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In this paper, a method for H_∞ observer design for linear systems with multiple delays in state and output variables is proposed. The designing method involves attenuating of the disturbance to a pre-specified level. The observer design requires solving certain algebraic Riccati equation. An example is given in order to illustrate the proposed method.

INTRODUCTION

Many papers have dealt with the problem of stabilizing a linear system with delays in state and/or input variables independently or dependently of the delay (see for example Furukawa et al [6], Lee et al [10], Niculescu et al [11], Choi et al [3] and the references therein). All these papers assume that the state variables are measurable entirely. However, this is not the case in practical situations.

Few papers have considered the problem of designing an asymptotic observer for time-delay systems.

This problem has been solved by Watanabe et al [15], [16] by introducing a distributed time-delay in the observer dynamic system. Also, Pearson et al [12] and Ramos et al [14] have built observers for delayed-state systems by computing the unstable eigenvalues of the system under consideration which, in some case, may be a very difficult numerical problem [9]. In those papers the robustness issues have not been considered.

Yao et al [17] have used a factorization approach in order to parameterize an observer for time-delay systems, however, no explicit procedure is given to obtain the observer.

Some papers have developed observer-based controllers for time-delay systems (see for example Choi et al [2]), but the delay is considered in state variables only and the observer design requires the solution of a pair of algebraic Riccati-like equations.

The authors have already obtained some results on observer design for time delay systems using two different approaches. The first one [4] uses the Razumikhin theorem to give a sufficient condition for the existence of an H_∞ observer. While the second one [5] gives some criteria for the existence of an exponential observer.

This paper is concerned with the problem of H_∞ observer design for linear systems with multiple delays in state and output variables using the Krasovskii theorem.

An extended Luenberger-type observer is firstly proposed for the system under consideration. The estimated error dynamics (which is the difference between the state of the system and its estimate) is a multiple delayed-state system. In order to stabilize it and to get a pre-specified attenuation level between the disturbance and the estimated error, the technique proposed by Lee et al [10] for H_∞ stabilization of single delayed-state systems is generalized to the case of multiple delays and used to study the stability of the estimated error system. It leads to a Riccati equation to be solved.

The paper is organized as follows. The problem statement is presented in Section 1. Section 2 is devoted to the H_∞ observer design. An example illustrates the proposed method in Section 3. The paper concludes with Section 4.

Notation. $\mathbb{R}$ is the set of real numbers, $\mathbb{R}^+$ is the set of real non-negative numbers, $\mathbb{R}^n$ denotes the n dimensional Euclidean space, $\mathbb{R}^{n\times m}$ is the set of all $n \times m$ real matrices, s is the Laplace variable, I_n is the $n \times n$ identity matrix, 0_n is the $n \times n$ zero matrix, $\mathcal{L}_2$ is the space of square integrable functions on $[0,\infty)$, $\mathcal{C}[t_1,t_2]$ is the space of continuous functions on $[t_1,t_2]$ and $\|.\|_\infty$ denotes the H_∞-norm defined as: $\|T(s)\|_\infty = \max\ \{\bar{\sigma}(T(j\omega)) : \omega \in \mathbb{R}\}$; $\bar{\sigma}(T)$ denotes the maximum singular value of the matrix T, $j = \sqrt{-1}$ and ω denotes the frequency.

For $X \in \mathbb{R}^{n\times n}$, the notation $X > 0$ (respectively, $X \geq 0$) means that the matrix X is real symmetric positive definite (respectively, positive semi-definite), X^* denotes the adjoint matrix of X ($X^*(j\omega) := X^T(-j\omega)$), $\lambda_i(X)$ denotes the i^{th} eigenvalue of X.

1. PROBLEM STATEMENT

Consider the following linear time-delay system:

$$\begin{cases} \dot{x}(t) &= Ax(t) + \sum_{i=1}^{m} A_i x(t-ih) + Ed(t) \\ y(t) &= Cx(t) + \sum_{i=1}^{m} C_i x(t-ih) + Fd(t) \\ x(t) &= \phi(t); \qquad t \in [-mh, 0] \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $y(t) \in \mathbb{R}^p$ is the measured output vector, $d(t) \in \mathbb{R}^q$ is the $\mathcal{L}_2$ disturbance vector, $\phi(t) \in \mathcal{C}[-mh, 0]$ is the initial functional condition vector, $h \in \mathbb{R}^+$ is the fixed, known delay duration, m is a positive integer such that mh represents the maximal delay in the system and A, A_i, C, C_i, E and F, $i = 1,\ldots,m$, are real matrices with appropriate dimensions.

