Mathematica Slovaca

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Mathematica Slovaca, Vol. 56 (2006), No. 2, 213--221

Persistent URL: http://dml.cz/dmlcz/129979

Math. Slovaca, **56** (2006), No. 2, 213–221

Mathematica Slovaca
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Slovak Academy of Sciences

ON INTERVALS AND THE DUAL OF A PSEUDO MV-ALGEBRA

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(*Communicated by Anatolij Dvurečenskij*)

ABSTRACT. For each pseudo MV-algebra $\mathcal{A}$ we define the pseudo MV-algebra $\mathcal{A}^{\mathrm{dual}}$. We apply this notion for considering the system of intervals of a pseudo MV-algebra; the earlier results concerning intervals of MV-algebras are generalized. For any pseudo MV-algebra there exists a one-to-one correspondence between internal direct product decompositions of $\mathcal{A}$ and internal direct product decompositions of $\mathcal{A}^{\mathrm{dual}}$.

I. Introduction

The system of intervals of an MV-algebra has been dealt with in [9].

The notion of pseudo MV-algebra was defined independently by Georgescu and Iorgulescu [5], [6] and by Rachůnek [11] (in [11], the term "generalized MV-algebra" was applied).

Dvurečenskij [4] proved that for each pseudo MV-algebra $\mathcal{A}$ there exists a lattice ordered group G with a strong unit u such that the underlying set A of $\mathcal{A}$ is equal to the interval $[0,u]$ of G and that the operations of $\mathcal{A}$ can be defined by means of the operations of G (for details, cf. Section 2 below). In this situation we write $\mathcal{A}=\Gamma(G,u)$.

Dvurečenskij's result generalize the well-known theorem of Mundici dealing with MV-algebras (Mundici [10]; cf. also the monograph Cignoli, D'Ottaviano and Mundici [3]).

Let us apply the notation as above; let $\mathcal{A}=\Gamma(G,u)$. For $x,y\in G$ we put $x+_1 y = x-u+y$; further, we set $x\leqq_1 y$ iff $x\geqq y$. Then $G_1=(G;+_1,\leqq_1)$ is

2000 Mathematics Subject Classification: Primary 06D35.
Keywords: pseudo MV-algebra, dual pseudo MV-algebra, system of intervals, internal direct product.

Supported by Science and Technology Assistance Agency under the contract No. APVT-51-032002.

a lattice ordered group and 0 is a strong unit of G_1. Denote $\mathcal{A}^{\text{dual}} = \Gamma(G_1, 0)$. We say that $\mathcal{A}^{\text{dual}}$ is the *pseudo MV-algebra dual* to $\mathcal{A}$. We clearly have $\mathcal{A}^{\text{dual dual}} = \mathcal{A}$.

In the particular case when $\mathcal{A}$ is an MV-algebra, another definition of the duality has been used in [9]. We show that both the definitions are equivalent for the case of MV-algebras. We use the notion of duality for investigating the system of intervals of a pseudo MV-algebra; we generalize the results of [9] concerning intervals of MV-algebras. Further, we consider the relations between internal direct product decompositions of a pseudo MV-algebra $\mathcal{A}$ and internal direct product decompositions of $\mathcal{A}^{\text{dual}}$.

2. Preliminaries

We recall the definition of a pseudo MV-algebra.

DEFINITION 2.1. Let $\mathcal{A} = (A; \oplus, \neg, \sim, 0, 1)$ be an algebra of type $(2, 1, 1, 0, 0)$. For $x, y \in A$ we put $x \odot y = \sim(\neg x \oplus \neg y)$. Then $\mathcal{A}$ is called a *pseudo MV-algebra* if the following identities are valid:

(A1) $x \oplus (y \oplus z) = (x \oplus y) \oplus z$;
(A2) $x \oplus 0 = 0 \oplus x = x$;
(A3) $x \oplus 1 = 1 \oplus x = 1$;
(A4) $\neg 1 = 0$; $\sim 1 = 0$;
(A5) $\neg(\sim x \oplus \sim y) = \sim(\neg x \oplus \neg y)$;
(A6) $x \oplus (y \odot \sim x) = y \oplus (x \odot \sim y) = (\neg y \odot x) \oplus y = (\neg x \odot y) \oplus x$;
(A7) $x \odot (\neg x \oplus y) = (x \oplus \sim y) \odot y$;
(A8) $\sim\neg x = x$.

