Jan Němec

An alternative proof of Painlevé's theorem

Applications of Mathematics, Vol. 45 (2000), No. 4, 291–299

Persistent URL: `http://dml.cz/dmlcz/134440`

AN ALTERNATIVE PROOF OF PAINLEVÉ'S THEOREM*

Jan Němec, Praha

(Received October 20, 1998)

Abstract. In this article we show some aspects of analytical and numerical solution of the n-body problem, which arises from the classical Newtonian model for gravitation attraction. We prove the non-existence of stationary solutions and give an alternative proof for Painlevé's theorem.

Keywords: n-body problem, ordinary differential equations, Painlevé's theorem

MSC 2000: 70F10

The gravitation model, suggested by Isaac Newton, describes the behaviour of mass points which attract each other only by gravitational forces. The mathematical formulation is given by $3n$ ordinary differential equations in the three-dimensional space with some initial conditions. The equations (which are derived in many publications, cf. [1], [8] and [4]) are

$$\ddot{\mathbf{u}}_i(t) = \mathbf{f}_i\big(\mathbf{u}_1(t), \ldots, \mathbf{u}_n(t)\big) = \varkappa \sum_{\substack{j=1 \\ j\neq i}}^{n} m_j \frac{\mathbf{u}_j(t) - \mathbf{u}_i(t)}{\|\mathbf{u}_j(t) - \mathbf{u}_i(t)\|^3}, \quad i = 1, \ldots, n. \tag{1}$$

We keep the following notation:

- **bold symbol**: vector function, variable or constant,
- *italic symbol*: scalar function, variable or constant,
- $\Omega \subset \mathbb{R}^{3n}$: $\Omega = \{(\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_n);\ \mathbf{u}_i \in \mathbb{R}^3, \mathbf{u}_i \neq \mathbf{u}_j, i \neq j\}$ open set,
- $\Gamma \subset \mathbb{R}$: open interval,
- $n > 1$: number of bodies,
- $\varkappa \doteq 6.67{\cdot}10^{-11}\mathrm{kg}^{-1}\mathrm{m}^3\mathrm{s}^{-2}$: gravitational constant,

* This research was supported by grant No. 201/98/1452 of the Grant Agency of the Czech Republic.

- $\mathbf{u}_i(t)$: position of the i-th body at time t,
- m_i: mass of the i-th body,
- $\|\mathbf{u}\| = \sqrt{{}_1u^2 + {}_2u^2 + {}_3u^2}$: the Euclidean norm for $\mathbf{u} = ({}_1u, {}_2u, {}_3u) \in \mathbb{R}^3$
- $\|\mathbf{u}\| = \sum\limits_{i=1,\dots,n} \|\mathbf{u}_i\|$: for $\mathbf{u} = (\mathbf{u}_1, \dots, \mathbf{u}_n) \in \mathbb{R}^{3n}$.

First of all we have to say that this model contains several simplifications of the reality. The first simplification is neglecting finite velocity of the information spread. It is clear from equation (1) that any information about masses and position is available immediately in the whole space, which means that the velocity of the information spread is infinite. To avoid this simplification we would have to consider delay differential equations, but for these equations there are problems with the existence and uniqueness of the solution. Some numerical experience with a model considering delay is given in [6]. Another simplification against the theory of relativity is the independence of mass on velocity. This simplification is not so serious, because in the usual situations the mutual velocities are small with respect to the light speed. And finally we consider only a mass point instead of a body. This simplification is correct for a spherically homogeneous body. Of course, this is not true for real planets, stars and other objects computed, but the difference from reality is small, too. Now we explore equations (1) from a mathematical point of view. This is a set of ordinary differential equations of the second order. We can use many results from literature given for this class of equations. The most important thing is as usual the existence and uniqueness of the solution. To obtain these assertions we will use a well-known theorem.

Theorem 1. *Let a function* $\mathbf{f}\colon \Omega \mapsto \mathbb{R}^{3n}$ *be locally Lipschitz continuous. Then for each* $\mathbf{u_0} \in \Omega$, $\mathbf{v_0} \in \mathbb{R}^{3n}$ *there exist a unique* $\Gamma \subset \mathbb{R}$ *and a function* $\mathbf{u} \in C(\Gamma, \Omega)$ *which solves equations* (1), *where* Γ *is an open set (finite or infinite),* $0 \in \Gamma$, *and* $\mathbf{u}$ *satisfies the initial conditions* $\mathbf{u}(0) = \mathbf{u_0}$, $\dot{\mathbf{u}}(0) = \mathbf{v_0}$.

