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LOCALLY MOST POWERFUL RANK TESTS FOR TESTING RANDOMNESS AND SYMMETRY

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Abstract. Let X_i, $1 \leqslant i \leqslant N$, be N independent random variables (i.r.v.) with distribution functions (d.f.) $F_i(x, \Theta)$, $1 \leqslant i \leqslant N$, respectively, where Θ is a real parameter. Assume furthermore that $F_i(\cdot, 0) = F(\cdot)$ for $1 \leqslant i \leqslant N$.

Let $R = (R_1, \ldots, R_N)$ and $R^+ = (R_1^+, \ldots, R_N^+)$ be the rank vectors of $X = (X_1, \ldots, X_N)$ and $|X| = (|X_1|, \ldots, |X_N|)$, respectively, and let $V = (V_1, \ldots, V_N)$ be the sign vector of X. The locally most powerful rank tests (LMPRT) $S = S(R)$ and the locally most powerful signed rank tests (LMPSRT) $S = S(R^+, V)$ will be found for testing $\Theta = 0$ against $\Theta > 0$ or $\Theta < 0$ with F being arbitrary and with F symmetric, respectively.

Keywords: locally most powerful rank tests, randomness, symmetry

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1. Introduction and notation

Let

$$\mathscr{F}_0 = \{F\colon\ F \quad \text{is an absolute continuous d.f. on } \mathbb{R}\},$$
$$\mathscr{F}_1 = \{F\colon\ F \in \mathscr{F}_0,\ F(-x) = 1 - F(x),\ x \in \mathbb{R}\}.$$

Let $X = (X_1, \ldots, X_N)$ be a vector of N i.r.v's. The hypothesis $\mathscr{H}_0$ ($\mathscr{H}_1$) means that $X_1, \ldots, X_N$ have the same d.f. $F \in \mathscr{F}_0 (F \in \mathscr{F}_1)$.

For $h = 0, 1$ let us consider the following alternatives:

$$\text{(1.1)}\ \mathscr{K}_h^1(\Delta) = \Big\{X \quad \text{has a density} \quad q_\Theta(x) = \prod_{i=1}^N f_i(x_i; \Theta),\ \Theta \in \Delta\Big\},$$

$$\text{(1.2)}\ \mathscr{K}_h^2(\Delta) = \Big\{X \quad \text{has a d.f.} \quad Q_\Theta(x) = \prod_{i=1}^N G_i(F(x_i); \Theta),\ F \in \mathscr{F}_h,\ \Theta \in \Delta\Big\}$$

where $\Delta = \Delta^+ = (0, a)$ or $\Delta = \Delta^- = (-a, 0)$ for some $a \in (0, \infty]$, and for each $\Theta \in \widetilde{\Delta} = \Delta \cup \{0\}$ we have:

(i) $f_i(x, \Theta)$ is a density on $\mathbb{R}$ such that $f_i(x, 0) = f(x)$, $1 \leqslant i \leqslant N$, and for the case $h = 1$, $f(-x) = f(x)$.

(ii) $G_i(y, \Theta)$ is a d.f. on $(0, 1)$ such that $G_i(y, 0) = y$, $1 \leqslant i \leqslant N$.

Recall that

$$P(R = r \big| \mathscr{H}_0) = 1/N! \tag{1.3}$$

for each $r \in \mathscr{R}$—the space of $N!$ permutations of $(1, \dots, N)$,

$$P(R^+ = r, V = v \big| \mathscr{H}_1) = 1/2^N \cdot N! \tag{1.4}$$

for $r \in \mathscr{R}$, $v \in \mathscr{V}$—the space of 2^N sequences $v = (v_1, \dots, v_N)$ with $v_i = 1$ or -1.

Let $X_{(1)} \leqslant \dots \leqslant X_{(N)}$ ($|X|_{(1)} \leqslant \dots \leqslant |X|_{(N)}$) be the order statistics of X (of $|X|$). Then $X_{(\cdot)} = (X_{(1)}, \dots, X_{(N)})$ and R are mutually independent under $\mathscr{H}_0$. The same conclusion is true for $|X|_{(\cdot)} = (|X|_{(1)}, \dots, |X|_{(N)})$, R^+ and V under $\mathscr{H}_1$.