If the operation $\oplus$ is commutative, then $\mathcal{A}$ is an *MV-algebra* (in this case $\neg x = \sim x$ for each $x \in A$).

Let $\mathcal{A}$ be a pseudo MV-algebra. For $x, y \in A$ we put $x \leqq y$ iff $\neg x \oplus y = 1$. Then $(A; \leqq)$ is a distributive lattice with the least element 0 and with the greatest element 1; we denote $(A; \leqq) = \ell(\mathcal{A})$.

If $a, b \in A$ and $a \leqq b$, then the set $\{c \in A : a \leqq c \leqq b\}$ is an *interval* of $\mathcal{A}$; we denote it by $[a, b]$. Let $\operatorname{Int} \mathcal{A}$ be the system of all intervals of $\mathcal{A}$.

Let G be a lattice ordered group with a strong unit u. Put $A = [0, u]$; for each $x, y \in A$ we set

$$x \oplus y = (x + y) \wedge u\,, \qquad \neg x = u - x\,, \qquad \sim x = -x + u\,, \qquad 1 = u\,.$$

Then the structure $(A; \oplus, \neg, \sim, 0, 1)$ is a pseudo MV-algebra; it will be denoted by $\Gamma(G, u)$.

Dvurečenskij [4] proved that for each pseudo MV-algebra $\mathcal{A}$ there exists a lattice ordered group G with a strong unit u such that

$$\mathcal{A} = \Gamma(G,u)\,. \tag{1}$$

Throughout the present paper we suppose that the relation (1) is valid.

The partial order of G induces a partial order on the set A; this partial order coincides with the partial order $\leqq$ defined above.

Let $a \in A$. For $x, y \in [0,a]$ we put

$$x \oplus_a y = (x+y) \wedge a\,, \qquad \neg_a x = a - x\,, \qquad \sim_a x = -x + a\,.$$

Then the structure $\mathcal{A}_a = \big([0,a], \oplus_a, \neg_a, \sim_a, 0, a\big)$ is a pseudo MV-algebra. In fact, we have $\mathcal{A}_a = \Gamma(G_a, a)$, where G_a is the convex ℓ-subgroup of G generated by the element a. We say that $\mathcal{A}_a$ is an *interval subalgebra* of $\mathcal{A}$.

3. The structure $\mathcal{A}^{\mathrm{dual}}$ and systems of intervals

Assume that G is a lattice ordered group with a strong unit u and let G_1 be as in Section 1.

LEMMA 3.1. *G_1 is a lattice ordered group with a strong unit 0.*

Proof. The algebraic structures G and G_1 have the same underlying set. Since $\leqq$ is a lattice order and $\leqq_1$ is dual to $\leqq$, we conclude that $(G_1, \leqq_1)$ is a lattice. We denote by $\vee^1$ and $\wedge^1$ the lattice operations in G_1. Further, it is easy to verify that $(G_1, +_1)$ is a group with the neutral element u. Let n be a positive integer and $x \in G_1$. The expression $x +_1 x +_1 \cdots +_1 x$ (n times) will be denoted by $n^{(1)}x$. Then $2^{(1)}0 = 0 - u + 0 = -u$,

$$3^{(1)}0 = 2^{(1)}0 +_1 0 = -u - u + 0 = -2u\,;$$

by induction we obtain

$$n^{(1)}0 = -(n-1)u\,.$$

Let $y \in G$. There exists a positive integer n such that $-y \leqq nu$. Hence $y \geqq -nu$, thus

$$y \leqq_1 -nu = (n+1)^{(1)}0\,.$$

Thus 0 is a strong unit of the lattice ordered group G_1. □

For $x, y \in G_1$ with $x \leqq_1 y$ we denote by $[x,y]_1$ the corresponding interval in G_1. Hence we have