It should be noted that matrices E and F may also include parameter uncertainties or modelling errors.

An extended Luenberger-type observer for system (1) is given by the following dynamical system:

$$\left\{ \begin{array}{rcl} \dot{\hat{x}}(t) & = & A\hat{x}(t) + \sum_{i=1}^{m} A_i\hat{x}(t-ih) - L(\hat{y}(t) - y(t)) \\ \hat{y}(t) & = & C\hat{x}(t) + \sum_{i=1}^{m} C_i\hat{x}(t-ih) \end{array} \right. \tag{2}$$

where $\hat{x}(t) \in I\!R^n$ is the estimated state of $x(t)$, $\hat{y}(t) \in I\!R^p$ is the estimated output of $y(t)$ and L is the observer gain matrix of appropriate dimension to be designed.

The estimated error, defined as $e(t) = x(t) - \hat{x}(t)$, obeys the following dynamical system, obtained from equations (1) and (2):

$$\dot{e}(t) = (A - LC)\,e(t) + \sum_{i=1}^{m}(A_i - LC_i)\,e(t-ih) + (E - LF)\,d(t). \tag{3}$$

Remark 1. In this paper no control input is considered in the system (1) since the same control input can be considered in the system (2). Those two terms cancel each other in the estimated error system (3).

The next definition introduces the concept of γ-*observer* which will be used throughout this paper.

Definition 2. Given a positive scalar γ, the system (2) is said to be a γ-*observer* for the associated system (1) if the solution of the functional differential equation (3) with $d(t) \equiv 0$ converges to zero asymptotically and, under zero initial condition, the H_∞ norm of the transfer function between the disturbance and the estimated error is bounded by γ, that is:

1. $\lim_{t\to\infty} e(t) \to 0$ for $d(t) \equiv 0$,
2. $\|T_{ed}(s)\|_\infty \leq \gamma$ under zero initial condition,

where $T_{ed}(s)$ is given by:

$$T_{ed}(s) = \left[sI_n - (A - LC) - \sum_{i=1}^{m}(A_i - LC_i)\,e^{-sih} \right]^{-1} (E - LF).$$

The purpose of this paper is to design a constant gain matrix L such that system (2) is a γ-*observer* for the associated system (1) for some positive scalar γ.

2. THE OBSERVER DESIGN

In this section, a sufficient condition for the existence of a γ-*observer* for time-delay systems of the form (1) is provided. Furthermore, an explicit calculation of the observer gain matrix L is given.

Firstly, a result concerning the H_∞ asymptotic stability of multiple time-delay systems is stated in Proposition 3. This result is an extension of the main result of Lee et al [10] concerning the H_∞ stabilization of single delayed-state systems. Then, this Proposition will be used to derive the main result of this paper in Theorem 4.

Proposition 3. Consider the following linear multiple delayed-state system:

$$\begin{cases} \dot{x}(t) & = & Ax(t) + \sum_{i=1}^{m} A_i x(t-ih) + Dd(t) \\ x(t) & = & \phi(t); \quad t \in [-mh, 0] \end{cases} \tag{4}$$

where $x(t) \in I\!R^n$ is the state vector, $d(t) \in I\!R^q$ is the $\mathcal{L}_2$ disturbance vector, $\phi(t) \in \mathcal{C}[-mh, 0]$ is the initial functional condition vector, $h \in I\!R^+$ is the fixed delay duration (could be unknown) and A, A_i and D, $i = 1, \ldots, m$, are real matrices with appropriate dimensions.

