Vladimír Kučera

Deadbeat control, pole placement, and LQ regulation

Kybernetika, Vol. 35 (1999), No. 6, [681]--692

Persistent URL: http://dml.cz/dmlcz/135318

KYBERNETIKA — VOLUME 35 (1999), NUMBER 6, PAGES 681–692

DEADBEAT CONTROL, POLE PLACEMENT, AND LQ REGULATION[1]

VLADIMÍR KUČERA

Deadbeat control, a typical example of linear control strategies in discrete-time systems, is shown to be a special case of the linear-quadratic regulation. This result is obtained by drawing on the parallels between the state-space and the transfer-function design techniques.

1. INTRODUCTION

The aim of this paper is to show that new results can be obtained by examining the parallels between the state-space and the transfer-function techniques in the design of linear control systems.

Deadbeat control is a typical example of linear control strategies in discrete-time systems. It consists of driving each initial state of the system to zero in shortest time possible. Two design procedures are presented, one based on the state-space techniques while the other on the transfer-function techniques. These procedures, obtained in isolation, are related using pole placement, a useful reference design problem. This provides further insight and reveals that the deadbeat control is a special case of the pole placement problem and, strikingly, also a special case of the LQ regulator problem.

2. DEADBEAT CONTROL PROBLEM

We consider a linear system described by the equation

$$x_{k+1} = Fx_k + Gu_k, \quad k = 0, 1, \ldots \tag{1}$$

where $u_k \in \mathbb{R}^m$ and $x_k \in \mathbb{R}^n$. The objective of *deadbeat control* is to determine a linear state feedback of the form

$$u_k = -Lx_k \tag{2}$$

[1]This research was supported by the Ministry of Education of the Czech Republic under project VS97–034 and by the Grant Agency of the Czech Republic under contract 102/97/0861.

which drives each initial state x_0 to the origin in a least number of steps.

We define the reachability subspaces of the system (1) by

$$\begin{aligned} R_0 &= 0 \\ R_k &= \text{image}[G \;\; FG \; \ldots \; F^{k-1}G], \quad k = 1, 2, \ldots . \end{aligned}$$

Hence R_k is the set of states of (1) that can be reached from the origin in k steps by applying an input sequence $u_0, u_1, \ldots, u_{k-1}$. When $R_n = \mathbb{R}^n$, system (1) is said to be *reachable.*

Define the integers

$$q_k = \text{dimension } R_k - \text{ dimension } R_{k-1}$$

and for $k = 1, 2, \ldots, m$ let

$$r_i = \text{cardinality } \{q_k : q_k \geq i\}.$$

The integers $r_1 \geq r_2 \geq \ldots \geq r_m$ are the reachability indices of system (1). Clearly the system (1) is reachable if and only if

$$\sum_{i=1}^{m} r_i = n.$$

We further define the controllability subspaces for (1) by

$$\begin{aligned} C_0 &= 0 \\ C_k &= \{x \in \mathbb{R}^n : F^k x \in R_k\}, \quad k = 1, 2, \ldots . \end{aligned}$$

Thus C_k is the set of all states of (1) that can be steered to the origin in k steps by an appropriate control sequence $u_0, u_1, \ldots, u_{k-1}$. When $C_n = \mathbb{R}^n$, system (1) is said to be *controllable.* It is clear that reachability implies controllability and the converse is true whenever F is non-singular.

If one is able to find a matrix L with the property that

$$(F - GL)\, C_k \subset C_{k-1}$$

for each $k = 1, 2, \ldots$ then every state of the closed-loop system

$$x_{k+1} = (F - GL)\, x_k \tag{3}$$

belonging to C_k is brought to the origin precisely in step k. Hence this L defines a deadbeat gain in (2).

3. STATE SPACE SOLUTION

The existence and construction of deadbeat control laws is described below using the state-space techniques.

For each $k = 1, 2, \ldots$ let $S_1, S_2, \ldots, S_k$ be a sequence of $m \times q_1,\ m \times q_2, \ldots, m \times q_k$ matrices such that

$$\begin{aligned} \text{image} \quad & [\, GS_1 \;\; FGS_2 \;\ldots\; F^{k-1}GS_k \,] \\ & = \text{image}\, [\, G \;\; FG \;\ldots\; F^{k-1}G \,] . \end{aligned}$$

Therefore $S_1, S_2, \ldots, S_k$ serve to select a basis for R_k.