$$[0,u] = [u,0]_1\,. \tag{1}$$

In view of 3.1 we can construct the pseudo MV-algebra $\Gamma(G_1, 0) = \mathcal{A}_1$. According to (1), the structures $\mathcal{A}$ and $\mathcal{A}_1$ have the same underlying sets and the lattice $\ell(\mathcal{A}_1)$ is dual to the lattice $\ell(\mathcal{A})$. The corresponding operations in $\mathcal{A}_1$ will be denoted by $\oplus_1, \neg_1, \sim_1, 0_1, 1_1$. Hence we have

$$0_1 = u = 1, \qquad 1_1 = 0.$$

Similarly as in $\mathcal{A}$ we put $x \odot_1 y = \sim_1(\neg_1 x \oplus_1 \neg_1 y)$.

Now let us assume that the operation $\oplus$ of $\mathcal{A}$ is commutative, i.e., that $\mathcal{A}$ is an MV-algebra. Consider the algebraic structure $\mathcal{A}_2 = (A; \oplus_2, \neg_2, 0_2, 1_2)$, where

$$\oplus_2 = \odot, \qquad \neg_2 = \neg, \qquad 0_2 = 1, \qquad 1_2 = 0.$$

There are several equivalent definitions of the notion of the MV-algebra. From the system of axioms used by Chang [2] (cf. also Cattano and Lombardo [1], and the author [9]) we obtain:

LEMMA 3.2. (Cf. [9; Lemma 2.3].) *The algebraic structure $\mathcal{A}_2$ is an MV-algebra. Moreover, if $\vee^2$ and $\wedge^2$ are the corresponding lattice operations, then $\vee^2 = \wedge$ and $\wedge^2 = \vee$.*

Let us investigate the relation between $\mathcal{A}_1$ and $\mathcal{A}_2$. Both these structures have the same underlying set, namely A. Further, $0_1 = u = 0_2$ and $1_1 = 0 = 1_2$. Also, $\vee^1 = \vee^2$ and $\wedge^1 = \wedge^2$.

Let G_1 be as above and let $x, z \in G_1$ such that $x +_1 z = u$. Then $x - u + z = u$, whence $z = u - x + u = 2u - x$. Thus we can write $-_1 x = 2u - x$, where $-_1$ denotes the corresponding subtraction operation in the group G_1.

PROPOSITION 3.3. *Let $\mathcal{A}$ be an MV-algebra. Then $\mathcal{A}_1 = \mathcal{A}_2$.*

Proof. In view of the above remarks it remains to verify that $\oplus_1 = \oplus_2$ and $\neg_1 = \neg_2$.

a) Let $x \in A$. In view of the definition of $\mathcal{A}_2$ we have

$$\neg_2 x = \neg x = u - x.$$

Further,

$$\neg_1 x = 0 +_1 (\neg_1 x) = 0 - u + (\neg_1 x) = -u + (2u - x) = u - x.$$

Hence $\neg_1 = \neg_2$.

b) Let $x, y \in A$. Then

$$\begin{aligned} x \oplus_2 y = x \odot y = \neg(\neg x \oplus \neg y) &= u - \big((u - x) \oplus (u - y)\big) \\ &= u - \big(((u - x) + (u - y)) \wedge u\big) = u - \big((2u - x - y) \wedge u\big) \\ &= u + \big((-2u + x + y) \vee (-u)\big) = (-u + x + y) \vee 0. \end{aligned}$$

Next, we have

$$x \oplus_1 y = (x +_1 y) \wedge_1 0 = (x - u + y) \vee 0 .$$

Thus $\oplus_1 = \oplus_2$. □

In view of Section 1 we get $\mathcal{A}_1 = \mathcal{A}^{\mathrm{dual}}$ for any pseudo MV-algebra $\mathcal{A}$. In [9], we defined $\mathcal{A}^{\mathrm{dual}} = \mathcal{A}_2$ for any MV-algebra. Hence according to 3.3, the definition of $\mathcal{A}^{\mathrm{dual}}$ from Section 1 coincides with the definition from [9] for any MV-algebra.