Given a positive scalar γ, if there exists two symmetric positive definite matrices Q and P such that

$$A^T P + PA + P\left(\sum_{i=1}^{m} A_i Q^{-1} A_i^T\right) P + mQ + \frac{1}{\gamma^2} I_n + PDD^T P < 0 \tag{5}$$

then the multiple delayed-state system (4) is asymptotically stable for any value of the delay and the following inequality holds:

$$\left\| \left(sI_n - A - \sum_{i=1}^{m} A_i e^{-sih} \right)^{-1} D \right\|_\infty \leq \gamma. \tag{6}$$

Proof. See Appendix A. □

Using the above Proposition, the gain matrix L can be designed such that the system (2) is a γ-*observer* for system (1) as follows.

Theorem 4. Consider the time-delay system (1). Given a positive scalar γ, there exists a γ-*observer* of the form (2) for the system (1) if the following algebraic Riccati equation

$$\begin{aligned} &A^T P + PA + 2P\left(\gamma^2 \sum_{i=1}^{m} A_i A_i^T + EE^T\right) P \\ &- \frac{2}{\epsilon} C^T \left[I_p - \frac{1}{\epsilon}\left(\gamma^2 \sum_{i=1}^{m} C_i C_i^T + FF^T\right)\right] C + \frac{1+m}{\gamma^2} I_n = 0 \end{aligned} \tag{7}$$

has a symmetric positive definite solution P for some positive scalar ϵ.

In this case the required observer gain is given by

$$L = \frac{1}{\epsilon} P^{-1} C^T. \tag{8}$$

Proof. Suppose that Riccati equation (7) has a symmetric positive definite solution P for a given γ and some appropriate positive scalar ϵ. Choosing $L = \frac{1}{\epsilon} P^{-1} C^T$, Riccati equation (7) can be rewritten as follows:

$$A^T P + PA + 2P\left(\gamma^2 \sum_{i=1}^{m} A_i A_i^T + EE^T\right) P - PLC - C^T L^T P$$
$$+ 2\gamma^2 PL\left(\sum_{i=1}^{m} C_i C_i^T\right) L^T P + 2PLFF^T L^T P + \frac{1+m}{\gamma^2} I_n = 0$$

or

$$(A - LC)^T P + P(A - LC) + P\left(\gamma^2 \sum_{i=1}^{m} A_i A_i^T + EE^T\right) P + \frac{1+m}{\gamma^2} I_n$$
$$+ \gamma^2 PL(\sum_{i=1}^{m} C_i C_i^T)L^T P + PLFF^T L^T P$$
$$= -\gamma^2 PL\left(\sum_{i=1}^{m} C_i C_i^T)L^T P - \gamma^2 P(\sum_{i=1}^{m} A_i A_i^T\right) P - PLFF^T L^T P - PEE^T P. \quad (9)$$

Using the following inequality [7]:

$$-XX^T - YY^T \leq XY^T + YX^T$$

where X and Y are any two matrices with suitable dimensions, it follows that

$$-PL\left(\sum_{i=1}^{m} C_i C_i^T\right) L^T P - P\left(\sum_{i=1}^{m} A_i A_i^T\right) P$$
$$\leq \; PL\left(\sum_{i=1}^{m} C_i A_i^T\right) P + P\left(\sum_{i=1}^{m} A_i C_i^T\right) L^T P \quad (10)$$

and

$$-PLFF^T L^T P - PEE^T P \leq PLFE^T P + PEF^T L^T P. \quad (11)$$

By (10) and (11), equation (9) leads to the following inequality:

$$(A - LC)^T P + P(A - LC) + \gamma^2 P\left(\sum_{i=1}^{m} A_i A_i^T\right) P + PEE^T P$$
$$+ \gamma^2 PL\left(\sum_{i=1}^{m} C_i C_i^T\right) L^T P + PLFF^T L^T P - \gamma^2 PL\left(\sum_{i=1}^{m} C_i A_i^T\right) P$$
$$- \gamma^2 P\left(\sum_{i=1}^{m} A_i C_i^T\right) L^T P - PLFE^T P - PEF^T L^T P + \frac{1+m}{\gamma^2} I_n \leq 0$$

or

$$(A - LC)^T P + P(A - LC) + \gamma^2 P\left[\sum_{i=1}^{m} (A_i - LC_i)(A_i - LC_i)^T\right] P$$
$$+ \frac{m}{\gamma^2} I_n + \frac{1}{\gamma^2} I_n + P(E - LF)(E - LF)^T P \leq 0. \quad (12)$$