Proof is found in almost every book on ordinary differential equations, cf. [7], p. 200–204, or [10]. □

To use this theorem, we have to prove that the function $\mathbf{f}(\mathbf{u}) = \big(\mathbf{f}_1(\mathbf{u}), \dots, \mathbf{f}_n(\mathbf{u})\big)$ in (1) is locally Lipschitz continuous on Ω. From formula (1) we see that the function $\mathbf{f}(\mathbf{u})$ is well-defined on Ω. Moreover, the partial derivatives of $\mathbf{f}(\mathbf{u})$ with respect to each variable exist and are continuous. Now we can use the next theorem.

Theorem 2. *Let* $\mathbf{f}\colon \Omega \mapsto \mathbb{R}^{3n}$, *let* $\partial\mathbf{f}/\partial_i u$ *exist for each* $i = 1, \dots, 3n$, *and let these derivatives be continuous. Then* $\mathbf{f}$ *is locally Lipschitz continuous.*

Proof. [7], p. 210. □

The above two theorems establish the existence and uniqueness of the solution. The next important question is the dependence of a solution on the initial conditions and the right-hand side. This is important mainly in practical computations, because the initial conditions are usually obtained from measuring or experiment, which include some observation error. For this reason we need continuous dependence on the initial conditions at least. A similar situation is with the dependence on perturbations of the right-hand side, because we consider an isolated system, which means we neglect the influence of distant space bodies and other non-gravitational forces (light pressure, electromagnetic forces etc.). The following theorem solves this situation.

Theorem 3. *Let the assumptions of Theorems* 1, 2 *be satisfied and let* $\mathbf{f}(\mathbf{u}, \varepsilon)\colon \Omega \times \mathbb{R} \mapsto \mathbb{R}^{3n}$ *be from* $C^2(\Omega \times \mathbb{R}, \mathbb{R}^{3n})$. *Then there exist*

$$\frac{\partial \mathbf{u}(t, \mathbf{u_0}, \mathbf{v_0}, \varepsilon)}{\partial_j u_0}, \quad \frac{\partial \mathbf{u}(t, \mathbf{u_0}, \mathbf{v_0}, \varepsilon)}{\partial_j v_0}, \quad \frac{\partial \mathbf{u}(t, \mathbf{u_0}, \mathbf{v_0}, \varepsilon)}{\partial \varepsilon}$$

for every $t \in \Gamma$ *and* $j = 1, \ldots, 3n$, *where the symbol* $\mathbf{u}(t, \mathbf{u_0}, \mathbf{v_0}, \varepsilon)$ *stands for the solution* $\mathbf{u}(t)$ *satisfying the initial conditions* $\mathbf{u}(0) = \mathbf{u_0}$, $\dot{\mathbf{u}}(0) = \mathbf{v_0}$ *and the ordinary differential equation* (1) *with the right-hand side* $\mathbf{f}(\mathbf{u}, \varepsilon)$.

P r o o f. [7], p. 243. □

Now we present theorems considering autonomous differential equations and their application to our system.

Theorem 4. *Let* $\mathbf{u}(t)$ *solve equation* (1) *on an interval* (a, b). *Then the function* $\mathbf{v}(t) = \mathbf{u}(t - d)$ *is a solution of equation* (1) *on the interval* $(a + d, b + d)$.

P r o o f. We can easily differentiate the function $\mathbf{v}(t)$ with respect to t. □

Theorem 5. *Only one trajectory of the solution passes through each point of the set* $\{(\mathbf{u}, \mathbf{v});\, \mathbf{u} \in \Omega,\, \mathbf{v} \in \mathbb{R}^{3n}\}$.

P r o o f. [3], p. 257. □

We have to point out that the term trajectory is considered in the sense of the phase space of positions and velocities and hence this theorem does not eliminate the possibility of intersection of trajectories (in the natural sense), provided the velocities are different in the common point.

