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ON A THEOREM OF POPA AND NOIRI ON MULTIFUNCTIONS

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ABSTRACT. The aim of this note is to correct a recent result of Noiri and Popa from [Popa, V.—Noiri, T.: *On upper and lower α-continuous multifunctions*, Math. Slovaca **43** (1993), 477–491] on multifunctions.

In [3], Theorem 4.7 claims that if $F, G\colon X \to Y$ are upper α-continuous and Y is Hausdorff, then $A = \{x \in X : F(x) \cap G(x) \neq \emptyset\}$ is α-closed in X. Unfortunately, this result is true only under some additional assumptions. We have a counterexample to Theorem 4.7 from [3].

In what follows, all spaces are assumed to be topological. Recall first that a subset A of a topological space X is called an *α-set* or *α-open* ([2]) if $A = U \backslash N$, where U is open and N is nowhere dense. Complements of α-open sets are called *α-closed*. The family $\alpha(X)$ of all α-open sets in X is a topology for X ([2]). A multifunction $F\colon X \to Y$ is called *upper α-continuous* ([3; Theorem 3.3] if $F^+(V)$ is α-open in X for any open set V of Y. A subset A of a space X is called *α-paracompact* ([1]) if for every open cover $\mathcal{V}$ of A in X, there exists a locally finite open cover $\mathcal{W}$ of A which refines $\mathcal{V}$. Furthermore, a multifunction $F\colon X \to Y$ is called *punctually α-paracompact* ([3]) if $F(x)$ is α-paracompact for each point $x \in X$.

EXAMPLE 1. Let $\mathbb{R}$ be the real line with the usual topology. Let $\mathbb{Q}$ and $\mathbb{P}$ be the sets of all rational and irrational numbers, respectively.

Define $F, G\colon \mathbb{R} \to \mathbb{R}$ as follows: $F(0) = \mathbb{Q}$ and $F(x) = x$ for every $x \neq 0$; $G(0) = \mathbb{P}$ and $G(x) = x$ for every $x \neq 0$. Clearly the range is Hausdorff. It is not difficult to verify that F and G are upper α-continuous. However, the set

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$A = \{x \in X : F(x) \cap G(x) \neq \emptyset\} = \mathbb{R} \setminus \{0\}$ is not α-closed, since no singleton in the real line is α-open.

Now, we present the correct version of [3; Theorem 4.7].

LEMMA 2. *In Hausdorff spaces, disjoint α-paracompact sets can be separated by disjoint open sets.*

THEOREM 3. *Let $F, G \colon X \to Y$ be both upper α-continuous and punctually α-paracompact. If Y is Hausdorff, then $A = \{x \in X : F(x) \cap G(x) \neq \emptyset\}$ is α-closed in X.*

Proof. Let $x \notin A$. Then $F(x) \cap G(x) = \emptyset$. Since F and G are punctually α-paracompact, then $F(x)$ and $G(x)$ are α-paracompact subsets of Y. Since Y is a Hausdorff space, then by Lemma 2, there exist open sets U and V such that $F(x) \subseteq U$, $G(x) \subseteq V$ and $U \cap V = \emptyset$. Since F and G are upper α-continuous, $F^+(U)$ and $G^+(V)$ are α-open sets. Put $W = F^+(U) \cap G^+(V)$. Then $x \in W$, $W \in \alpha(X)$ and $W \cap A = \emptyset$. Thus A is α-closed in X. $\square$

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