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THE LIFE AND WORK OF ZBYNĚK ŠIDÁK (1933–1999)

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Abstract. Zbyněk Šidák, the chief editor of the Applications of Mathematics, an outstanding Czech statistician and probabilist, died on November 12, 1999, aged 66 years. This article is devoted to memory of him and outlines his life and scientific work.

Keywords: Markov chains, rank tests, multivariate distribution theory, multiple comparison methods

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1. Life

Zbyněk Šidák was born October 24, 1933 in a small East Bohemia town Golčův Jeníkov where he also spent his childhood. Unfortunately, his youth was strongly influenced by a congenital heart disease and his health improved substantially only after a successful heart operation in 1955. However, a severe control of all physical activities remained his whole-life destiny. He studied mathematical statistics at Charles University in Praha (1951–1956) and achieved the diploma by a Thesis on martingales and submartingales. In 1956, he became a fellow researcher in the Mathematical Institute of the Czechoslovak Academy of Sciences, Praha, and continued to work there till his death; for several years as the head of the Department of Probability Theory and Mathematical Statistics. He published numerous research papers covering a broad range of topics among which Markov chains, multivariate statistical problems and rank tests constituted the centre of his attention. Markov chains were the topic of Šidák's CSc. (PhD.) Thesis (1961) and also of his DrSc. Thesis (1973). Well known is the basic inequality [28] for multivariate normal distribution bearing his name, unique and widely cited is also his book (with J. Hájek) on the theory of rank tests [30]. He was awarded the Bernard Bolzano Medal, the highest award given by the Academy of Sciences of the Czech Republic for a contribution to the development of mathematics. He held visiting positions at a number of universities: University of Stockholm (1961), Michigan State University (1966–1967), Indian Statistical Institute (1968, 1980), Moscow State University (1974), University of California (1986), University of North Caroline (1994–1995). Although his frail heart regrettably weakened in the last years, he was still able to prepare (in collaboration with P. K. Sen) a new revised and modernized edition of his and Hájek's famous monograph on the theory of rank tests, which appeared shortly before his death [73].

Zbyněk Šidák was an excellent specialist with a clear logical mind, conscientious and accurate in his work. He married Krista Štěpničková in 1958; they had three children, one daughter and two sons. He was a quiet, reliable, reserved and honest man with a deep feeling for his family. Unfortunately, many human sorrows combined with his poor health influenced his life, in particular its last years.

2. Work

2.1. Markov chains.

Šidák's interest in stochastic processes was awoken as early as during his studies at Charles University. In the sixties, due to the effort of many mathematicians (e.g. S. Orey, N. Jain, B. Jamison, M. Rosenblatt, S. Foguel ...), the theory of Markov chains was extended almost completely from countable state spaces to arbitrary measurable spaces, and the attention paid to operator theoretic methods in the Markov chains theory increased considerably. Two famous books [93] and [78] may be consulted for an overview of these achievements. Eleven Šidák's papers on the operator theoretic treatment of Markov chains on general state spaces represent an important early contribution to the field. (It is quite typical of him that, albeit no specific applications are either discussed or even mentioned in his papers, he was attracted to general Markov chains by the study of a highly applied topic—the mathematical learning theory as developed in the book [76].)

The starting point for most of Šidák's work concerning Markov chains was D. G. Kendall's paper [87]. This author considered a transition matrix $(p_{ij})_{i,j=1}^{\infty}$ of an irreducible homogeneous Markov chain with a countable state space. He proved that there exists at least one (generally, infinite) subinvariant measure $m = (m_j)$ and showed that

$$T\colon\ \ell_2 \longrightarrow \ell_2,\ x \longmapsto \left(\sum_{j=1}^{\infty}\Big(\frac{m_j}{m_k}\Big)^{1/2} p_{jk}x_j\right)_{k=1}^{\infty}$$

is a linear contraction. Employing Sz.-Nagy's theory of unitary dilations and the spectral theorem he obtained an integral representation

$$p_{jk}^{n} = \sqrt{\frac{m_k}{m_j}} \int_{\mathbb{S}} \mathrm{e}^{\mathrm{i}n\theta}\, \mathrm{d}\mu_{jk}(\theta), \quad n \geqslant 0, \tag{1}$$

with complex measures μ_{jk} on $\mathbb{S} = \{z \in \mathbb{C};\ |z| = 1\}$ satisfying $\overline{\mu}_{jk} = \mu_{kj}$.