Theorem 6. *Let* $\mathbf{u}(t)$ *solve equation* (1). *Then one of the following situations arises:*

(i) *Constant solution:* $\mathbf{u}(t) = \mathbf{u_0}$, $\mathbf{f}(\mathbf{u_0}) = 0$ *(later we shall see that this situation cannot occur in our problem for* $n > 1$*).*

(ii) *Periodical solution: Non-constant solution and there exist t_1, t_2, $t_1 \neq t_2$ for which $\mathbf{u}(t_1) = \mathbf{u}(t_2)$ and also $\dot{\mathbf{u}}(t_1) = \dot{\mathbf{u}}(t_2)$. In this case the solution exists on the whole real axis.*

(iii) *Non-periodical solution: For all t_1, t_2, $t_1 \neq t_2$, either $\mathbf{u}(t_1) \neq \mathbf{u}(t_2)$ or $\dot{\mathbf{u}}(t_1) \neq \dot{\mathbf{u}}(t_2)$. This is the only case, which can include collisions.*

P r o o f. [3], p. 258. □

Now we exclude case (i) from the previous theorem.

Theorem 7. *There exists no constant solution for the n-body problem ($n > 1$).*

P r o o f. The proof will be given by way of contradiction. We assume that there exists $\mathbf{u} \in \Omega$ such that $\mathbf{f}(\mathbf{u}) = 0$. We can suppose without loss of generality that $\|\mathbf{u}_1\| = \max\limits_{j=1,\dots,n} \|\mathbf{u}_j\|$. Otherwise we can always change the numeration. Next we assume that $\mathbf{u}_1 = ({}_1u_1, {}_2u_1, {}_3u_1) = (\|\mathbf{u}_1\|, 0, 0)$. If this is not satisfied, we rotate the axes. Now we can compute the first component of the vector function $\mathbf{f}$,

$$
{}_1f_1 = \varkappa \sum_{j=2}^{n} m_j \frac{{}_1u_j(t) - {}_1u_1(t)}{\|\mathbf{u}_j(t) - \mathbf{u}_1(t)\|^3}. \tag{2}
$$

From our assumption ${}_1u_1 = \|\mathbf{u}_1\| = \max\limits_{j=1,\dots,n} \|\mathbf{u}_j\|$ we obtain ${}_1u_j - {}_1u_1 \leqslant 0$. However for the equality we have $\mathbf{u}_1 = \mathbf{u}_j$ and this cannot be true because $\mathbf{u} \in \Omega$. This implies that all the terms on the right-hand-side of formula (2) are less than zero and consequently the whole right-hand side is less than zero. This contradicts our assumption that $\mathbf{f}(\mathbf{u}) = 0$ and the theorem is proved. □

From Theorem 1 we find that the solution exists on an open interval $\Gamma \in \mathbb{R}$ so that the following situations can occur:

(a) $\Gamma = (-\infty, \infty)$,
(b) $\Gamma = (-\infty, \beta)$; $\beta \in \mathbb{R}$,
(c) $\Gamma = (\alpha, \infty)$; $\alpha \in \mathbb{R}$,
(d) $\Gamma = (\alpha, \beta)$; $\alpha, \beta \in \mathbb{R}$.

Let us consider the situation when the interval Γ is bounded from at least one side, that means situations (b), (c), (d). From equation (1) we see that this situation may occur when the bodies collide. The question is whether all solutions on bounded intervals are terminated by collisions. An incomplete answer to this question is given by the next theorem. (We will deal with an interval bounded from the right-hand side, that means cases (b) and (d). The left-hand-side bounded intervals could be investigated in the same way.)

Theorem 8 (Painlevé). *Let $\alpha \in \mathbb{R} \cup \{-\infty\}$, $\beta \in \mathbb{R}$ and let $\mathbf{u}(t)\colon (\alpha, \beta) \mapsto \Omega$ be a solution of equation* (1). *Then*

$$\lim_{t \to \beta^-} \min_{i \neq j} \|\mathbf{u}_i(t) - \mathbf{u}_j(t)\| = 0. \tag{3}$$

In the first place we should point out that (3) is weaker than the following assertion (4) which means a real collision:

$$\exists i, j\colon \quad i \neq j, \quad \lim_{t \to \beta^-} \|\mathbf{u}_i(t) - \mathbf{u}_j(t)\| = 0. \tag{4}$$