Now let us return to the case when the operation $\oplus$ need not be abelian.

Let $\mathcal{P}$ be the class of all pseudo MV-algebras and let $\mathcal{A}$ be an element of $\mathcal{P}$. We denote

$$\begin{aligned} M_1(\mathcal{A}) &= \{\mathcal{A}_1 \in \mathcal{P} : \ \operatorname{Int} \mathcal{A}_1 = \operatorname{Int} \mathcal{A}\} ; \\ M_2(\mathcal{A}) &= \{\mathcal{A}_1 \in \mathcal{P} : \ \ell(\mathcal{A}_1) = \ell(\mathcal{A})\} ; \\ M_3(\mathcal{A}) &= \{\mathcal{A}_1 \in \mathcal{P} : \ \ell(\mathcal{A}_1) = \ell(\mathcal{A}^{\mathrm{dual}})\} . \end{aligned}$$

It is obvious that $M_2(\mathcal{A}) \subseteq M_1(\mathcal{A})$. Further, from the definition of $\mathcal{A}^{\mathrm{dual}}$ we obtain $M_3(\mathcal{A}) \subseteq M_1(\mathcal{A})$. Hence we have

$$M_2(\mathcal{A}) \cup M_3(\mathcal{A}) \subseteq M_1(\mathcal{A}) .$$

The direct product of pseudo MV-algebras is defined in the usual way. A pseudo MV-algebra $\mathcal{A}$ is *directly indecomposable* if, whenever $\mathcal{A} \simeq \mathcal{A}_1 \times \mathcal{A}_2$, then either A_1 or A_2 is a one-element set.

Direct products of MV-algebras have been investigated in [7]; for more general case of pseudo MV-algebras, cf. [8].

THEOREM 3.4. *Let $\mathcal{A}$ be a pseudo MV-algebra. Then the following conditions are equivalent:*

(i) $M_2(\mathcal{A}) \cup M_3(\mathcal{A}) = M_1(\mathcal{A})$.

(ii) *The pseudo MV-algebra $\mathcal{A}$ is directly indecomposable.*

Proof. In view of the fact that the lattice $\ell(\mathcal{A})$ is dual to the lattice $\ell(\mathcal{A}^{\mathrm{dual}})$ we can apply the same argument as in [9; Sec. 3] with the distinction that instead of [7] (which is denoted as [8] in the article [9]), the result of the paper [8] is used now. □

According to Proposition 3.3, Theorem 3.4 is a generalization of the result of [9], which was denoted as $(*)$.

4. Some further results on $\mathcal{A}^{\mathrm{dual}}$

Let L be a lattice; the corresponding dual lattice will be denoted by L^d. The lattice L is said to be *self-dual* if $L \simeq L^d$.

Lemma 4.1. *Let G be a partially ordered group, $0 < a \in G$. Then the interval $[0,a]$ of G is self-dual.*

Proof. For each $x \in [0,a]$ we put $\varphi_1(x) = -x$. Then φ_1 is a bijection of $[0,a]$ onto $[-a,0]$ and for any $x_1, x_2 \in [0,a]$ we have

$$x_1 \leqq x_2 \iff \varphi_1(x_1) \geqq \varphi_1(x_2).$$

Thus φ_1 is an isomorphism of $[0,a]^d$ onto $[-a,0]$.

Further, for each $y \in [-a,0]$ we put $\varphi_2(y) = y + a$. Hence φ_2 is an isomorphism of $[-a,0]$ onto $[0,a]$. Therefore $[0,a]$ is isomorphic to $[0,a]^d$. □

As a corollary we obtain:

Proposition 4.2. *Let $\mathcal{A}$ be a pseudo MV-algebra. Then $\ell(\mathcal{A})$ is isomorphic to $\ell(\mathcal{A}^{\mathrm{dual}})$.*

For the case when $\mathcal{A}$ is an MV-algebra we have a stronger result.