To extend these results to general state spaces, Z. Šidák adopted the setting from Nelson's seminal paper [92]. Let p be a Markov transition kernel on a measurable space (X, Σ). Set

$$P\colon\ f \longmapsto \int_X f(y)p(\cdot, \mathrm{d}y), \quad f\colon\ X \longrightarrow \mathbb{R} \text{ bounded } \Sigma\text{-measurable.}$$

Nelson proved that if there is a subinvariant measure μ for p, $P^*\mu \leqslant \mu$, then P may be viewed as a contraction in the space $L^q(\mu)$, $q \in [1, \infty]$. (We will write $P = P_q$, if the dependence on q is important.) Šidák, applying to P_2 the theory of unitary dilations, showed in [9] that supposing the existence of a subinvariant measure μ for p one may find for all $A, B \in \Sigma$, $\mu A < \infty$, $\mu B < \infty$ functions $\boldsymbol{v}_A\colon\ [0, 2\pi] \longrightarrow L^2(\mu; \mathbb{C})$, $v_{A,B}\colon\ [0, 2\pi] \longrightarrow \mathbb{C}$ of bounded variation such that

$$p^n(\cdot, A) = \int_0^{2\pi} \mathrm{e}^{\mathrm{i}nt}\, \mathrm{d}\boldsymbol{v}_A(t), \quad \int_B p^n(x, A)\, \mathrm{d}\mu(x) = \int_0^{2\pi} \mathrm{e}^{\mathrm{i}nt}\, \mathrm{d}v_{A,B}(t), \quad n \geqslant 0,$$

the first identity being valid in $L^2(\mu; \mathbb{C})$. He investigated the continuity properties of the functions $\boldsymbol{v}_A$, $v_{A,B}$ and derived several related representation results, including a sufficient condition for reversibility generalizing the one proved by D. Kendall for countable state spaces.

Closely related results on integral representations are given in [10] for processes with complete connections.

Let us return to the introductory section of [9], devoted to the existence of invariant and subinvariant measures, which is of independent interest. Z. Šidák showed that there exists a finite finitely additive invariant measure for every Markov transition kernel, and, via Yosida-Hewitt decomposition, obtained results on σ-additive invariant measures (analogous procedure was employed later e.g. by S. Foguel, cf. [79]). Šidák's finitely additive invariant measure, which does not look like a very useful object by itself, has been used recently by Zhdanok and Belyakov (see e.g. [105]) to obtain nontrivial results in the ergodic theory of Markov chains. Under Nelson's irreducibility hypothesis,

(2) *all measures* $\nu_x = \sum_{k=1}^{\infty} \frac{1}{2^k} p^k(x, \cdot)$, $x \in X$, *are equivalent*,

Z. Šidák gave a sufficient condition for the existence of a subinvariant measure; the final form of this result may be found in [25]: for any sub-Markovian transition kernel p satisfying (2) there exists a subinvariant measure. (In [25], the σ-algebra Σ is assumed to be countably generated; this hypothesis may be relaxed owing to a result from [85] as noted in [56].) The main goal of the paper [25] is to establish the following dichotomy result describing all possible types of the long time behaviour of an irreducible Markov chain. (This theorem, announced in [23], is likely to be the most influential Šidák's contribution to the Markov chains theory.) Let p be an irreducible sub-Markovian transition kernel and μ its subinvariant measure, which may be chosen equivalent to all ν_x's. Set $\Sigma_0 = \{A \in \Sigma;\ 0 < \mu A < \infty\}$. Then either

$$\sum_{n=1}^{\infty} p^n(x, A) = \infty \quad \text{for all } x \in X \text{ and all } A \in \Sigma_0 \tag{3}$$