R e m a r k . Here we introduce two proofs of Theorem 8. The first is commonly used and is given for example in [8]. The present proof has arisen from strengthening Theorem 1. In literature the existence of an interval Γ on which we have a unique solution is proved, and moreover, the minimal length of the interval Γ is estimated. This length depends on $\max_i \|\dot{\mathbf{u}}_i(0)\|$, $\min_{i\neq j} \|\mathbf{u}_i(0) - \mathbf{u}_j(0)\|$, $\|\mathbf{f}(0)\|$[1]. From definition (1) of the function $\mathbf{f}(\mathbf{u}(t))$ it is clear that $\|\mathbf{f}(0)\|$ can be estimated by (from (1) using the triangle inequality)

$$\|\mathbf{f}(0)\| \leqslant \frac{q}{\min_{i \neq j} \|\mathbf{u}_i(0) - \mathbf{u}_j(0)\|^2}, \tag{5}$$

where q is a constant depending on masses of bodies, but not on their positions and velocities. In the next theorem we estimate $\max_i \|\dot{\mathbf{u}}_i(0)\|$ similarly by some constant depending on $\min_{i\neq j} \|\mathbf{u}_i(0) - \mathbf{u}_j(0)\|$ which together gives us that the minimum length of the interval Γ depends only on $\min_{i\neq j} \|\mathbf{u}_i(0) - \mathbf{u}_j(0)\|$. And that is what we wanted to prove: if the interval Γ is bounded then (3) holds.

By the energy conservation law (cf. [1] or [8]) the total energy of the n-body problem is constant.

Theorem 9. *Let $E \in \mathbb{R}$ be the total energy of the n-body system and let for a time $t \in \mathbb{R}$ there exist $\varepsilon > 0$ such that*

$$\min_{i \neq j} \|\mathbf{u}_i(t) - \mathbf{u}_j(t)\| \geqslant \frac{\varepsilon}{2}. \tag{6}$$

[1] We mean the length of interval containing the point zero. This restriction can be made in accordance with Theorem 4.

Then

$$\max_i \|\dot{\mathbf{u}}_i(t)\| \leqslant D = \sqrt{\frac{2}{m}\Big(E + \frac{2\varkappa}{\varepsilon}\sum_{i\neq j} m_j m_i\Big)}, \tag{7}$$

where $m = \min_{j=1,\ldots,n} m_j$.

P r o o f. We start from the formula for the total energy (see [8])

$$E = \frac{1}{2}\sum_{i=1}^{n} m_i \|\dot{\mathbf{u}}_i(t)\|^2 - \varkappa \sum_{i\neq j} \frac{m_i m_j}{\|\mathbf{u}_i(t) - \mathbf{u}_j(t)\|}. \tag{8}$$

The second term on the right-hand side can be estimated using (6) and we move this term to the left-hand side:

$$E + \frac{2\varkappa}{\varepsilon}\sum_{i\neq i} m_i m_j \geqslant \frac{1}{2}\sum_{i=1}^{n} m_i\|\dot{\mathbf{u}}_i(t)\|^2 \geqslant \frac{1}{2} m \sum_{i=1}^{n} \|\dot{\mathbf{u}}_i(t)\|^2 \geqslant \frac{1}{2} m \max_i \|\dot{\mathbf{u}}_i(t)\|^2. \tag{9}$$

The required inequality can be obtained after dividing both sides of the inequality (9) by $\frac{1}{2}m$ and extracting the roots. □

At this moment it is possible to say that according to Remark following Theorem 8, Painlevé's theorem is proved.

In the next part we present another, a more natural proof (from the author's point of view). This proof is based on the theorem on a compact set.

Theorem 10 (on a compact set). *Let a set $\Lambda \subset \Omega$ be compact, $\alpha \in \mathbb{R} \cup \{-\infty\}$, $\beta, B \in \mathbb{R}$ and let $\mathbf{u}\colon (\alpha, \beta) \mapsto \Omega$ be the maximal solution of problem* (1)*. Then there exists $\beta_1 \in (\alpha, \beta)$ such that $\mathbf{u}(t) \notin \Lambda$ or $\max_{j=1,\ldots,n} \|\dot{\mathbf{u}}(t)\| > B$ for all $t \in (\beta_1, \beta)$.*