The LMPRT's for testing $\mathscr{H}_0$ against $\mathscr{K}_0^j(\Delta)$ (abbr. for $\{\mathscr{H}_0, \mathscr{K}_0^j(\Delta)\}$), $j = 1, 2$, are investigated in Section 2, and the LMPSRT for $\{\mathscr{H}_1, \mathscr{K}_1^j(\Delta)\}$, $j = 1, 2$, in Section 3.

2. The locally most powerful rank tests of randomness

Two theorems will be given in this section for $\{\mathscr{H}_0, \mathscr{K}_0^1(\Delta)\}$ and $\{\mathscr{H}_0, \mathscr{K}_0^2(\Delta)\}$, respectively. These results generalize Theorem II.4.8. [3] as well as those of Lehmann [5], Gibbons [1].

Theorem 2.1. *Let $\mathscr{K}_0^1(\Delta)$ be defined by* (1.1). *Suppose for $1 \leqslant i \leqslant N$*

(i) *$f_i'(x, \Theta) = \partial f_i(x, \Theta)/\partial\Theta$ exists, $\Theta \in \widetilde{\Delta}$, and it is continuous at $\Theta = 0$ for a.e. $x \in \mathbb{R}$, where $f_i'(x, 0)$ is understood to be a one-sided derivative.*

(ii) $\lim\limits_{\Theta \to 0} \int_{-\infty}^{\infty} |f_i'(x, \Theta)| \, \mathrm{d}x = \int_{-\infty}^{\infty} |f_i'(x, 0)| \, \mathrm{d}x < \infty$.

Denote

$$A(k, i) = E\{f_i'(X_{(k)}, 0)/f(X_{(k)})\} \tag{2.1}$$

where $X_{(1)}, \dots, X_{(N)}$ are order statistics of N i.r.v.'s with the same density $f(x)$.

Then the test with critical region

$$S(R) = \sum_{i=1}^{N} A(R_i, i) \geqslant \lambda \quad (\textit{resp.} \ \leqslant \lambda) \tag{2.2}$$

is the LMPRT for $\{\mathscr{H}_0, \mathscr{K}_0^1(\Delta^+)\}$ *(for* $\{\mathscr{H}_0, \mathscr{K}_0^1(\Delta^-)\}$*) at the corresponding level.*

P r o o f. This theorem generalizes Th.II.4.8 in [3], and it is proved similarly. One must replace in the proof of the latter the density $\mathrm{d}(x, \Delta c_i)$ by $f_i(x, \Theta)$, $\dot{\mathrm{d}}$ by f_i', Δ by Θ, and note that $f_i(x,0) = f(x)$, $1 \leqslant i \leqslant N$. □

Theorem 2.2. *Let* $\mathscr{K}_0^2(\Delta)$ *be defined by* (1.2)*. Suppose for* $1 \leqslant i \leqslant N$

(iii) $g_i(y,\Theta) = \partial G_i(y,\Theta)/\partial y$ *exists for* $\Theta \in \widetilde{\Delta}$, $0 < y < 1$,

(iv) $g_i'(y,\Theta) = \partial g_i(y,\Theta)/\partial\Theta$ *exists for* $\Theta \in \widetilde{\Delta}$, $0 < y < 1$, *and it is continuous at* $\Theta = 0$ *for a.e.* $y \in (0,1)$, *where* $g_i'(y,0)$ *is the one-sided derivative,*

(v) $\lim\limits_{\Theta\to 0} \int_0^1 |g_i'(y,\Theta)| \,\mathrm{d}y = \int_0^1 |g_i'(y,0)| \,\mathrm{d}y < \infty$.

Denote

$$a(k,i) = E\{g_i'(U_{(k)}, 0)\}, \quad 1 \leqslant i \leqslant N \tag{2.3}$$

where $U_{(1)}, \ldots, U_{(N)}$ *are order statistics of* N *i.r.v.'s with the same uniform distribution on* $(0,1)$.