(recurrence), or

$$\sum_{n=1}^{\infty} p^n(x, A) < \infty \quad \text{for every } A \in \Sigma_0 \text{ and } \mu\text{-almost all } x \in X \tag{4}$$

(transience). If (3) holds then either

$$\lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} p^k(x, A) > 0 \quad \text{for all } x \in X \text{ and all } A \in \Sigma_0 \tag{5}$$

(positive recurrence) or

$$\lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} p^k(x, A) = 0 \quad \text{for all } x \in X \text{ and all } A \in \Sigma_0$$

(null recurrence). In [56], an example is given showing that (4) need not hold true for all $x \in X$. In the recurrent case, any subinvariant measure is invariant and uniqueness up to a multiplicative constant holds for the invariant measure. Moreover, if p is positive recurrent, any invariant measure is finite, the limit in (5) exists for each $A \in \Sigma$ and defines an invariant measure for p. Consecutively, the problem to obtain an analogous classification of all possible types of limit behaviour under weaker irreducibility hypotheses turned out to be a challenging one. Let us quote at least the papers [84], [103] for results related directly to Šidák's work.

Another problem, also related to the paper [87], which attracted Šidák's attention, is the structure of the spectrum of the operators P_q introduced above. Let p be an irreducible Markov transition kernel with the period d on a countable state space.

Let μ be a subinvariant measure for p, denote by $\sigma_p(P_q)$ the point spectrum of the operator P_q in $L^q(\mu;\mathbb{C})$. It is shown in [18], [20] that

$$\sigma_p(P_q) \cap \mathbb{S} = \bigl\{\mathrm{e}^{2\pi \mathrm{i} k/d};\ k = 0, \dots, d-1\bigr\}, \quad 1 \leqslant q \leqslant \infty,$$

in the positive recurrent case,

$$\sigma_p(P_q) \cap \mathbb{S} = \emptyset, \quad 1 \leqslant q < \infty, \quad \sigma_p(P_\infty) \cap \mathbb{S} = \bigl\{\mathrm{e}^{2\pi \mathrm{i} k/d};\ k = 0, \dots, d-1\bigr\}$$

in the null recurrent case, and

$$\sigma_p(P_q) \cap \mathbb{S} = \emptyset, \quad 1 \leqslant q < \infty, \quad \sigma_p(P_\infty) \cap \mathbb{S} \supset \bigl\{\mathrm{e}^{2\pi \mathrm{i} k/d};\ k = 0, \dots, d-1\bigr\}$$

if p is transient; the last inclusion may be sharp. The same assertions hold true for a sub-Markovian transition kernel satisfying (2) on a general state space X, provided a suitable cyclic decomposition of X exists [29]. In the paper [15], written unfortunately in Czech, several examples are given in which the whole spectrum $\sigma(P_2)$ is computed, not only the unimodular eigenvalues. In particular, it is shown that the situation may be rather complex in the transient case: it may happen that the spectral radius $r(P_2) < 1$ although $\|P_2\| = 1$, or one may have $r(P_2) < 1$, $\|P_2\| < 1$ (even if the measure μ is invariant), or $r(P_2) = \|P_2\| = 1$ may hold; this phenomenon was later rediscovered by Holmes [83] and Vere-Jones (see e.g. [104]). Results on the spectrum of the operator P are based on a thorough analysis of subharmonic functions of P.

An impression cannot be avoided on reading carefully written and detailed Šidák's papers, namely that he chose the functional analytic approach to Markov chains not only due to the strength of the operator theoretic methods but even more because of the clarity of exposition attainable by their use. This impression is further confirmed by the paper [22] (in Czech) showing that ergodic theorems for positive contractions yield ergodic theorems for Markov chains in a very transparent way.