Theorem 1. [3] There exists a deadbeat control law (2) if and only if the system (1) is controllable. Let

$$\begin{aligned} L_0 &= 0 \\ L_k &= L_{k-1} + L'_k (F - GL_{k-1})^k , \; k = 1, 2, \ldots \end{aligned} \tag{4}$$

where L'_k satisfies

$$L'_k \, [\, GS_1 \;\; FGS_2 \;\ldots\; F^{k-1}\, GS_k \,] = [\, 0 \;\ldots\; 0 \; S_k \,] .$$

Then $L = L_n$ is a deadbeat gain.

The theorem identifies a deadbeat gain via the recursive procedure (4). Actually this procedure can be terminated in p steps, where

$$p = \min\{ k : C_{k+1} = C_k \} .$$

The resulting closed-loop system matrix is nilpotent with index p,

$$(F - GL)^p = 0 , \tag{5}$$

and every initial state of the system is driven to the origin in no more than p steps.

If F is non-singular, the recursive procedure (4) can be shortcut by setting $L_{p-1} = 0$, thus giving

$$L = L'_p F^p$$

or

$$L \, [\, F^{-p} GS_1 \;\ldots\; F^{-1} GS_p \,] = [\, 0 \;\ldots\; 0 \, S_p \,] .$$

4. TRANSFER FUNCTION SOLUTION

Let us now analyze the problem by using transfer function techniques. We start with the transfer function of (1) in the matrix fraction form

$$(zI_n - F)^{-1} G = B(z)\, A^{-1}(z) \tag{6}$$

where A and B are right coprime polynomial matrices in z of respective size $m \times m$ and $n \times m$. Furthermore, let A be column reduced with highest-column-degree coefficient matrix A_H and column-degree ordered with column degrees $r_1 \geq r_2 \geq \ldots \geq r_m$. These integers are the reachability indices of (1).

Define the matrices

$$\begin{aligned}\bar{A}(z^{-1}) &= A(z)\,\mathrm{diag}\,[z^{-r_1}, \ldots, z^{-r_m}] \\ \bar{B}(z^{-1}) &= B(z)\,\mathrm{diag}\,[z^{-r_1}, \ldots,, z^{-r_m}].\end{aligned}$$

Clearly, $\bar{A}$ and $\bar{B}$ are right coprime polynomial matrices in z^{-1} with the properties

$$(I_n - Fz^{-1})^{-1} G z^{-1} = \bar{B}(z^{-1})\,\bar{A}^{-1}(z^{-1}) \tag{7}$$

and

$$\bar{A}(0) = A_H, \quad \bar{B}(0) = 0.$$

Theorem 2. [2] There exists a deadbeat control law (2) if and only if the system (1) is controllable. Let P, Q be a polynomial solution pair of the equation

$$(I_n - Fz^{-1})\,P(z^{-1}) + Gz^{-1}\,Q(z^{-1}) = I_n \tag{8}$$

such that $\begin{bmatrix} P \\ Q \end{bmatrix}$ has lowest column degrees among all polynomial solution pairs of (8). Then P is non-singular and

$$L = Q(z^{-1})\,P^{-1}(z^{-1}) \tag{9}$$

is a deadbeat gain.

The striking claim that L given by (9) is a constant matrix results from the fact that system controllability implies the matrices $I_n - Fz^{-1}$ and Gz^{-1} are left coprime. Then associated with equations (7) and (8) are constant matrices X and Y such that the following Bézout identity holds.

$$\begin{bmatrix} I_n - Fz^{-1} & -Gz^{-1} \\ Y & X \end{bmatrix} \begin{bmatrix} P(z^{-1}) & \bar{B}(z^{-1}) \\ -Q(z^{-1}) & \bar{A}(z^{-1}) \end{bmatrix} = I_{n+m}. \tag{10}$$

The resultant L is indeed constant,

$$L = Q(z^{-1})\,P^{-1}(z^{-1}) = X^{-1} Y.$$

Equation (8) yields

$$I_n - (F - GL)\,z^{-1} = P^{-1}(z^{-1}) \tag{11}$$

so that the response of the closed-loop system (3) is

$$x(z^{-1}) = P(z^{-1})\,x_0.$$

The column degrees of P being minimal, $x(z^{-1})$ is of least degree for each x_0. According to (5) and (11), the highest power of z^{-1} that occurs in P is $p - 1$.