Then the test with critical region

$$S(R) = \sum_{i=1}^{N} a(R_i, i) \geqslant \lambda \quad (\mathit{resp.}\ \leqslant \lambda) \tag{2.4}$$

is the LMPRT for $\{\mathscr{H}_0, \mathscr{K}_0^2(\Delta^+)\}$ $(\{\mathscr{H}_0, \mathscr{K}_0^2(\Delta^-)\})$ *at the respective level.*

P r o o f. It follows from Th.2.1. In fact, for

$$f_i(x,\Theta) = g_i(F(x),\Theta) f(x), \quad \text{where} \quad f(x) = \mathrm{d}F(x)/\,\mathrm{d}x,$$

the conditions (iv)–(v) are equivalent to (i)–(ii). Since $G_i(y,0) = y$, $g(y,0) = 1$, $0 < y < 1$, then $f_i'(x,0)/f(x) = g_i'(F(x),0)$. Therefore $A(k,i) = a(k,i)$. □

E x a m p l e 2.1. Let, for $0 < y < 1$,

$$G_i(y,\Theta) = \begin{cases} (1-\Theta)y + \Theta y^2, & 1 \leqslant i \leqslant m, \\ y, & m+1 \leqslant i \leqslant N. \end{cases}$$

Then, for $1 \leqslant k \leqslant N$,

$$a(k,i) = \begin{cases} -1 + 2k/(N+1), & 1 \leqslant i \leqslant m, \\ 0, & m+1 \leqslant i \leqslant N, \end{cases}$$

because $E\{U_{(k)}\} = k/(N+1)$, $1 \leqslant k \leqslant N$.

Theorem 2.2 implies that the two-sample test with critical region

$$S(R) = \sum_{i=1}^{m} R_i \geqslant \lambda \tag{2.5}$$

is the LMPRT for testing $\mathscr{H}_0$ against

$$\mathscr{K}_0^2(\Delta^+) = \Big\{Q_\Theta^F(x) = \prod_{i=1}^{m}[(1-\Theta)F(x_i)+\Theta F^2(x_i)]\cdot \prod_{i=m+1}^{N} F(x_i),\ 0 < \Theta < 1,\ F \in \mathscr{F}_0\Big\}$$

at the respective level.

This is the case considered by Lehmann [5].

Example 2.2. If $G_i(y,\Theta) = (1-\Theta c_i)y + \Theta c_i y^2$, $0 < \Theta c_i < 1$, then $a(k,i) = c_i[2k/(N+1)-1]$. Theorem 2.2 implies that the test of Wilcoxon type with critical region

$$S(R) = \sum_{i=1}^{N} c_i R_i \geqslant \lambda \tag{2.6}$$

is the LMPRT for testing $\mathscr{H}_0$ against

$$\mathscr{K}_0^2(\Delta^+) = \Big\{Q_\Theta^F = \prod_{i=1}^{N}[(1-\Theta c_i)F(x_i) + \Theta c_i F^2(x_i)],\ \Theta > 0,\ 0 < \Theta c_i < 1,\ F \in \mathscr{F}_0\Big\}$$

at the respective level.

Example 2.3. For

$$G_i(y,\Theta) = \begin{cases} y^{1+\Theta}, & 1 \leqslant i \leqslant m, \\ 1-(1-y)^{1+\Theta}, & m+1 \leqslant i \leqslant N, \end{cases}$$

noting that

$$E\{\ln U_{(k)}\} = -\sum_{j=0}^{N-k} 1/(N-j),$$

$$E\{\ln\,(1-U_{(k)})\} = -\sum_{j=0}^{k-1} 1/(N-j) \text{ (see (25)–(26), p. 83, [3])}$$

and

$$\sum_{i=1}^{N}\sum_{j=0}^{i-1} 1/(N-j) = N,$$

one obtains from Theorem 2.2 that the test with critical region

$$S(R) = \sum_{i=1}^{m} a(R_i) \geqslant \lambda, \tag{2.7}$$

where

$$a(k) = \sum_{j=0}^{k-1}[1/(N-j)] - \sum_{j=0}^{N-k}[1/(N-j)], \; 1 \leqslant k \leqslant N, \tag{2.8}$$

is the LMPRT for testing $\mathscr{H}_0$ against

$$\mathscr{K}_0^2(\Delta^+) = \Big\{Q_\Theta^F(x) = \prod_{i=1}^{m}[F(x_i)]^{1+\Theta} \prod_{i=m+1}^{N}[1-(1-F(x_i))^{1+\Theta}], \\ \Theta > 0, \; F \in \mathscr{F}_0\Big\}$$

at the corresponding level.