In the sixties, Z. Šidák followed very carefully new results on Markov chains on general state spaces as is witnessed by his rich collection of reprints with copious handwritten remarks on the margins. However, he did not seem to be deeply influenced by them and remained interested mainly in the topics he addressed in his very first papers, looking for strengthened results or shorter and more lucid proofs. In this sense, all the papers on Markov chains treated up to now may be viewed as parts of one whole. The paper [14] stands a bit alone, nonetheless, it contains one of the most interesting Šidák's results. Let $(p_t)_{t \geqslant 0}$ be a semigroup of Markov kernels on a measurable space (X, Σ). Then there exist a zero-dimensional compact Hausdorff space S, a semigroup $(\varrho_t)_{t \geqslant 0}$ of Feller Markov kernels on $(S, \mathrm{Ba}(S))$, $\mathrm{Ba}(S)$ denoting the Baire σ-algebra over S, a mapping $\psi\colon X \longrightarrow S$ and an isomorphism Ψ of Σ onto the algebra of open-and-closed sets in S such that

$$p_t(x, A) = \varrho_t\bigl(\psi(x), \Psi(A)\bigr), \quad t \geqslant 0,\ x \in X,\ A \in \Sigma.$$

In [56] then Z. Šidák showed, using a result of Rosenblatt [95], that the Baire sets may be replaced by the Borel σ-algebra of S, $\varrho_t(x, \cdot)$ becoming regular Borel measures.

The paper [56], published in 1977 but written seven years earlier, has already been mentioned several times above; besides the cited results, other minor corrections and amendments to the previously published papers may be found there.

2.2. Multiple comparisons methods.

Šidák's thoroughly developed understanding of the needs of statistical applications is clearly discernible on reading almost any of his papers on statistics. He liked to feel possible applications even under purely theoretical results. This led him to solving problems of multiple comparisons, to the selection of the best one from several populations and also to the discrimination analysis and cluster analysis.

His first paper [11] on multiple comparisons was motivated by problems that he encountered in medical applications. He proved that the probabilities $P\{\xi_1 < d_1, \dots, \xi_p < d_p\}$ and $P\{|\xi_1| < d_1, \dots, |\xi_p| < d_p\}$, where $\xi_1, \dots, \xi_p$ is a multivariate normal variable with zero expectations and with correlations $\varrho_{ij} = b_i b_j$, $0 \leqslant b_k < 1$, $k = 1, 2, \dots, p$, are non-decreasing functions of b_i for arbitrary positive $d_1, \dots, d_p$.

He further showed that

a) the probabilities $P\{\frac{\xi_1}{s} < d_1, \dots, \frac{\xi_p}{s} < d_p\}$ and $P\{|\frac{\xi_1}{s}| < d_1, \dots, |\frac{\xi_p}{s}| < d_p\}$ are also non-decreasing; here s^2 is an estimate of the assumed common variance of the variables ξ_k and independent of them,

b) the following inequalities hold true for the supposed correlation structure:

$$P\bigl\{\xi_1 < d_1, \dots, \xi_p < d_p\bigr\} \geqslant \prod_{k=1}^{p} P\{\xi_k < d_k\}, \tag{6}$$

$$P\bigl\{|\xi_1| < d_1, \dots, |\xi_p| < d_p\bigr\} \geqslant \prod_{k=1}^{p} P\{|\xi_k| < d_k\}, \tag{7}$$

$$P\Bigl\{\frac{\xi_1}{s} < d_1, \dots, \frac{\xi_p}{s} < d_p\Bigr\} \geqslant \prod_{k=1}^{p} P\Bigl\{\frac{\xi_k}{s} < d_k\Bigr\}, \tag{8}$$

$$P\Bigl\{\Bigl|\frac{\xi_1}{s}\Bigr| < d_1, \dots, \Bigl|\frac{\xi_p}{s}\Bigr| < d_p\Bigr\} \geqslant \prod_{k=1}^{p} P\Bigl\{\Bigl|\frac{\xi_k}{s}\Bigr| < d_k\Bigr\}. \tag{9}$$

His approach to the comparison of several treatments with control in the case of unequal number of observations in groups is based on the above results.