Proposition 4.3. *Let $\mathcal{A}$ be an MV-algebra. Then $\mathcal{A}$ is isomorphic to $\mathcal{A}^{\mathrm{dual}}$.*

Proof. For each $x \in A$ we put $\varphi(x) = \neg x$. The MV-algebras $\mathcal{A}$ and $\mathcal{A}^{\mathrm{dual}}$ have the same underlying set, and in view of Section 3 we obtain

$$\mathcal{A} = (A; \oplus, \neg, 0, u),$$
$$\mathcal{A}^{\mathrm{dual}} = (A; \odot, \neg, u, 0).$$

Obviously, $\varphi(0) = u$ and $\varphi(u) = 0$. It remains to verify that for each $x, y \in A$ the relation

$$\varphi(x \oplus y) = \varphi(x) \odot \varphi(y)$$

is valid. We have

$$\begin{aligned} \varphi(x \oplus y) &= \neg(x \oplus y) = u - \big((x+y) \wedge u\big) \\ &= u + \big((-x-y) \vee (-u)\big) = (u - x - y) \vee 0; \end{aligned}$$

$$\begin{aligned} \varphi(x) \odot \varphi(y) &= (\neg x) \odot (\neg y) = (u - x) \odot (u - y) \\ &= (u - x - u + u - y) \vee 0 = (u - x - y) \vee 0. \end{aligned}$$

□

The question whether 4.3 is valid also for pseudo MV-algebras remains open. Let us express the hypothesis that the answer is "No".

Now we want to investigate the relations between direct product decompositions of a pseudo MV-algebra and direct product decompositions of its dual.

The internal direct factors of a pseudo MV-algebra have been dealt with in [8]; we recall some definitions (with a slightly modified notation).

Let I be a nonempty set of indices and for each $i \in I$ let L_i be a lattice. Consider the direct product $L^1 = \prod_{i\in I} L_i$. Let φ be an isomorphism of a lattice L onto L^1. For $x \in L$ we denote by $\big(x(i)\big)_{i\in I}$ the image of x under φ. Further, let x_0 be a fixed element of L. For each $i_0 \in I$ we denote by $L_{i_0}[x_0]$ the set of all $x \in L$ such that, whenever $i \in I \setminus \{i_0\}$, then $x(i) = x_0(i)$. Hence $L_{i_0}[x_0]$ is a sublattice of L with $x_0 \in L_{i_0}[x_0]$.

For $y \in L$ and $i \in I$ we denote by $yL_i[x_0]$ the element $z \in L_i[x_0]$ such that $y(i) = z(i)$. Then the mapping $\varphi[x_0]$ defined by

$$\varphi[x_0](y) = \big(yL_i[x_0]\big)_{i\in I}$$

is an isomorphism of L onto the direct product

$$L^2 = \prod_{i\in I} L_i[x_0]\,.$$

We say that $L_i[x_0]$ is an internal direct factor of L and that $\varphi[x_0]$ is an internal direct product decomposition of L (with the central element x_0).

For each $i \in I$, the lattices L_i and $L_i[x_0]$ are isomorphic. Hence for any $i \in I$ and any $x_1 \in L$, the lattices $L_i[x_0]$ and $L_i[x_1]$ are isomorphic.

An analogous notation can be applied for MV-algebras. Let I be as above and for each $i \in I$ let $\mathcal{A}_i$ be an MV-algebra. Assume that ψ is an isomorphism of an MV-algebra $\mathcal{A}$ onto the direct product $\mathcal{A}^1 = \prod_{i\in I} \mathcal{A}_i$. For $i_0 \in I$ we define the element $u^0_{i_0} \in A$ as follows:

$$u^0_{i_0}(i) = \begin{cases} 0 & \text{if } i \neq i_0\,, \\ u^{i_0} & \text{if } i = i_0 \end{cases}$$

where u^{i_0} is the greatest element of $\mathcal{A}_{i_0}$. Further, we put

$$\mathcal{A}_{i_0}(0) = \big([0, u^0_{i_0}], \oplus_{i_0}, \neg_{i_0}, \sim_{i_0}, 0, u^0_{i_0}\big)\,,$$

where for each $x, y \in [0, u^0_{i_0}]$ we put

$$x \oplus_{i_0} y = (x + y) \wedge u^0_{i_0}\,, \qquad \neg_{i_0} x = u^0_{i_0} - x\,, \qquad \sim_{i_0} x = -x + u^0_{i_0}\,.$$