Applying now the "Bounded Real Lemma" (see Appendix B) leads to

$$
\begin{aligned}
&(A-LC)^T\bar{P}+\bar{P}(A-LC)+\gamma^2\bar{P}\left[\sum_{i=1}^{m}(A_i-LC_i)(A_i-LC_i)^T\right]\bar{P}\\
&\quad+\frac{m}{\gamma^2}I_n+\frac{1}{\gamma^2}I_n+\bar{P}(E-LF)(E-LF)^T\bar{P}<0
\end{aligned}
\tag{13}
$$

for some $\bar{P}=\bar{P}^T>0$ with $\bar{P}\geq P$ (Note that $\bar{P}=P$ if (12) is a strict inequality).

Using now Proposition 3, inequality (13) directly corresponds to (5) applied on estimated error system (3) with $Q=\frac{1}{\gamma^2}I_n$. By proposition 3, it follows that system (3) is asymptotically stable independently of the delay and

$$
\left\|\left[sI_n-(A-LC)-\sum_{i=1}^{m}(A_i-LC_i)\,e^{-sih}\right]^{-1}(E-LF)\right\|_\infty\leq\gamma
$$

i.e.

$$
\|T_{ed}(s)\|_\infty\leq\gamma.
$$

This ends the proof. □

It should be noted that Theorem 4 provides a delay independent criterion for the existence of a γ-*observer*. However, the delay value must be known for the observer implementation.

Remark 5. Some sufficient conditions for the existence of a symmetric positive definite solution to the Riccati equation (7) are presented. These conditions link the existence of a γ-*observer* (given in Theorem 4) with some stabilizability properties on the time-delay system.

Let us define the following matrices:

$$
\begin{aligned}
K &:= \frac{2}{\epsilon}C^T\left[I_p-\frac{1}{\epsilon}\left(\gamma^2\sum_{i=1}^{m}C_iC_i^T+FF^T\right)\right]C-\frac{1+m}{\gamma^2}I_n\\
H &:= [\gamma A_1,\ \gamma A_2,\ \dots\ \gamma A_m,\ E].
\end{aligned}
$$

Using the above definitions, Riccati equation (7) can be rewritten as:

$$
\bar{A}^TP+P\bar{A}-2PHH^TP+K=0 \tag{14}
$$

with $\bar{A}:=-A$.

For a given $\gamma>0$, if there exists a positive scalar ϵ such that K is symmetric and positive definite, then there exists a full rank matrix $J\in\mathbb{R}^{n\times n}$ such that $K=J^TJ$. In this case, Riccati equation (14) has a symmetric positive definite solution P if $(\bar{A},H)$ is stabilizable and $(J,\bar{A})$ is detectable (see [18] Corollary 13.8, p. 338).

Now this solution P is symmetric positive definite since K is symmetric positive definite. Since J has full rank then $(J,\bar{A})$ is always detectable. If (A,H) is stabilizable, then $(\bar{A},H)$ is also stabilizable.

So Riccati equation (7) has a symmetric positive definite solution P if:

(i) there exists a positive scalar ϵ such that K is symmetric positive definite,

(ii) the pair (A, H) is stabilizable.

3. ILLUSTRATIVE EXAMPLE

In order to illustrate the applicability of the proposed method, let us consider the following time-delay system:

$$\begin{aligned}\dot{x}(t) &= \begin{bmatrix} -1 & 2 \\ 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} x(t-1) + \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} x(t-2) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} d(t) \\ y(t) &= \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} x(t) + \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} x(t-1) + \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x(t-2) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} d(t).\end{aligned}$$

Note that this time-delay system is weakly observable but is not strongly observable as (C, A) is not observable (see Lee and Olbrot [9] for the definitions).

For this example two γ-*observers* will be constructed for different values of γ using the proposed method.

(i) $\gamma = 1$: in this case, solving Riccati equation (7) for $\epsilon = 0.1$ gives:

$$P = \begin{bmatrix} 9.2532 & -1.6586 \\ -1.6586 & 2.0263 \end{bmatrix}$$

and the corresponding gain is:

$$L = \begin{bmatrix} 0 & 1.0368 \\ 0 & 5.7838 \end{bmatrix}.$$

The estimated errors are shown in Figure 1 for some initial condition $\phi(t)$. This figure shows that the estimated errors converge asymptotically to zero for $d(t) \equiv 0$. In Figure 2, the singular value of $T_{ed}(j\omega)$ is traced versus the frequency. It shows that $||T_{ed}(j\omega)||_\infty$ is less than one (less than 0.27 in this case).