At the session of ISI in Beograd, 1965, Šidák for the first time presented [21] his proof that inequality (7) holds true for a general covariance structure of multivariate normal distribution and that its "studentized" analogue (9) holds true, too. The proof was published in 1967 [28] and almost simultaneously and independently was the inequality proved by Scott [98] and Khatri [88]. Consequently, the inequality bears the names of all these three authors.

During his appointment at Michigan State University, Šidák published several papers concerning this subject. The paper [24] presented a survey of some methods of constructing confidence intervals for the means of multivariate normal distributions. [27] is devoted to the multivariate studentized analogue to inequality (7) in the case of a correlation structure of the form $\varrho_{ij} = \lambda_i \lambda_j$, $0 \leqslant \lambda_i$, $\lambda_j \leqslant 1$. In [33], [26] he extended Slepian's result [99] to rectangular regions (Slepian proved that the probability $P\{\xi_1 < d_1, \dots, \xi_p < d_p\}$ is a non-decreasing function of correlations for multivariate normal variables). Šidák gave counterexamples demonstrating that the complete analogue of Slepian's result does not hold true in general for the "two-sided" probability $P\{|\xi_1| < d_1, \dots, |\xi_p| < d_p\}$. On the other hand, it is true in an important special case, where the correlations are of the form $\lambda_i \lambda_j \varrho_{ij}$, ϱ_{ij} being a fixed correlation matrix, and it is also true locally in the case of equicorrelated variables. In [35] he presented an analogue to the previous result for the Student distribution and showed that Scott's [98] proof of the inequality

$$P\left\{\left|\frac{\xi_1}{s_1}\right| < d_1, \dots, \left|\frac{\xi_p}{s_p}\right| < d_p\right\} \geqslant \prod_{k=1}^{p} P\left\{\left|\frac{\xi_k}{s_k}\right| < d_k\right\},$$

where s_k^2 are estimates of variances independent of ξ_k, is incorrect for the general Student distribution. In the paper [47] written in 1971 but published in 1975, he also pointed out that "the method used by C. G. Khatri [89] and by A. Scott [98] for proving certain inequalities for multivariate normal distribution seems to be powerful, but is incorrect". In [40], he extended some of his previous results to the case of intersection of symmetric convex regions. He proved that for any partition $\Xi_1, \Xi_2, \dots, \Xi_q$ of the vector of normal variables $\xi_1, \dots, \xi_p$, with $E(\xi_k) = 0$ and with correlations $\varrho_{ij} = b_i b_j$, $-1 \leqslant b_i \leqslant 1$ $i, j = 1, \dots, p$, the inequalities $P\{\Xi_1 \in \mathscr{A}_1, \dots, \Xi_q \in \mathscr{A}_q\} \geqslant P\{\Xi_1 \in \mathscr{A}_1\} P\{\Xi_2 \in \mathscr{A}_2, \dots, \Xi_q \in \mathscr{A}_q\} \geqslant \prod_{i=1}^{q} P\{\Xi_i \in \mathscr{A}_i\}$ hold; here $\mathscr{A}_1, \mathscr{A}_2, \dots, \mathscr{A}_q$ either are symmetric convex sets or they all are complements of such sets. For equicorrelated normal variables with equal variances and equal expectations he in particular proved that for any Borel set $\mathscr{E}$ in $\mathbb{R}_1$, the probability $P\{\xi_1 \in \mathscr{E}, \xi_2 \in \mathscr{E}, \dots, \xi_p \in \mathscr{E}\}$ is a non-decreasing function of the correlation coefficient ϱ.

Y. L. Tong [101] proved for a non-negative random variable X that

$$EX^k \geqslant (EX^{k/s})^s \geqslant (EX)^k + \left[EX^{k/s} - (EX)^{k/s}\right]^s; \quad k \geqslant s \geqslant 1.$$

Šidák in [46] and [39] generalized these inequalities and applied them to the comparison of treatments with one control. He also presented their interesting applications to multivariate equicorrelated normal, t, χ^2, Poisson and exponential distributions. [46] is his last paper written on multiple comparisons. His methods of constructing rectangular confidence regions have been naturally incorporated in many statistical software packages.