5. POLE PLACEMENT

We consider a linear system described by equation (1),

$$x_{k+1} = Fx_k + Gu_k, \quad k = 0, 1, \ldots$$

where $u_k \in \mathbb{R}^m$ and $x_k \in \mathbb{R}^n$. The objective of *pole placement* design is to determine a linear state feedback of the form (2),

$$u_k = -Lx_k$$

such that the closed-loop system (3),

$$x_{k+1} = (F - GL)x_k$$

has a prespecified set of invariant polynomials $c_1, c_2, \ldots, c_n$ where c_i contains c_{i+1} as factor, $i = 1, 2, \ldots, n-1$ and

$$\sum_{i=1}^{n} \text{degree } c_i = n\,.$$

Therefore the goal is to assign a complete set of similarity invariants to $F - GL$: the position as well as the Jordan structure of the closed-loop poles. The characteristic polynomial of $F - GL$ is given by

$$\chi_{F-GL}(z) = c_1(z) \ldots c_n(z)\,.$$

This polynomial captures the positions of the poles; the structure of the repeated poles is given by the invariant factors.

Theorem 3. [4] Suppose that system (1) is reachable with reachability indices $r_1 \geq r_2 \geq \ldots \geq r_m$. The pole placement problem is solvable if and only if the following set of inequalities is satisfied,

$$\sum_{i=1}^{j} \text{degree } c_i \geq \sum_{i=1}^{j} r_i, \quad j = 1, 2, \ldots, m \tag{12}$$

with equality holding when $j = m$.

Thus state feedback (2) can place the poles of (1) at arbitrary positions but the structure of each multiple pole is limited: one cannot split it into as many cyclic chains (Jordan blocks) as one might wish.

If the inequalities (12) are verified, then a feedback gain matrix L that achieves the similarity invariants desired, can be constructed in three steps ([1]). Firstly, factorize the transfer function of (1) as in (6),

$$(zI_n - F)^{-1}G = B(z)A^{-1}(z)$$

where A and B are right coprime polynomial matrices with A column reduced and column-degree ordered, so that the column degrees of A equal $r_1, r_2, \ldots, r_m$. Secondly, form a column reduced $m \times m$ polynomial matrix C with invariant polynomials $c_1, c_2, \ldots, c_m$ that has the same column degrees as A. Such a matrix exists whenever the inequalities (12) are verifed. Thirdly, let X, Y be a constant solution of the polynomial matrix equation

$$XA(z) + YB(z) = C(z). \tag{13}$$

Such a solution pair X, Y is unique if (1) is reachable and, by construction, X is non-singular. Then

$$L = X^{-1}Y$$

assigns to $F - GL$ the invariant polynomials $c_1, c_2, \ldots, c_m$; the remaining invariant polynomials are trivial, $c_{m+1} = \ldots = c_n = 1$.

The aim of this section is to show that the deadbeat control is a special case of pole placement.

To this end, consider the equation

$$X\bar{A}(z^{-1}) + Y\bar{B}(z^{-1}) \quad = \quad I_m \tag{14}$$

which is associated with equation (8) via the Bézout identity (10),

$$\begin{bmatrix} I_n - Fz^{-1} & -Gz^{-1} \\ Y & X \end{bmatrix} \begin{bmatrix} P(z^{-1}) & \bar{B}(z^{-1}) \\ -Q(z^{-1}) & \bar{A}(z^{-1}) \end{bmatrix} = I_{n+m}\,.$$