This is the case considered by Gibbons [1].

3. The locally most powerful signed rank tests of symmetry

The following theorems for the symmetry hypothesis generalize the results in [4] and Theorems II.4.9–10 [3].

Theorem 3.1. *Let* $\mathscr{K}_1^1(\Delta)$ *be defined by* (1.1) *with* f_i *satisfying* (i)–(ii) *of Th.* 2.1. *For* $1 \leqslant i \leqslant N$, $j = 1, 2$ *denote*

$$f_{j,i}(x) = (1/2)[f_i'(x,0) + (-1)^j f_i'(-x,0)], \tag{3.1}$$

$$A_j(k,i) = E\{f_{j,i}(|X|_{(k)})/f(|X|_{(k)})\} \tag{3.2}$$

where $|X|_{(1)}, \ldots, |X|_{(N)}$ *are order statistics in absolute value of* N *i.r.v.'s with the same symmetric density* $f(x)$. *Then the test with critical region*

$$S(R^+, V) = \sum_{i=1}^{N}[A_1(R_i^+, i)V_i + A_2(R_i^+, i)] \geqslant \lambda \quad (\leqslant \lambda) \tag{3.3}$$

is the LMPSRT for $\{\mathscr{H}_1, \mathscr{K}_1^1(\Delta^+)\}$ *(for* $\{\mathscr{H}_1, \mathscr{K}_1^1(\Delta^-)\}$*) at the corresponding level.*

P r o o f. The proof of Theorem 3.1 is similar to that of Theorems 1–2 in [4]. Therefore we outline only its principal steps.

Denote

$$\begin{aligned} \mathrm{d}Q_\Theta &= q_\Theta \,\mathrm{d}x = \prod_{i=1}^{N} f_i(x,\Theta)\,\mathrm{d}x_i, \quad \Theta \in \widetilde{\Delta}, \\ B(r,v) &= \{x\colon\ R^+ = r,\ V = v\},\ r \in \mathscr{R},\ v \in \mathscr{V}, \\ Q_\Theta(r,v) &= Q_\Theta\{B(r,v)\}. \end{aligned}$$

Note that $Q_0 \in \mathscr{H}_1$ and $\mathrm{d}Q_0 = \prod\limits_{i=1}^{N} f(x_i)\,\mathrm{d}x_i$. Then

$$\begin{aligned} L_\Theta(r,v) &= (1/\Theta)[Q_\Theta(r,v) - Q_0(r,v)] \\ &= \int_{B(r,v)} \cdots \int \frac{1}{\Theta}\Big[\prod_{i=1}^{N} f_i(x_i,\Theta) - \prod_{i=1}^{N} f(x_i)\Big]\,\mathrm{d}x \\ &= \sum_{i=1}^{N} \int_{B(r,v)} \cdots \int L_i(x,\Theta)\,\mathrm{d}x, \end{aligned} \tag{3.4}$$

where

$$L_i(x,\Theta) = (1/\Theta)[f_i(x_i,\Theta) - f(x_i)] \prod_{j=1}^{i-1} f(x) \prod_{s=i+1}^{N} f_s(x_s,\Theta),\ \prod_{j=1}^{0} = \prod_{s=N+1}^{N} = 1,$$

with

$$\begin{aligned} \limsup_{\Theta\to 0} \int \cdots \int |L_i(x,\Theta)|\,\mathrm{d}x &\leqslant \int |f_i'(x,0)|\,\mathrm{d}x_i \\ &= \int \cdots \int |f_i'(x,0)/f(x_i)|\,\mathrm{d}Q_0. \end{aligned} \tag{3.5}$$

The convergence theorem of Scheffé [6] (see also Theorem II.4.2. [3]) implies

$$\lim_{\Theta\to 0} L_\Theta(r,v) = \sum_{i=1}^{N} \int_{B(r,v)} \cdots \int \{f_i'(x,0)/f(x_i)\}\,\mathrm{d}Q_0. \tag{3.6}$$