(ii) $\gamma = 0.1$: solving Riccati equation (7) for $\epsilon = 0.0019$ gives:

$$P = \begin{bmatrix} 274.2521 & -13.0704 \\ -13.0704 & 21.0615 \end{bmatrix}$$

the corresponding gain is:

$$L = \begin{bmatrix} 0 & 1.2273 \\ 0 & 25.7511 \end{bmatrix}.$$

The estimated errors are shown in Figure 3 for the same initial condition $\phi(t)$ as above. This figure shows that the estimated errors converge asymptotically to zero for $d(t) \equiv 0$. In Figure 4, the singular value of $T_{ed}(j\omega)$ is traced versus the frequency. It shows that $||T_{ed}(j\omega)||_\infty$ is less than 0.1 (less than 0.045 in this case).

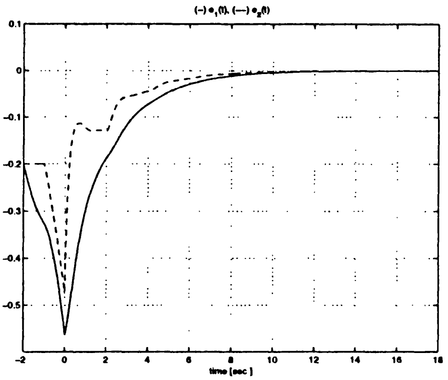

Fig. 1. Estimated errors for $\gamma = 1$.

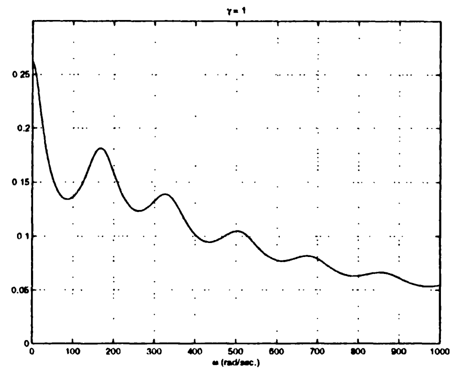

Fig. 2. $\bar{\sigma}(T_{ed}(j\omega))$ versus ω.

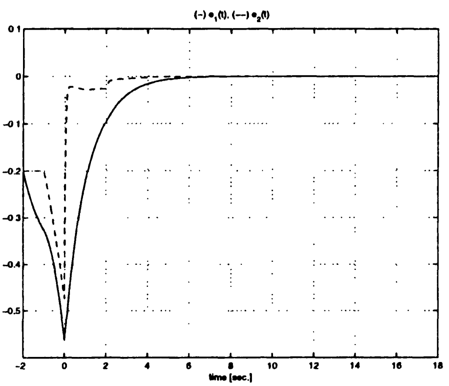

Fig 3. Estimated errors for $\gamma = 0.1$.

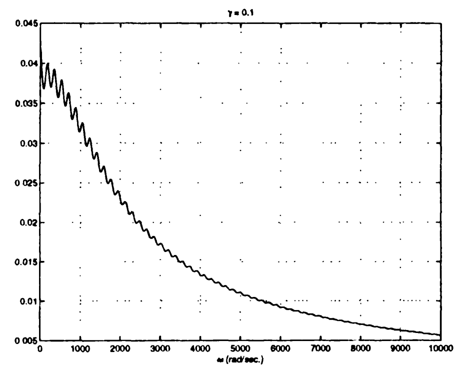

Fig 4. $\bar{\sigma}(T_{ed}(j\omega))$ versus ω.

Note that in the both cases, the curve between -1 and 0 sec. represents the initial function condition of the given system. For simulation purposes, an initial value at time $t = -2$ sec. is used to generate an initial value function on $t \in [-1, 0]$.

It should be noted that the sufficient conditions given in Remark 5 are not satisfied for this example although some efficient γ-*observer* can be constructed.

