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THE STRONG LAW OF LARGE NUMBERS FOR O-CONVERGENT RANDOM VARIABLES

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This paper is concerned with random variables which take values in a Dedekind σ-complete vector lattice. A function is termed a random variable if it is the o-limit (i.e. the limit with respect to the order) both of an increasing sequence of elementary random variables and a decreasing sequence of elementary random variables. An elementary random variable is defined in the usual manner, i.e. as a function which takes on only a countable number of values, each on a measurable set. The purpose of the paper is to find sufficient conditions for a sequence of random variables to obey the strong law of large numbers with respect to the o-convergence, i.e. to ensure the o-convergence of the partial sums to 0 on a set of probability 1.

Unlike the majority of papers dealing with the validity of the strong law of large numbers for random variables with respect to a topology (cf. [1], [2], [3], [4]), we do not assume very much about random variables. They need not be independent and in many cases we are not even interested whether their expectations (i.e. an integral) exist or not. On the other hand, however, we pay much attention to the type of the image space of random variables. Briefly speaking we consider two cases: the case, when in addition to the order a topology of the image space is given and the case without topology.

There are two possible approaches to the problem we are dealing with. The first of them is to compare the o-convergence and the topological convergence in the image space, the other is to give sufficient conditions purely in the terms of the o-convergence.

The first approach makes it necessary to explore relations between random variables as defined above (i.e. with respect to the o-convergence) and random variables with respect to a topology. (In what follows the latter are termed random elements to avoid a possible misunderstanding). Some results in this direction are given in [5] and will be mentioned below.

As to the integral (i.e. the expectation) of a random variable with respect to the order, [5] gives two wide types of spaces in which such an integral is well defined. It follows that discu sing the validity of the strong law of large numbers in such spaces we may distinguish between the case when random variables are integrable and the case when the integral does not exist.

In order to make this paper more self-contained, the first part contains the basic notions, definitions and results which are required throughout the paper. The second part discusses the case when the image space is a vector lattice with order-units. In particular, the strong law of large numbers for random variables with values in the space of continuous functions on an extremally disconnected compact topological space is given. The cases of normed linear spaces, metrizable linear spaces and locally convex linear topological spaces are then discussed. The third part deals with spaces which are, in a sense, dual to those discussed in the previous chapter. It is shown that both Toeplitz' and Kronecker's Lemmas hold in such spaces. As a consequence we may prove the strong law of large numbers by using methods which are known in the real case. Finally, the case when no topology is given is considered. In particular, we obtain the strong law of large numbers for random variable with value in regular spaces.

I

We mention the basic definitions and notions which will be used throughout the paper. We also present a brief survey of results on which our theory is based.

A vector lattice X is called Dedekind σ-complete if every non-empty at most countable subset of X which is bounded from above has a supremum.

An element e of a vector lattice X is said to be an order-unit if, given x in X, there exists a positive integer n such that $-ne \leq x \leq ne$.

Let (Ω, S, P) be a probability space, X be any Dedekind σ-complete vector lattice. A function $f: \Omega \to X$ is said to be an elementary random variable if there are a sequence of pairwise disjoint sets E_i, $E_i \in S$, $\cup E_i = \Omega$ and a sequence $\{x_i\}$ of elements of X such that $f(\omega) = x_i$ for every $\omega \in E_i$. A function $f: \Omega \to X$ is said to be a random variable if there exist an increasing sequence f_n of elementary random variables and a decreasing sequence g_n of elementary random variables such that $f_n(\omega) \to f(\omega)$ and $g_n(\omega) \to f(\omega)$ for every $\omega \in \Omega$.

If a topology on X is given, we may consider random variables with respect to this topology. (From now on they will be called random elements.) For our purposes it is enough to consider the case of a linear metric space.

Let X be a linear metric space. *A function $f: \Omega \to X$ is called a random element if $f^{-1}(B) \in S$ for every Borel set B of X.*

Since only strong laws of large numbers for random elements can be found in literature, the investigation of the relationship between random variables and random elements may be of certain value. In this direction we are able to prove the following propositions. We recall to the first of them that *a subset H of a cone K in a vector lattice exhausts K if for every $x \in K$ there exists $h \in H$ and a natural number n such that $x \leqslant nh$.*

Proposition 1. *Let X be a complete separable linear metric space ordered by a closed cone K such that a countable set exhausts K and the mapping $x \to |x|$ is continuous at 0. Then every random element is a random variable.*

For the proof see [5].

Proposition 2. *Let X be a complete separable linear metric space ordered by a closed cone with non-empty interior. Then every random element is a random variable.*

The proof is based on the fact that in such spaces the topological boundedness implies o-boundedness.

