Mathematica Slovaca

Book Reviews

*Mathematica Slovaca*, Vol. 42 (1992), No. 2, 249--250

Persistent URL: http://dml.cz/dmlcz/133023

Mathematica Slovaca
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Mathematical Institute
Slovak Academy of Sciences

Math. Slovaca, **42** (1992), No. 2, 249–250

# BOOK REVIEWS

COLLECTED PAPERS OF PAUL TURÁN. VOL. 1–3. Edited by Paul Erdös. Akadémiai Kiadó, Budapest 1990. Volume 1: xxxviii+1–838 pp.; Volume 2: xii+847–1750 pp.; Volume 3: xii+1751–2666 pp. ISBN 963 05 4298 6

Paul Turán (1910–1976) is widely regarded as one of the great mathematicians to command a comprehensive knowledge of mathematical science. The mathematical works of Turán, whose deep and original ideas have had a great influence on the development of various fields in mathematics. Some of his major contributions are: The power sum method; analytic and probabilistic number theory; combinatorics and graph theory; statistical group theory; and interpolation theory; but it was the number theory that captivated his interest unabatedly throughout his life.

The present collection contains almost all the works of Turán (228 articles from 246 entries) in unchanged form as photographic copies of the originals, except English translations of the papers published only in Hungarian. Notes on further developments are added to many of the papers. The collected papers are ordered chronologically and are numbered according to the complete list of publications with the original pagination. The papers are published in three volumes.

Volume 1 contains 71 mathematical papers published between 1933 and 1955, for example: *On interpolation* I–III; *Über die Primzahlen der arithmetischen Progression* I–II; *On Riemann's hypothesis*; *On the distribution of roots of polynomials*; *On a certain point of the kinetic gas theory*; etc.. Several papers in this volume are already well known. Among the best known are the 1936 paper [6] *Über einige Verallgemeinerungen eines Satzes von Hardy und Ramanujan*, the 1946 paper [28] *On a theorem of Littlewood*, and the 1948 paper [38] *On a problem in the theory of uniform distribution* (with Paul Erdös). In [6] he found a very simple proof of Hardy and Ramanujan's theorem, according to which almost all integers $n$ have asymptotically $\log\log n$ prime factors. Further developments led to the so-called *Turán-Kubilius inequality*. In [28] he found general inequalities for sums of powers of complex numbers which is what Turán calls the first main theorem of his power sum theory. Probably the power sum method and its applications are the most important and original results of Turán's work. He wrote a book on the power sum method entitled *On a New Method in Analysis and its Applications* in 1952 in Hungarian and German. A Chinese edition with some new results appeared in 1954, and a completely rewritten and essentially expanded English edition has appeared in 1984. The book is not reprinted in these collected papers. In joint work with Erdös [38] they had the following quantitative form of Weyl's criterion for uniform distribution: For an arbitrary set of $N$ real numbers $x_n$ and for all $0 \leq \alpha < \beta \leq 2\pi$ , $1 \leq m \leq N$

$$\left| \sum_{\alpha < x_n < \beta} 1 - \frac{\beta - \alpha}{2\pi} N \right| \leq c \left( \frac{N}{m} + \sum_{k=1}^{m} \frac{|s_k|}{k} \right),$$

where $s_k = \sum_{n=1}^{N} e^{2\pi i k x_n}$ and $c$ is an absolute constant.

In addition, this volume also contains the information about his scientific life, *Personal reminisences of the work of Paul Turán* by P. Erdös, and *Letter to Professor Paul Turán* by Gábor Halász. The list of publications, list of papers according to topics, list of publications dealing with Paul Turán's work, and index to coauthors are also included. There is a photograph of the author.

Volume 2 contains 80 mathematical papers published between 1955 and 1966. In addition, there are two biography papers, *Leopold Fejér* (1880-1959) *His life and work*, and *The fiftieth anniversary of Pál Erdös*. The vast majority of the papers are in comparative theory of primes (15 items), and interpolation theory (9 items). As to comparison with the main term of the prime number theorem, Littlewood proved the infiniteness of sign changes of $\pi(x) - Li(x)$. Knapowski observed that Turán's power sum method can also yield such effective bounds, even effective lower bounds for the number of sign changes up to $x$. It was they who made a systematic study of discrepancies in the distribution of primes in different arithmetic progressions. A typical result is the following: For $(k,l) = 1$, $l \not\equiv 1$ and every sufficiently large $T$ there exist then $x_1$ and $x_2$ in the interval $[\log_3 T, T]$ such that

$$\pi(x_1,k,1) - \pi(x_1,k,l) > \sqrt{x}/\log x_1, \pi(x_2,k,1) - \pi(x_2,k,l) < -\sqrt{x}/\log x_1,$$

provided that no Dirichlet $L$-function mod $k$ vanishes for some region.

Volume 3 presents a collection of 74 papers published between 1967 and 1980. Included here are: *Life and work of Kató Rényi; On the work of Alan Baker* (Rapports sur les Médailles Fields); *The work of Alfréd Rényi; Stanislav Knapowski; Commemoration of mathematicians who were victims of fascism; Ein sonderbarer Lebensweg, Ramanujan.* The mathematical papers concern various areas including statistical group-theory and partitions (17 items), applications of the graph theory (6 items), the twin-prime problem, the distribution of roots of Riemann zeta function, and the sign changes of $\pi(x) - Li(x)$. A systematic statistical theory of groups and partitions is mainly the creation of Turán and Erdös. The group they were mostly concerned with is $S_n$, the symmetric group of order $n$. They showed that the order $O(P)$ of almost all, i.e. all but $o(n!)$, elements $P \in S_n$ satisfies

$$\left|\log O(P) - \frac{1}{2}\log^2 n\right| < \omega(n)\log^{3/2} n,$$

whenever $\omega(n) \to \infty$. In this volume the sequences of papers by Erdös and Turán contains a number of statistical results on the arithmetical structure of $O(P)$ for $P \in S_n$, on the possible different values of $O(P)$ for $P \in S_n$, on the cardinalities of the conjugacy classes of $S_n$ and on the common orders of the elements in a random conjugacy class of $S_n$. Turán also started the field of extremal problems in graph theory. He posed and completely solved the following problem. Let $f(n, K_p)$ be the smallest integer for which every graph on $n$ vertices and $f(n, K_p)$ edges contains a complete graph on $p$ vertices. He determined $f(n, K_p)$ explicitly. Turán later found beautiful applications to discrete geometry and potential theory partly in joint work with Erdös, Meir and Vera T. Sós, included in this volume. These applications led to the investigation of the so-called Turán-Ramsey theorems, full of interesting new phenomena.

In addition to the collected papers, Volume 3 also contains *Errata et corrigenda* on the basis of the author's private copies.

This excellent collection will be valued by the mathematical community. It is only when we see Turán's papers collected in one place that we can begin to appreciate the scope and profundity of his influence on mathematical thought and, in particular, on the theory of numbers. A credit to its publisher, it is a fitting memorial to its author.

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