4. CONCLUSION

In this paper, a method for a γ-*observer* design for time-delay systems has been developed. The design method not only stabilizes the observer but also guarantees an H_∞ norm bound between the disturbance and the estimated error. The designed observer uses the past values of the estimated state but the observer gain matrix is designed independently of the delay (i.e. the presented method is a delay-independent one but the delay must be known for the construction of the observer). This observer gain is obtained by solving a modified Riccati equation. Sufficient conditions for solving this modified Riccati equation have also been provided.

APPENDIX A: PROOF OF PROPOSITION 3

The proof is divided into two parts: in the first part, the asymptotic stability of the system (4) is demonstrated, and, in the second part, the inequality (6) is proved.

Consider that for a given $\gamma > 0$ there exists $Q = Q^T > 0$ and $P = P^T > 0$ such that Riccati equation (5) holds.

First part. Define the Lyapunov functional V as follows:

$$V(t) := x^T(t)\,Px(t) + \sum_{i=1}^{m} \int_{t-ih}^{t} x^T(\theta)\,Qx(\theta)\,\mathrm{d}\theta.$$

It is easy to verify that there exist two positive scalars β_1 and β_2 such that

$$\beta_1 \|x(t)\|^2 \leq V(t) \leq \beta_2 \|x(t+\theta)\|^2. \tag{15}$$

The corresponding Lyapunov derivative along the trajectory of $x(t)$ is given by

$$\frac{\mathrm{d}V(t)}{\mathrm{d}t} = \bar{x}^T(t)\,\mathcal{A}\bar{x}(t) \tag{16}$$

where

$$\bar{x}(t) := \begin{bmatrix} x(t) \\ x(t-h) \\ x(t-2h) \\ \vdots \\ x(t-mh) \end{bmatrix}$$

$$\mathcal{A} := \begin{bmatrix} A^TP + PA + mQ & PA_1 & PA_2 & \cdots & PA_m \\ A_1^TP & -Q & 0_n & \cdots & 0_n \\ A_2^TP & 0_n & -Q & \cdots & 0_n \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ A_m^TP & 0_n & 0_n & \cdots & -Q \end{bmatrix}.$$

Since $\frac{1}{\gamma^2}I_n + PDD^TP > 0$, inequality (5) implies that

$$A^TP+PA+P[A_1,\ A_2,\ \ldots,\ A_m]\ \mathrm{diag}\{Q^{-1}\}_m\ [A_1,\ A_2,\ \ldots,\ A_m]^TP+mQ<0 \tag{17}$$

where $\mathrm{diag}\{Q^{-1}\}_m$ denotes a $m \times m$ block-diagonal matrix with elements Q^{-1}.

Using the Schur complement [1] and (17) it follows that $\mathcal{A}$ is negative definite. Hence

$$\frac{\mathrm{d}V(t)}{\mathrm{d}t} \leq -\lambda_{\min}(\mathcal{A})\,\|\bar{x}(t)\|^2. \tag{18}$$

Combining (15) and (18), one can prove the asymptotic stability of system (4) using the Krasovskii theorem (see Appendix C).

Second part. Let us define the positive definite matrix S as follows:

$$\begin{aligned} S &:= -(A^T P + PA + P[A_1,\ A_2,\ \dots,\ A_m]\,\mathrm{diag}\{Q^{-1}\}_m\,[A_1,\ A_2,\ \dots,\ A_m]^T P \\ &\quad + mQ + \frac{1}{\gamma^2} I_n + PDD^T P) \end{aligned}$$

then

$$\begin{aligned} &A^T P + PA + P[A_1,\ A_2,\ \dots,\ A_m]\,\mathrm{diag}\{Q^{-1}\}_m\,[A_1,\ A_2,\ \dots,\ A_m]^T P \\ &\quad + mQ + \frac{1}{\gamma^2} I_n + PDD^T P + S = 0. \end{aligned} \tag{19}$$

Define the following matrices

$$\begin{aligned} W(j\omega) &:= P[A_1,\ A_2,\ \dots,\ A_m]\,\mathrm{diag}\{Q^{-1}\}_m\,[A_1,\ A_2,\ \dots,\ A_m]^T P \\ &\quad + mQ - \sum_{i=1}^{m} A_i^T P e^{j\omega i h} - \sum_{i=1}^{m} P A_i e^{-j\omega i h} \\ X(j\omega) &:= \left(j\omega I_n - A - \sum_{i=1}^{m} A_i e^{-j\omega i h} \right)^{-1}. \end{aligned}$$

Note that $W(j\omega)$ is non-negative definite for all $\omega \in \mathbb{R}$.