A linear functional T on X is called

a) *monotone if $Tx \geqslant 0$ for all $x \geqslant 0$;*

b) *order continuous if for each sequence x_n in X with order limit x, Tx_n converges to Tx;*

c) *o-bounded if it maps o-bounded sets into bounded sets;*

d) *a lattice homomorphism if* $\inf(Tx, Ty) = 0$ *for every pair of elements x, $y \in X$ satisfying* $\inf(x, y) = 0$;

e) *a lattice σ-homomorphism if it follows from* $x = \sup x_n$, $n = 1, 2, \ldots$, $x, x_n \in X$ *that* $Tx = \sup Tx_n$.

In what follows the set of all o-bounded linear functionals and the set of all linear functionals continuous with respect to a topology on X will be denoted X^+ and X^*, respectively.

Proposition 3. *Let X be a separable locally convex metrizable linear space with a closed ordering. Let a countable set exhaust the cone. Let $x_n \to x$ imply $T(x_n) \to T(x)$ for every $T \in X^*$. Then the uniform o-limit of every sequence of random elements is a random element.*

Proposition 4. *Let X be a separable locally convex metrizable linear space with a cone K such that X^* has a countable basis. Let $x_n \to x$ imply $T(x_n) \to T(x)$ for every $T \in X^*$. If $V_n(\omega) \to V(\omega)$ for every $\omega \in \Omega$, V_n random elements, then V is a random element.*

Proposition 5. *Let X be a separable locally bounded locally convex metrizable linear space, X^* be separable in the strong topology. Let $x_n \to x$ imply $T(x_n) \to T(x)$ for every $T \in X^*$. If $V_n(\omega) \to V(\omega)$ for every $\omega \in \Omega$, V_n a sequence of random elements, then V is a random element.*

For random variables this paper is dealing with we can define an integral of the Daniel type. For elementary random variables it can be done as follows:

An elementary random variable is said to be integrable if the series $\Sigma x_i P(E_i)$ *is absolutely o-convergent.* (See the above definition of an elementary random variable.) *The integral of f is then defined as follows* $\int f \, \mathrm{d}P = \Sigma x_i P(E_i)$.

A straightforward adaptation of the Daniel extension method is possible provided the integral as defined above is continuous under monotone limits, i.e. $f_n \downarrow 0$ implies $\int f_n \, \mathrm{d}P \downarrow 0$. In [5] two types of spaces in which this condition is satisfied are given. We refer to the corresponding properties of these spaces as (A) and (B), respectively.

(A) X is a locally convex space with an ordering given by a closed cone such that $x_n \xrightarrow{o} x$ *implies* $Tx_n \to Tx$ *for every* $T \in X^*$.

(B) X is a vector lattice regularly ordered by a cone such that every o-bounded linear functional is o-continuous.

Let (Ω, S, P) *be a probability space, X be a Dedekind* σ*-complete vector lattice ordered by a cone such that either (A) or (B) holds. A random variable* $f\colon \Omega \to X$ *is called integrable if there exist an increasing sequence* $\{f_n\}$ *of elementary integrable random variables, such that* $f_n \downarrow f$ *and a decreasing sequence* $\{g_n\}$ *of elementary integrable random variables such that* $g_n \downarrow f$, *both with uniformly bounded integrals*

The integral of f is defined by $\int f \, \mathrm{d}P = \lim_n \int f_n \, \mathrm{d}P = \lim_n \int g_n \, \mathrm{d}P$.

Proofs of the above stated results and more facts on the inte ration of random variables can be found in [5].

II

Definition 2.1. *Let* (Ω, S, P) *be a probability space, X be any Dedekind* σ*-complete vector lattice. We say that a sequence* f_n *of random variables obeys the strong law of large numbers (SLLN) if* $S_n(\omega) = \frac{1}{n} \sum_1^n f_k(\omega) \to 0$ *on a set of probability* 1.

In the following lemma sufficient conditions for a sequence f_n of random variables to obey *SLLN* are given.

Lemma 2.1. *Each of the following conditions is sufficient for a sequence* f_n *of random variables to obey SLLN.*

a) $P\{\omega\,;\ \limsup |S_n(\omega)| = 0\} = 1$

b) $\forall z > 0 \,\forall n \geq 1 \,\exists o(n, z) \to 1 \,\forall m \geq n$

$$P \bigcap_{k=n}^{m} \{\omega\,;\ |S_k(\omega)| \leq z\} \geq o(n, z).$$

Proof. Obvious.

We begin discussing *SLLN* with the case when the image space X contains an order-unit. An example of such a space is $B(S)$ — the space of all bounded real-valued functions on a set S with the usual ordering. We recall that *a norm is induced by an order unit* e *if* $\|x\| = \inf\{a>0;\ -ae \leqslant x \leqslant ae\}$ *for all* $x \in X$.