Multiply (14) on the right by the matrix diag $[z^{r_1}, \ldots, z^m]$ to obtain

$$XA(z) + YB(z) = C(z) \tag{15}$$

where

$$C(z) = \operatorname{diag}[z^{r_1}, \ldots, z^{r_m}].$$

We recognize in (15) a special case of equation (13) for pole placement. In the light of (15), the deadbeat control stategy is one which calls for a nilpotent matrix $F - GL$: the closed-loop system has all its poles at the origin. But this is not a complete inference. To ensure that each x_0 is driven to the origin in a least number of steps, each cyclic component of $F - GL$ must be of least size. This is equivalent to $F - GL$ having similarity invariants precisely $c_i(z) = z^{r_i}$, $i = 1, 2, \ldots, m$ where r_i are the reachability indices of the system (1). Thus the inequalities (12) must be satisfied with equality for each $k = 1, 2, \ldots, m$.

The pole placement interpretation provides further insight as well as a simple alternative construction of a deadbeat gain. This interpretation, however, is limited to reachable systems. For systems that are controllable but not reachable, equation (15) possesses many constant solution pairs X, Y among which only some define deadbeat gains, and (15) alone is not sufficient to identify them. Equation (8), however, does the job neatly. This difficulty disappears when F is non-singular. Under this assumption, the system (1) is reachable if and only if it is controllable.

6. LINEAR QUADRATIC REGULATOR

We consider a linear system described by equation (1),

$$x_{k+1} = Fx_k + Gu_k, \quad k = 0, 1, \dots$$

where $u_k \in I\!R^m$ and $x_k \in I\!R^n$. The objective of *LQ regulation* is to find a linear state feedback of the form (2),

$$u_k = -Lx_k$$

which stabilizes the closed-loop system (3),

$$x_{k+1} = (F - GL)\, x_k$$

and, for every initial state x_0, minimizes the quadratic cost

$$\sum_{k=0}^{\infty} y_k^T y_k$$

for some $y_k \in I\!R^l$ of the form

$$y_k = Hx_k + Ju_k. \tag{16}$$

The general form (16) of the quadratic cost accounts for cross weighting between x_k and u_k and allows considering $J^T J$ to be singular or zero.

Theorem 4. [5] Suppose that system (1) is stabilizable and the fictitious system with output (16) is left invertible. Let W be the largest symmetric non-negative definite solution of the algebraic Riccati equation

$$\begin{aligned} W \;=\; & F^T W F + H^T H \\ & - (F^T W G + H^T J)\, (J^T J \; + \; G^T W G)^{-1}\, (J^T H + G^T W F) \end{aligned} \tag{17}$$

and define the feedback gain matrix

$$L = (J^T J + G^T W G)^{-1} (J^T H + G^T W F). \tag{18}$$

If L stabilizes $F - GL$, then (18) is the LQ regulator gain.

It is evident that stabilizability of (1) is a necessary condition for an LQ regulator gain to exist. The sufficient condition has to do with detectability of (16) and is not easy to express in terms of the given data. However, if L defined by (18) does not stabilize $F - GL$, then no LQ regulator exists. Finally, if an LQ regulator gain does exist, then it is unique.

It comes as no surprise that LQ regulation can be interpreted also as a particular pole placement. This can be seen from the return-difference indentity ([6])

$$\begin{aligned} & [\, A(z^{-1}) + LB(z^{-1})\,]^T \; (J^T J \; + \; G^T W G)^{-1} \; [\, A(z) + LB(z)\,] \\ = \;\; & [\, HB(z^{-1}) + JA(z^{-1})\,]^T \; [\, HB(z) + JA(z)\,] \end{aligned} \tag{19}$$

which follows from (17) and (18) on introducing the polynomial matrix factorizations (6).

Define a polynomial $m \times m$ matrix C, which is column reduced and column-degree ordered with column degrees $r_1 \geq r_2 \geq \ldots \geq r_m$, by the equation

$$C^T(z^{-1})\, C(z) = [HB(z^{-1}) + JA(z^{-1})]^T\, [HB(z) + JA(z)] \tag{20}$$

and whose inverse C^{-1} is analytic in the domain $|z| > 1$. This matrix is referred to as the *spectral factor* and is determined uniquely up to multiplication on the left by a constant orthogonal matrix.