Since $f(-x_i) = f(x_i)$ and by (3.1) we have

$$\begin{aligned} f_{1,i}(-x_i) &= -f_{1,i}(x_i), \\ f_{2,i}(-x_i) &= f_{2,i}(x_i), \\ f_i'(x_i,0) &= f_{1,i}(x_i) + f_{2,i}(x_i), \end{aligned}$$

it follows from (1.4) and (3.6) that

$$\lim_{\Theta\to 0} L_\Theta(r,v) = (1/2^N N!)\sum_{i=1}^{N}\int\cdots\int\Big\{\frac{f_i'(x_i,0)}{f(x_i)}\Big\}\,\mathrm{d}Q_0(x\big|R^+=r,V=v)$$

$$= (1/2^N N!)\sum_{i=1}^{N}\int\cdots\int\Big\{\Big[\frac{f_{1,i}(|x_i|)}{f(|x_i|)}\Big]v_i+\Big[\frac{f_{2,i}(|x_i|)}{f(|x_i|)}\Big]\Big\}\,\mathrm{d}Q_0(x|R^+=r,V=v)$$

$$= (1/2^N N!)\sum_{i=1}^{N}E\{[f_{1,i}(|X_i|)/f(|X_i|)]V_i+[f_{2,i}(|X_i|)/f(|X_i|)]|R^+=r,V=v\}$$

$$= (1/2^N N!)\sum_{i=1}^{N}E\{[f_{1,i}(|X|_{(r_i)})/f(|X|_{(r_i)})]V_i+[f_{2,i}(|X|_{(r_i)})/f(|X|_{(r_i)})]\}$$

$$= (1/2^N N!)\sum_{i=1}^{N}[A_1(r_i,i)v_i+A_2(r_i,i)].$$

This implies the conclusion of Theorem 3.1 in the same manner as in [4]. □

Theorem 3.2. *Let* $\mathscr{K}_1^2(\Delta)$ *be defined by* (1.2). *Let the conditions* (iii)–(v) *of Theorem* 2.2 *be satisfied. Denote for* $j=1,2$, $1\leqslant i\leqslant N$

(3.7) $$g_{j,i}(u) = (1/2)\{g_i'[(1+u)/2;0]+(-1)^j g_i'[\tfrac{1}{2}(1-u);0]\},$$

(3.8) $$a_j(k,i) = E\{g_{j,i}(U_{(k)})\},$$

where $U_{(1)},\dots,U_{(N)}$ *are order statistics of* N *i.r.v.'s with the same uniform distribution on* $(0,1)$.

Then the test with critical region

(3.9) $$S(R^+,V) = \sum_{i=1}^{N}\{a_1(R_i^+,i)V_i+a_2(R_i^+,i)\}\geqslant\lambda\quad(\leqslant\lambda)$$

is the LMPSRT for $\{\mathscr{H}_1,\mathscr{K}_1^2(\Delta^+)\}$ *(for* $\{\mathscr{H}_1,\mathscr{K}_1^2(\Delta^-)\}$*) at the respective level.*

P r o o f. Note that

$$f_i(x,0) = f(x) = \mathrm{d}F(x)/dx,\ F\in\mathscr{F}_1,$$
$$f_i(x,\Theta) = g_i(F(x),\Theta)\cdot f(x),$$
$$f_i'(x,\Theta) = g_i'(F(x),\Theta)\cdot f(x),$$
$$f_i'(x,0)/f(x) = g_i'(F(x),0).$$

Then, by (3.1) and $F(-x) = 1 - F(x)$,

$$f_{j,i}(x)/f(x) = (1/2)[g_i'(F(x), 0) + (-1)^j g_i'(1 - F(x), 0)]. \tag{3.10}$$

Since $|X|$ has a d.f. $2F(x) - 1$ provided X has a d.f. $F(x)$, hence setting

$$2F(|X|) - 1 = U,$$

we see that U has the uniform distribution on $(0, 1)$. Therefore

$$F(|X|_{(k)}) = [\tfrac{1}{2}(1 + U_{(k)})],$$

and $1 - F(|X|_{(k)})$ has the same distribution as $\frac{1}{2}(1 - U_{(k)})$ and, by (3.2), (3.8), (3.10),

$$A_j(k, i) = a_j(k, i), \ j = 1, 2, \ 1 \leqslant i, \ k \leqslant N.$$

Thus Theorem 3.1 implies Theorem 3.2. □

Example 3.1. For $\mathscr{K}_1^2(\Delta)$ with $G_i(y, \Theta)$ as in Example 2.1:

$$G_i(y, \Theta) = \begin{cases} (1 - \Theta)y + \Theta y^2, & 1 \leqslant i \leqslant m, \\ y, & m + 1 \leqslant i \leqslant N, \end{cases}$$

one has for $0 < u < 1$

$$g_i'([\tfrac{1}{2}(1 \pm u)], 0) = \begin{cases} \pm u, & 1 \leqslant i \leqslant m, \\ 0, & m + 1 \leqslant i \leqslant N, \end{cases}$$

$$g_{1,i}(u) = \begin{cases} u, & 1 \leqslant i \leqslant m, \\ 0, & m + 1 \leqslant i \leqslant N, \end{cases}$$

$$g_{2,i}(u) = 0, \qquad 1 \leqslant i \leqslant N.$$

Then the test with critical region

$$S(R^+, V) = \sum_{i=1}^{m} R_i^+ V_i \geqslant \lambda \tag{3.11}$$

is the LMPSRT for $\{\mathscr{H}_1, \mathscr{K}_1^2(\Delta^+)\}$ at the respective level.

E x a m p l e 3.2. For $\mathscr{K}_1^{\,2}(\Delta)$ with Q_Θ^F as in Example 2.2:

$$Q_\Theta^F(x) = \prod_{i=1}^{N} [(1-\Theta c_i)F(x_i) + \Theta c_i F^2(x_i)],\ F \in \mathscr{F}_1,\ 0 < \Theta c_i < 1,$$

one can verify that the LMPSRT for $\{\mathscr{H}_1, \mathscr{K}_1^{\,2}(\Delta^+)\}$ is determined by the critical region

$$S(R^+, V) = \sum_{i=1}^{N} c_i R_i^+ V_i \geqslant \lambda. \tag{3.12}$$

E x a m p l e 3.3. For $\mathscr{K}_1^{\,1}(\Delta^+)$ with $q_\Theta(x) = \prod\limits_{i=1}^{N} f(x_i - \Theta),\ \Theta > 0$, where f is symmetric and continuously differentiable, one has

$$\begin{aligned} &f_i'(x,\Theta) = -f'(x-\Theta),\ f_i'(x,0) = -f'(x),\ f_i'(-x,0) = f'(x),\\ &f_{1,i}(x) = -f'(x),\ f_{2,i} = 0,\\ &A_1(k,i) = -E\{f'(|X|_{(k)})/f(|X|_{(k)})\} = A_1^f(k), \quad A_2(k,i) = 0. \end{aligned}$$

It follows from Theorem 3.1 that the test with critical region

$$\sum_{i=1}^{N} A_1^f(R_i^+) \cdot V_i \geqslant \lambda$$

is the LMPSRT for $\{\mathscr{H}_1, \mathscr{K}_1^{\,1}(\Delta^+)\}$. This coincides with Th.II.4.9. [3].

E x a m p l e 3.4. For $\mathscr{K}_1^{\,1}(\Delta^+)$ with

$$q_\Theta(x) = \prod_{i=1}^{m} e^{-\Theta} f(e^{-\Theta}x_i) \prod_{i=m+1}^{N} f(x_i), \Theta > 0,$$

where f is symmetric and continuously differentiable, one has

$$f_i'(x,0) = \begin{cases} -f(x) - xf'(x), & 1 \leqslant i \leqslant m,\\ 0, & m+1 \leqslant i \leqslant N, \end{cases}$$

hence $f_{2,i} = f_i'$, $f_{1,i} = 0$, $1 \leqslant i \leqslant N$ and

$$\begin{aligned} &A_1(k,i) = 0, \quad 1 \leqslant i \leqslant N, \qquad A_2(k,i) = 0,\ m+1 \leqslant i \leqslant N,\\ &A_2(k,i) = E\{-1 - |X|_{(k)} \cdot f'(|X|_{(k)})/f(|X|_{(k)})\} = A_2^f(k), \quad 1 \leqslant i \leqslant m. \end{aligned}$$

This result is identical with Th. II.4.10. [3]: The test with critical region

$$\sum_{i=1}^{m} A_2^f(R_i^+) \geqslant \lambda$$

is the LMPSRT for $\{\mathscr{H}_1, \mathscr{K}_1^{\,1}(\Delta^+)\}$ at the respective level.

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