For $z \in A$ we denote by $z\big(\mathcal{A}_{i_0}(0)\big)$ the element $t \in \mathcal{A}_{i_0}(0)$ such that $z(i_0) = t(i_0)$. Consider the mapping

$$\psi^0 \colon \mathcal{A} \to \prod_{i\in I} \mathcal{A}_i(0)$$

defined by $\psi^0(z) = z\big(\mathcal{A}_i(0)\big)$ for each $z \in \mathcal{A}$ and $i \in I$. Then we have (cf. [8])

(a) ψ^0 is an isomorphism of $\mathcal{A}$ onto $\prod_{i\in I} \mathcal{A}_i(0)$;

(b) for each $i \in I$, $\mathcal{A}_i(0)$ is isomorphic to $\mathcal{A}_i$.

PROPOSITION 4.4. (Cf. [8].) *Let $\mathcal{A}$ be a pseudo MV-algebra. Put $L = \ell(\mathcal{A})$. Assume that we have an internal direct decomposition*

$$\varphi^0 \colon L \to \prod_{i \in I} L_i(0) .$$

Then all $L_i(0)$ are underlying lattices of internal subalgebras of $\mathcal{A}$ and the mapping φ^0 yields, at the same time, an internal direct product decomposition

$$\varphi^0 \colon \mathcal{A} \to \prod_{i \in I} \mathcal{A}_i(0) ,$$

of the pseudo MV-algebra $\mathcal{A}$, where $L_i(0) = \ell(\mathcal{A}_i(0))$ for each $i \in I$.

PROPOSITION 4.5. *Let $\mathcal{A}$ be a pseudo MV-algebra. Then there is a one-to-one correspondence between internal direct decompositions of $\mathcal{A}$ and internal direct product decompositions of $\mathcal{A}^{\mathrm{dual}}$.*

Proof. Assume that there is given an internal direct product decomposition

$$\psi^0 \colon \mathcal{A} \to \prod_{i \in I} \mathcal{A}_i(0) \tag{1}$$

of $\mathcal{A}$. Denote $\ell(\mathcal{A}_i(0)) = L_i(0)$. Hence $L_i(0)$ are sublattices of the lattice $L = \ell(\mathcal{A})$. It is obvious that the mapping ψ^0 yields, at the same time, an internal direct product decomposition

$$\psi^0 \colon L \to \prod_{i \in I} L_i(0) \tag{2}$$

of the lattice L.

For each $i \in I$ we have $L_i(0)(u) = L_i(u)$ (under the notation as above); moreover, from (2) we obtain the internal direct product decomposition

$$\psi^1 \colon L \to \prod_{i \in I} L_i(u) . \tag{3}$$

Then ψ^1 is, at the same time, an internal direct product decomposition of the corresponding dual lattice; we get

$$\psi^1 \colon L^d \to \prod_{i \in I} \big(L_i(u)\big)^d . \tag{4}$$

Now we apply Proposition 4.4. In view of (4) and of the fact that u is the zero element of $\mathcal{A}^{\mathrm{dual}}$ we conclude that each $\big(L_i(u)\big)^d$ is the underlying lattice of an interval subalgebra $\mathcal{A}'_i(0)$ of the pseudo MV-algebra $\mathcal{A}^{\mathrm{dual}}$; moreover, ψ^1 yields, at the same time, an internal direct product decomposition

$$\psi^1 \colon \mathcal{A}^{\mathrm{dual}} \to \prod_{i \in I} \mathcal{A}'_i(0) , \tag{5}$$

where $\ell(\mathcal{A}'_i(0)) = (L_i(u))^d$.

The internal direct product decomposition (5) of $\mathcal{A}^{\mathrm{dual}}$ corresponds to the internal direct product decomposition (1) of $\mathcal{A}$.

By applying reverse steps we can proceed from (5) to (1); therefore the correspondence under consideration is one-to-one. $\square$

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Received May 13, 2004

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