P r o o f. [7], p. 216. □

Theorem 11. *Let $\alpha \in \mathbb{R} \cup \{-\infty\}$, $\beta \in \mathbb{R}$ and let $\mathbf{u}\colon (\alpha, \beta) \mapsto \Omega$ be the maximal solution of problem* (1)*. Then there exists a sequence $\{t_i\}_{i=1}^{\infty}$, $\lim_{i\to\infty} t_i = \beta^-$, $t_i < \beta$ for all $i = 1, 2, \ldots$, and there exist $j, k \in \{1, \ldots, n\}$, $j \neq k$, such that*

$$\lim_{i\to\infty} \|\mathbf{u}_j(t_i) - \mathbf{u}_k(t_i)\| = 0. \tag{10}$$

P r o o f. The proof makes use of the previous theorem. We know that for all $\Lambda_i = \{\mathbf{u}\colon \|\mathbf{u}_k\| \leqslant i,\ \|\mathbf{u}_j - \mathbf{u}_k\| \geqslant \frac{1}{i}\ \forall k, j = 1, \ldots, n,\ k \neq j\}$ there exists β_i such that for all $t \in (\beta_i, \beta)$ we have $\mathbf{u}(t) \notin \Lambda_i$ or $\max\limits_{j=1,\ldots,n} \|\dot{\mathbf{u}}_i(t)\| > i$ (the sets Λ_i are closed and bounded which implies that they are compact and for B in Theorem 10 we take i). So we can select a subsequence which has at least one of the following properties:

(y) $\lim\limits_{i\to\infty} \|\mathbf{u}(t_i)\| = \infty$,

(yy) $\lim\limits_{i\to\infty} \|\dot{\mathbf{u}}(t_i)\| = \infty$,

(yyy) $\lim\limits_{i\to\infty} \|\mathbf{u}_k(t_i) - \mathbf{u}_j(t_i)\| = 0$.

Consequently, we prove that (y) and (yy) imply (yyy) and that is the assertion of the theorem. So let the case (y) be true. Because the function $\mathbf{u}$ has two continuous derivatives, we can estimate

$$\|\mathbf{u}_k(t_i)\| \leqslant \|\mathbf{u}_k(\tilde{\alpha})\| + (t_i - \tilde{\alpha}) \sup_{t\in(\tilde{\alpha},t_i)} \|\dot{\mathbf{u}}_k(t)\|. \tag{11}$$

(If $\alpha \in \mathbb{R}$ then $\tilde{\alpha} = \alpha$. Otherwise $\tilde{\alpha}$ is an arbitrary sufficiently small real number.) From this estimation we get the existence of a subsequence $\{s_i\}$ which satisfies (yy), since the term on the left-hand side tends to infinity for $i \longrightarrow \infty$, the first term on the right-hand side is bounded, and since $(t_i - \tilde{\alpha})$ tends to zero. Hence, the last term has to be infinity. In the sequel we denote this subsequence again by $\{t_i\}$. A similar procedure is used for the proof of the second implication:

$$\begin{aligned} \|\dot{\mathbf{u}}_k(t_i)\| &\leqslant \|\dot{\mathbf{u}}_k(\tilde{\alpha})\| + (t_i - \tilde{\alpha}) \sup_{t\in(\tilde{\alpha},t_i)} \|\ddot{\mathbf{u}}_k(t)\| \\ &\leqslant \|\dot{\mathbf{u}}_k(\tilde{\alpha})\| + \frac{(t_i - \tilde{\alpha})M}{\inf\limits_{t\in(\tilde{\alpha},t_i)} \min\limits_{j\neq k} \|\mathbf{u}_k(t) - \mathbf{u}_j(t)\|^2}. \end{aligned} \tag{12}$$

And we see that there exists a subsequence for which $\lim\limits_{i\to\infty} \min\limits_{k\neq j} \|\mathbf{u}_k(t_i) - \mathbf{u}_j(t_i)\| = 0$ holds (using the same considerations like after (11)). However, there is a finite number of bodies and so we can find at least one pair which has property (yyy). $\square$

P r o o f (Painlevé's theorem). Now we have all auxiliary assertions to be able to give an easy and straightforward proof of Painlevé's theorem. The proof will be given by way of contradiction. Let there exist $\varepsilon > 0$ such that for all $\delta > 0$ there is $\tilde{t} \in (\beta - \delta, \beta)$ for which

$$\min_{i\neq j} \|\mathbf{u}_i(\tilde{t}) - \mathbf{u}_j(\tilde{t})\| \geqslant \varepsilon. \tag{13}$$