Using the above definition, equation (19) can be rewritten as

$$(X^*(j\omega))^{-1} P + P X^{-1}(j\omega) - W(j\omega) - \frac{1}{\gamma^2} I_n - PDD^T P - S = 0. \tag{20}$$

Pre and post multiplying (20) by $D^T X^*(j\omega)$ and $X(j\omega)\,D$ respectively implies that

$$\begin{aligned} &D^T X^*(j\omega)\,PD + D^T P X(j\omega) D - D^T X^*(j\omega) PDD^T P X(j\omega)\,D - I_n \\ =&\ -I_n + D^T X^*(j\omega) \left\{ W(j\omega) + \frac{1}{\gamma^2} I_n + S \right\} X(j\omega)\,D. \end{aligned}$$

It follows that

$$\begin{aligned} &-(I_n - D^T P X(-j\omega)\,D)^T (I_n - D^T P X(j\omega)\,D) \\ =&\ -I_n + D^T X^*(j\omega)\,\{W(j\omega) + S\}\,X(j\omega)\,D \\ &+ \frac{1}{\gamma^2} D^T X^*(j\omega) X(j\omega)\,D. \end{aligned} \tag{21}$$

The left hand side of the above equation is non-positive definite for all $\omega \in \mathbb{R}$. Define $T(j\omega) := X(j\omega)\,D$, then equation (21) implies

$$-I_n + D^T X^*(j\omega)\,\{W(j\omega) + S\}\,X(j\omega)\,D + \frac{1}{\gamma^2} T^*(j\omega)\,T(j\omega) \le 0$$

and

$$T^*(j\omega)\,T(j\omega) \ \le\ \gamma^2 I_n - \gamma^2 D^T X^*(j\omega)\,\{W(j\omega) + S\}\,X(j\omega)\,D$$

for all $\omega \in \mathbb{R}$.

Since $W(j\omega)$ and S are non-negative and positive matrices for all $\omega \in \mathbb{R}$ respectively then

$$T^*(j\omega)T(j\omega) \leq \gamma^2 I_n$$

for all $\omega \in \mathbb{R}$, i.e.

$$\|(sI_n - A - \sum_{i=1}^{m} A_i e^{-sih})^{-1} D\|_\infty \leq \gamma.$$

This ends the proof. □

APPENDIX B: THE BOUNDED REAL LEMMA [13]

The following statements are equivalent:

(i) There exists a symmetric positive definite matrix $\bar{P}$ such that

$$A^T\bar{P} + \bar{P}A + \bar{P}BB^T\bar{P} + C^TC < 0.$$

(ii) The Riccati equation

$$A^TP + PA + PBB^TP + C^TC = 0$$

has a solution $P \geq 0$.

Furthermore, if these statements hold then $P < \bar{P}$.

APPENDIX C: THE KRASOVSKII THEOREM [8]

Consider the retarded functional differential equation

$$\dot{x}(t) = f(t, x_t), \quad t \geq t_0, \quad x_t(\theta) = x(t+\theta) \tag{22}$$
$$x_{t_0}(\theta) = \phi(\theta); \quad \theta \in [-h, 0].$$

Assume that $\phi(\theta) \in \mathcal{C}[-h, 0]$, the operator $f(t, \phi)$ is continuous and Lipschitzian in ϕ and $f(t, 0) = 0$.

If there exist a continuous functional $V(t, \phi)$ such that

$$w_1(\|\phi(0)\|) \leq V(t, \phi) \leq w_2(\|\phi(\theta)\|), \quad \dot{V} \leq -w_3(\|x(t)\|)$$

where $w_i(r)$, $r \geq 0$, are some scalar, continuous, nondecreasing functions such that $w_i(0) = 0$ and $w_i(r) > 0$ for $r > 0$.
Then the trivial solution of (22) is uniformly asymptotically stable (see [8] Theorem 5.3, p. 73).

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