If C^{-1} is actually analytic in $|z| \geq 1$, then it follows from (19) and (20) that the LQ regulator gain is given by

$$L = X^{-1}Y$$

where X, Y is a constant matrix solution pair of the equation

$$XA(z) + YB(z) = C(z).$$

This is the pole placement equation (13). Therefore, the LQ regulation is a special case of pole placement; it is the spectral factorization (20) that tells us which similarity invariants are LQ optimal.

This interpretation, however, is limited to reachable systems. If (1) is stabilizable but not reachable, then the polynomial matrices appearing on the left-hand side of (6) are not left coprime while those appearing on the right-hand side of (6) are right coprime. As a result, the pole placement equation (13) has many constant solution pairs X, Y among which only one is optimal, and equation (13) alone is not sufficient to identify it.

7. DEADBEAT CONTROL AS AN LQ REGULATION

The aim of this section is to show that the deadbeat control is LQ optimal, at least for reachable systems.

Let system (1) be reachable with reachability indices $r_1, r_2, \ldots, r_m$. Let T be a similarity transformation that brings (1) to the reachability standard form ([2])

$$F' = TFT^{-1}, \quad G' = TG \tag{21}$$

where F' is a top-companion matrix with non-zero entries in rows r_i, $i = 1, 2, \ldots, m$ and G' has non-zero entries only in rows r_i and columns $j \geq i$, $i = 1, 2, \ldots, m$.

Theorem 5. Let system (1) be reachable with reachability indices $r_1 \geq r_2 \geq \ldots \geq r_m$. Then the feedback gain L which is LQ optimal with respect to $H = T$, $J = 0$ in (16) is a deadbeat gain.

Proof. Write

$$(zI_n - F)^{-1}G = B(z)\, A^{-1}(z)$$

for the original system (1) and

$$(zI_n - F')^{-1}G' = B'(z)\, A'^{-1}(z)$$

for its associate in the reachability standard form, where the matrices on the right-hand sides are right coprime with A and A' normalized to be column reduced with column degrees $r_1, r_2, \ldots, r_m$. Then (21) implies that

$$A(z) = A'(z), \quad B(z) = T^{-1}B'(z) \tag{22}$$

and B' has the block-diagonal form

$$B'(z) = \begin{bmatrix} 1 & & \\ z & & \\ \vdots & & \\ z^{r_1-1} & & \\ & \ddots & \\ & & 1 \\ & & z \\ & & \vdots \\ & & z^{r_m-1} \end{bmatrix}.$$

The spectral factorization (20) reads

$$\begin{aligned} C^T(z^{-1})\,C(z) &= B^T(z^{-1})\,T^TTB(z) \\ &= B'^T(z^{-1})\,B'(z) \\ &= \operatorname{diag}\,[r_1, r_2, \ldots, r_m] \end{aligned}$$

so that

$$C(z) = \operatorname{diag}\,\left[\sqrt{r}_1 z^{r_1}, \ldots, \sqrt{r}_m z^{r_m}\right]. \tag{23}$$

Therefore, the LQ regulator with $H = T, J = 0$ induces the closed-loop similarity invariants $c_i(z) = z^{r_i}$, $i = 1, 2, \ldots, m$. It follows from (15) that it is a deadbeat control system. □

8. EXAMPLE

To illustrate, consider a reachable system (1) described by

$$F = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad G = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}.$$

A deadbeat gain (2) can be calculated using Theorem 1. One can take

$$S_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad S_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad S_3 = 0$$

thus obtaining, recursively,

$$L_1 = \begin{bmatrix} \beta & 1+\beta & 0 \\ \alpha & \alpha & 1 \end{bmatrix}, \quad L_2 = L_3 = \begin{bmatrix} 1 & 2 & 0 \\ \alpha & \alpha & 1 \end{bmatrix}$$

for any real numbers α and β. Any and all deadbeat gains are given as

$$L = \begin{bmatrix} 1 & 2 & 0 \\ \alpha & \alpha & 1 \end{bmatrix}.$$

An alternative construction results from Theorem 2. The polynomial equation (8) has the least-column-degree solution