So we take $\delta = \frac{\varepsilon}{8D}$, where D is defined in Theorem 9. Now we take the smallest η which satisfies

$$\min_{j\neq k} \|\mathbf{u}_j(\tilde{t} + \eta) - \mathbf{u}_k(\tilde{t} + \eta)\| = \|\mathbf{u}_{\tilde{j}}(\tilde{t} + \eta) - \mathbf{u}_{\tilde{k}}(\tilde{t} + \eta)\| = \frac{\varepsilon}{2}. \tag{14}$$

The existence of η is ensured by Theorem 11 and the continuity of the function $\mathbf{u}$. Moreover, it is clear that $\eta < \delta$. Now we obtain the inequality

$$\begin{aligned}(15)\qquad \frac{\varepsilon}{2} &= \|\mathbf{u}_{\tilde{j}}(\tilde{t}+\eta) - \mathbf{u}_{\tilde{k}}(\tilde{t}+\eta)\| \\ &\geqslant \|\mathbf{u}_{\tilde{j}}(\tilde{t}) - \mathbf{u}_{\tilde{k}}(\tilde{t})\| - \eta\Big(\sup_{\tau\in(\tilde{t},\tilde{t}+\eta)} \|\dot{\mathbf{u}}_{\tilde{j}}(\tau)\| + \sup_{\tau\in(\tilde{t},\tilde{t}+\eta)} \|\dot{\mathbf{u}}_{\tilde{k}}(\tau)\|\Big),\end{aligned}$$

where the second term on the right-hand side can be estimated using Theorem 9 via

$$(16)\qquad \sup_{\tau\in(\tilde{t},\tilde{t}+\eta)} \|\dot{\mathbf{u}}_{\tilde{j}}(\tau)\| + \sup_{\tau\in(\tilde{t},\tilde{t}+\eta)} \|\dot{\mathbf{u}}_{\tilde{k}}(\tau)\| \leqslant 2D.$$

Now we see that

$$(17)\qquad \frac{\varepsilon}{2} \leqslant 2\eta D < 2\delta D = \frac{\varepsilon}{4},$$

which is impossible and hence the theorem is proved. □

Now let us go back to our question whether the end of any interval means a collision. The answer for $n \leqslant 3$ is yes due to Painlevé (the proof is based on the triangle inequality). For $n \geqslant 5$ the answer is no and this result is described in [11]. For $n = 4$ this problem is still open.

From Theorems 1 and 3 it is clear that our problem is well-defined on some interval and can be correctly solved by numerical methods. Problems arising in numerical solution are described in details in [5] and [9]. The most common methods for solving this problem are the Runge–Kutta methods and their modifications for second order differential equations. In [9] special methods are given, which keep constant energy and momentum.

References

[1] *P. Andrle*: Foundation of Celestial Mechanics. Academia, Praha, 1971. (In Czech.)
[2] *P. Andrle*: Celestial Mechanics. Academia, Praha, 1987. (In Czech.)
[3] *J. Kofroň*: Ordinary Differential Equations in Real Space. Univerzita Karlova, Praha, 1990. (In Czech.)
[4] *M. Křížek*: On the three-body problem. Rozhledy mat.-fyz. *70* (1992), 105–112. (In Czech.)
[5] *M. Křížek*: Numerical experience with the three-body problem. Comput. Appl. Math. *63* (1995), 403–409.
[6] *M. Křížek*: Numerical experience with the finite speed of gravitational influence. Math. Comput. Simulation *50* (1999), 237–245.
[7] *J. Kurzweil*: Ordinary Differential Equations. Elsevier, Amsterdam, 1986.
[8] *Ch. Marchal*: The Three Body Problem. Elsevier, Amsterdam, 1990.
[9] *A. Marciniak*: Numerical Solution of the N-Body Problem. Reidel Publishing Company, Dordrecht, 1985.

[10] *P. Painlevé*: Leçons sur la Théorie Analytic des Equations Différentielles. Hermann, Paris, 1897.

[11] *D. G. Saari, Z. Xia*: Off to infinity in finite time. Notices Amer. Math. Soc. *42* (1995), 538–546.

Author's address: *Jan Němec*, Mathematical Institute, Academy of Sciences, Žitná 25, 115 67 Prague 1, Czech Republic, e-mail: `nemec@math.cas.cz`.