Z. Šidák worked on multiple comparisons having in mind statistical applications. It is worth noticing that these results have also found a new interpretation in the theory of Gaussian measures on Banach spaces. Let ν be a centred Gaussian measure on a real separable Banach space E; the Gaussian correlation conjecture states that

$$\nu(A \cap B) \geqslant \nu(A)\nu(B) \tag{10}$$

for any symmetric convex subsets A, B of E. In this setting, Šidák's results from [28], [33] say that (10) holds provided one of the sets A, B is a slab of the form $\{x \in E;\ |x^*(x)| \leqslant 1\}$ for a continuous linear functional x^* in the dual of E. The reader may find related results on Gaussian correlation inequalities, their applications and further references e.g. in the book [75], §4.6, or in the papers [97], [91].

2.3. Rank tests.

A two-sided improvement of Rosenbaum's [94] test of difference in location of two populations is the earliest Šidák's work [3] in nonparametric statistics. The idea of the test was independently used by Tukey [102]. The test appeared to have some optimum properties for uniform distributions. Šidák [58] later compared (by Monte Carlo methods) some rank tests that are optimal for uniform distributions and published the power functions of the Haga test (introduced in [80]), of the symmetrized E-test suggested in [30] and of the Wilcoxon test. The last one appeared to be never better than the other two tests in this situation.

In [32] he used an idea very similar to the alignment principle concept of Hodges and Lehman [82] and suggested a general distribution free procedure for testing the null hypothesis of two distributions differing at most by the value of some parameter against the alternative of some additional difference between them.

In the first half of the seventies, Šidák published a series of tables facilitating the use of rank tests for the two sample problem. They are as follows: tables [42] for the Fisher-Yates-Terry-Hoeffding test and for the van der Waerden test, tables [44] for the two-sample Savage rank test, tables [43] for the Capon [77] and for the Klotz [90] test for the two-sample scale problem, tables [49] for the two-sample median test and tables [55] for Hájek's E-test mentioned above.

The interest in pattern recognition, discrimination analysis and in the application of the methods developed led him to three particular problems, the solution of which he proposed to base on rank tests. The results were presented at DIANA conferences on discrimination analysis. In [59] he proposed a general principle for estimating the change point in a sequence of random variables and presented a distribution free estimate based on the Mann-Whitney test. Distribution free discriminant procedures for circular data are developed in [62]. The third of the papers [65] is devoted to nonparametric discrimination of shapes of response curves and to discrimination of permutations.

However, the most important Šidák's contribution to this research area is represented by the outstanding monograph Theory of rank tests [30], which was written jointly with his friend and former colleague Jaroslav Hájek. The book was published in 1967 and was several times reprinted in the U.S.A. The Russian translation [36] appeared in 1971. The monograph concentrates mainly on problems concerning location, scale parameters and contiguous alternatives. The authors managed to present the theory in a very compact, clear and lucid way. The book remained unique for years not only due to its rigorous and elegant style of exposition, but also due to the extensive range of issues covered. Unfortunately, the early death of J. Hájek in 1974 ended the fruitful cooperation of the authors.

For many years that followed, Šidák was pressed by statisticians from all over the world to prepare an updated edition. He hesitated because of his health but finally he took up the task and together with P. K. Sen prepared a new modernized edition in the second half of the nineties. It was published in 1999, a few months before Šidák's death. The authors offered the work as a tribute to the memory of Professor J. Hájek. Sadly, the book happened to become a memorial of Z. Šidák, as well.

The authors left the structure of the original text unchanged but incorporated new results into it and added two new chapters. Among others, Hájek's results [81] on asymptotic distribution theory of linear rank statistics for general alternatives were included. Martingale characterizations of various rank statistics were used for treating the properties of rank statistics in the finite case and also in asymptotics. Attention was newly paid to a treatment of rank tests under various types of censoring especially in connection with life testing. The authors concentrated mainly on the presentation of ideas and the more interested reader is invited, by many references in the text, to make a detour through the original papers.

