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Lecture Notes of the Seventh International School on "Mathematical Theory in Fluid Mechanics". Foreword

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LECTURE NOTES OF THE SEVENTH INTERNATIONAL SCHOOL ON "MATHEMATICAL THEORY IN FLUID MECHANICS"

FOREWORD

In June 2001, the Seventh International School on *Mathematical Theory in Fluid Mechanics* was held at the small village of Paseky in the northern part of the Czech Republic. The group of main speakers consisted of Eduard Feireisl, Ricardo Rosa, Martin Rumpf, Gregory Seregin and Vladimír Šverák.

During this school, *Eduard Feireisl* presented results on the global existence theory for barotropic compressible fluids, improving earlier results obtained by P. L. Lions in his book *Mathematical Topics in Fluid Mechanics, Volume 2: Compressible Models*, Oxford, 1998. To be more specific, considering the isentropic model with the constitutive equation for the pressure p of the form $p = a\varrho^{\gamma}$, ϱ being the density and $\gamma > 1$ the adiabatic constant ($a > 0$), the existence theory of P. L. Lions provides weak (renormalized) solutions if $\gamma \geqslant \frac{3d}{d+2}$, where d is the space dimension of the domain occupied by the fluid. E. Feireisl (in part with co-workers) has improved the assertion up to $\gamma > \frac{d}{2}$, covering thus several physically interesting cases. Supported by this result, E. Feireisl studies then the issues of large time behavior (global apriori estimates, the existence of bounded absorbing sets), problems of shape optimization, and flows of rigid bodies in a compressible medium. In this volume, E. Feireisl summarizes known recent results and main ideas concerning the last topic: flows of rigid bodies in viscous fluids. He provides us with his approach that allows him to establish the global-in-time existence of weak solutions for flows of a finite number of rigid bodies in both incompressible and compressible viscous fluids.

Ricardo Rosa introduced to the participants of the Paseky School, among others, the link between the Navier-Stokes equations and the statistical theory of turbulence and the rigorous framework of statistical solutions of the Navier-Stokes equations as well. These topics form a part of in the meantime published book by C. Foias, O. Manley, R. Rosa and R. Temam: *Navier-Stokes Equations and Turbulence*, Cambridge University Press, 2001. In this volume, R. Rosa focuses on some aspects of *stationary* statistical turbulence paying attention to the cases of non-smooth boundaries and forces with minimal regularity. He also considers arbitrary stationary statistical solutions, in contrast to the above book where statistical solutions obtained as limits of time-averages of individual weak solutions are taken into account only.

The last paper of this volume, written by J. Becker, G. Grün, M. Lenz and M. Rumpf, reflects the series of lectures on *thin film models* delivered at Paseky by *Martin Rumpf.* The article presents a derivation of the model system from the Navier-Stokes equations with a free boundary. An appropriate scaling leads to an equation of the fourth order for the height of the liquid film. This equation exhibits interesting mathematical properties. Based on them, the appropriate finite element and finite volume schemes were designed. The article gives a description of the numerical methods and emphasizes their potential in numerical simulations. Various possible extensions are also clearly outlined. The results are applicable in other areas, for example in dislocation theory in plasticity.

Gregory Seregin and *Vladimír Šverák* focused their Paseky lecture series on the regularity problem for the Navier-Stokes equations in three dimensions. While G. Seregin was discussing the results on partial regularity for suitable weak solutions and presented the results extending the theory up to the boundary and estimating the number of possible singularities, V. Šverák gave an overview of known results and open problems for systems of equations sharing certain common aspects (such as the scaling) with the Navier-Stokes system.

The reader will not find the contributions of G. Seregin and V. Šverák in this volume. Instead we suggest to look at their two joint articles prepared after the last Paseky school. In the first one, *On solutions to the Navier-Stokes equations with lower bounds on pressure*, Arch. Rational Mech. Anal. **163** (2002), 65–86, the authors show that if the solution exhibits a singularity, the pressure cannot be bounded from below. In the second contribution *The Navier-Stokes Equations and Backward Uniqueness*, in: Nonlinear Problems in Mathematical Physics and Related Topics II (eds. M. Sh. Birman, S. Hildebrandt, V. A. Solonnikov, N. N. Uraltseva), Kluwer, 2002, G. Seregin and V. Šverák show with a caveat added later that $L^\infty(0,T;L^3(R^3))$ is the regularity class.

All articles in this volume as well as those of Seregin and Šverák mentioned above present significant results and give better understanding to problems in areas that enjoy a significant attention of mathematicians in the theory of partial differential equations stemming from the mechanics of fluids.

We thank very much all the main speakers for their interesting talks and the authors of the articles in this volume for time and enthusiasm in the preparation of their written lecture notes. We are also very thankful to all participants for their interest, stimulating questions and discussions during the course. We would like to use this opportunity and call the attention to the school web-page

`www.karlin.mff.cuni.cz/paseky-fluid/`

where information on the previous Paseky schools as well as on the forthcoming Eighth Paseky School, June 8–14, 2003, can be found.

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