$$\begin{aligned} P(z^{-1}) &= \begin{bmatrix} 1-z^{-1} & -z^{-1} & 0 \\ z^{-1} & 1+z^{-1} & 0 \\ -\alpha z^{-1} & -\alpha z^{-1} & 1 \end{bmatrix} \\ Q(z^{-1}) &= \begin{bmatrix} 1+z^{-1} & 2+z^{-1} & 0 \\ \alpha-\alpha z^{-1} & \alpha-\alpha z^{-1} & 1 \end{bmatrix} \end{aligned}$$

for any real α and (9) gives

$$L = \begin{bmatrix} 1 & 2 & 0 \\ \alpha & \alpha & 1 \end{bmatrix}$$

as before.

Now recall that the reachability indices of (1) are $r_1 = 2$, $r_1 = 1$ and using Theorem 3, find a feedback gain which alters the invariant polynomials of (1) to be

$$c_1(z) = z^2, \quad c_2(z) = z\,.$$

One calculates a right coprime polynomial factorization (6),

$$A(z) = \begin{bmatrix} z^2-z-1 & 0 \\ 0 & z-1 \end{bmatrix}, \quad B(z) = \begin{bmatrix} z-1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

and forms the matrix

$$C(z) = \begin{bmatrix} z^2 & 0 \\ 0 & z \end{bmatrix}.$$

Equation (15) has the unique constant solution pair

$$X = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad Y = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

which yields the feedback gain

$$L = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

This is indeed a deadbeat gain. The other deadbeat gains can be obtained by modifying $C(z)$ to $C'(z)$ using unimodular transformations,

$$C'(z) = C(z) \begin{bmatrix} 1 & 0 \\ \alpha & 1 \end{bmatrix}.$$

Let us now transform (1) to its reachability standard form. The similarity transformation T is given by (22) as

$$T = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

This allows calculating a deadbeat gain as the LQ regulator gain for $H = T$, $J = 0$. The spectral factor (23) reads

$$C(z) = \begin{bmatrix} \sqrt{2}z^2 & 0 \\ 0 & z \end{bmatrix}$$

and the pole placement equation (13) has the unique constant solution

$$X = \begin{bmatrix} \sqrt{2} & 0 \\ 0 & 1 \end{bmatrix}, \quad Y = \begin{bmatrix} \sqrt{2} & 2\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

The resulting LQ regulator gain

$$L = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

is indeed a deadbeat gain. The other deadbeat gains, however, cannot be obtained using this approach.

9. CONCLUSIONS

The relationship of deadbeat, pole placement, and LQ regulator control laws, obtained here by examining close parallels between the state-space and the transfer-function design techniques, has several merits. Firstly it provides further insight. Secondly it provides alternative design procedures. Thirdly, and most importantly, it demonstrates the usefulness of connecting seemingly isolated results in obtaining new results.

(Received December 11, 1998.)

REFERENCES

[1] V. Kučera: Assigning the invariant factors by feedback. Kybernetika *17* (1981), 118–127.

[2] V. Kučera: Analysis and Design of Discrete Linear Control Systems. Prentice–Hall, London 1991.

[3] C. T. Mullis: Time optimal discrete regulator gains. IEEE Trans. Automat. Control *17* (1972), 265–266.

[4] H. H. Rosenbrock: State–space and Multivariable Theory. Nelson, London 1970.

[5] L. M. Silverman: Discrete Riccati equations – alternative algorithms, asymptotic properties and system theory interpretations. In: Control and Dynamic Systems, Advances in Theory and Applications *12*, Academic Press, New York 1976, pp. 313–386.

[6] U. Shaked: A transfer function approach to the linear discrete stationary filtering and the steady-state discrete optimal control problems. Internat. J. Control *29* (1979), 279–291.

Prof. Ing. Vladimír Kučera, DrSc., Institute of Information Theory and Automation, Academy of Sciences of the Czech Republic, Pod vodárenskou věží 4, 182 08 Prague and Trnka Laboratory for Automatic Control, Faculty of Electrical Engineering, Czech Technical University, Technická 2, 166 27 Prague. Czech Republic.
e-mail: kucera@utia.cas.cz