The topics considered in the two new chapters completing the book are closely related to the alignment principle. They reflect the developments of the theory of rank estimates and aligned rank tests that took place in the course of the 30 years long period between the two editions.

The papers of P. K. Sen and J. Jurečková are the main source of issues covered there, in particular both the results and proofs in the section on the asymptotic linearity of rank statistics in regression parameters are due to Jurečková [86].

Briefly treated are rank estimates of shift between two samples, of shift of one sample and of regression parameters. Deeper attention is paid to aligned rank tests in linear models, especially to the two-way layout and to aligned rank statistics for subhypothesis testing. Finally, the rank with other robust procedures are compared.

The newly treated topics and results constitute more than one third (168 pages) of the revised edition. The book represents not only a valuable up to date monograph and a basic reference book for specialists. Doubtlessly, it will also become one of the important sources of knowledge and inspiration to the coming generation of statisticians.

2.4. Miscellaneous results, other activities.

At the beginning of his scientific activity, Z. Šidák aimed at strengthening Bahadur's results [74] on the characterization of the conditional expectation operators among all bounded linear operators in $L^2(\mathscr{F}) = L^2(\Omega, \mathscr{F}, \mu)$, $(\Omega, \mathscr{F}, \mu)$ being a probability space. His approach described in the paper [4] (even recently frequently referred to) is based on the following elegant observation: a closed linear subspace $M \subseteq L^2(\mathscr{F})$ is of the form $M = L^2(\mathscr{G})$ for a sub-σ-algebra $\mathscr{G} \subseteq \mathscr{F}$ if and only if M is a Banach lattice containing constants.

Šidák's paper [31] reflects the outburst of interest in 3D interpretations of microscopic images in the sixties initiated perhaps by the constitution of stereology as an independent scientific branch in 1961 (stereology may be considered as a sampling theory of populations having geometric structure; the sampling means are intersection and projection). Šidák considered a rather general problem of a slice (thickness τ) through a transparent material embedding opaque spherical particles (such a situation is usually met in the transmission electron microscopy). Estimating the properties (size distribution, its moments etc.) of particles from the projection of such a slice requires a solution of an ill-posed problem which is typical for inversions of integral equations; the limit $\tau \to 0$ is the famous Wicksell's problem—"a playground for the application of numerical methods for solving ill-posed problems" [100]. Although preceded and followed by many similar solutions, Šidák's paper is still remembered and cited (see e.g. [96]) not only for its clarity and mathematical rigour but also for the ingenious proposal to combine estimates obtained from two slices of different thickness.

A small collection of papers [60] (survey paper), [64] and [70] demonstrates the last field of Šidák's interest—selecting and ordering of populations. From variable goals of this statistical branch with extremely wide scope of applications ranging from poultry science to quality control and psychology, Šidák focused mainly on the problem of selecting the better of two populations; trinomial populations are treated in [64], selection strategy for this problem is proposed and tested by simulation in [70], which is the last Šidák's scientific paper.

In the course of his professional life, Šidák was used to write also popular papers covering learning, mathematical education, linguistic and scientific terminology ([5], [6], [13], [16], [17], [45], [71], [72]) and statistical approaches to various competitions [66–69].

Šidák's contribution to both Czech and international scientific life was significant. For more then 30 years he edited the journal Applications of Mathematics. He founded and led the seminar on multivariate statistical methods in Mathematical Institute, Praha. A series of summer schools on pattern recognition (KOBRA, 1979), on analysis of real data (ANACONDA, 1981), on factor analysis (FATIMA, 1985; FATIMA II, 1986) and international conferences on discrimination analysis (DIANA, 1982; DIANA II, 1986; DIANA III, 1990) were based on the work of this seminar. He was the organizer and the good spirit of all these workshops and conferences. He worked in program committees of European Meeting of Statisticians (1979, Varna) and of COMPSTAT (1984, Praha); he also co-edited the proceedings of this congress [61].

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