Theorem 2.1. *Let X be an ordered normed linear space with a norm induced by an order-unit. If a sequence f_n of random variables obeys SLLN with respect to the norm, then f_n obeys SLLN with respect to the ordering.*

Proof. It is sufficient to show that, given any natural k, there exists for each natural n a nonnegative number $o\left(n, \frac{1}{k} e\right)$ with the property that $\lim_n o\left(n, \frac{1}{k} e\right) = 1$ for each k and such that for each $m \geqslant n$ $P \bigcap_{t=n}^{m} \left\{\omega;\ |S_t(\omega)| \leqslant \frac{1}{k} e\right\} \geqslant o\left(n, \frac{1}{k} e\right)$. If an element ω does not belong to $\left\{\omega;\ |S_t(\omega)| \leqslant \frac{1}{k} e\right\}$ for an index t, $n \leqslant t \leqslant m$, there exists a continuous lattice homomorphism T, $\|T\| = 1$, separating $S_t(\omega)$ and $\frac{1}{k} e$, i.e. $|TS_t(\omega)| > \frac{1}{k} Te$, since X contains an order-unit e. For such a functional we have $T(e) = \|T\|$, because the norm is induced by e. Hence $\|S_t(\omega)\| > \frac{1}{k}$, by definition of the norm. Hence we have the following set inclusion

$$\bigcap_{t=n}^{m} \left\{\omega;\ |S_t(\omega)| \leqslant \frac{1}{k} e\right\} \supset \bigcap_{t=n}^{m} \left\{\omega;\ \|S_t(\omega)\| < \frac{1}{k}\right\}.$$

Since f_n obeys *SLLN* with respect to the norm, there exists an $o\left(n, \frac{1}{k}\right)$ such that

$$P \bigcap_{t=n}^{m} \left\{\omega;\ \|S_t(\omega)\| < \frac{1}{k}\right\} \geqslant o\left(n, \frac{1}{k}\right) \to 1.$$

The rest of the proof follows easily.

Corollary 2.1. *Let the assumptions be as above. If $b\|f\|$ denotes essential supremum of a random variable $\|f\|$ and if $\frac{1}{n} \sum_1^n b\|f_i\| \to 0$, then $\{f_n\}$ obeys SLLN.*

Proof. Since $P \bigcup_i \{\omega;\ \|f_i\| > b\|f_i\|\} = 0$, we have $\limsup \left\|\frac{\Sigma f_i}{n}\right\| \leqslant \limsup \frac{\Sigma b\|f_i\|}{n}$ on a set of probability 1.

This corollary can be compared with [2], th. III. 13.

As mentioned above an example of the space from theorem 2.1. is $B(S)$. In this case o-norm is equivalent to the usual supremum norm. Once the ordering is changed, they may cease to be equivalent. (l^∞ with P_s-ordering provides a required counterexample.)

Corollary 2.2. *Let X be an ordered linear space in which o-units exist with o-bound topology (i.e. with the largest locally convex topology making all o-intervals bounded). If f_n obeys SLLN with respect to the norm, then f_n obeys SLLN with respect to the ordering.*

Proof. The o-bound topology is the topology induced by each of its o-units. A natural example is l^∞.

Corollary 2.3. *Let X be an M-normed Banach lattice (i.e. such that $\|x \vee y\| = \|x\| \vee \|y\|$, $x, y \in P$). If f_n obeys SLLN with respect to the norm, then f_n obeys SLLN with respect to the ordering.*

Examples of such spaces are L^∞, $D_\varphi(X)$. (See [6].)

Having in mind a key role which the spaces $C(S)$ play in the theory of above spaces we present the following.

Corollary 2.4. *If f_n is a sequence of random variables with values in $C(S)$, S an extremally disconnected compact Hausdorff space, then f_n obeys SLLN with respect to the ordering whenever it satisfies SLLN with respect to the norm.*

Sometimes it is more suitable to find sufficient conditions in terms of linear functionals which belong to a fundamental set. For this purpose we slightly adapt definition 2.1.

Definition 2.2. *A sequence f_n obeys SLLN uniformly with respect to $\mathcal{T}$ (a set of linear functionals) if for each $a > 0$*

$$\lim_n P\left(\bigcap_{k=n}^{\infty} \{\omega\,;\, |TS_k(\omega)| < a\}\right) = 1$$

uniformly for $T \in \mathcal{T}$.

Theorem 2.2. *Let X be a normed linear space with an o-unit norm and with a separable dual space with respect to the weak topology. Let f_n obey SLLN uniformly for $\mathcal{T} = \{T;\ T\text{-continuous linear functional},\ \|T\| \leq 1\}$. Then f_n obeys SLLN with respect to the ordering.*

It is well known that if a space X has an o-unit, then lattice homomorphisms separate points of X. We also know that so-called locally directed spaces (i.e. spaces in which the upward-directed neighbourhoods of 0 form a local base) have the latter property. *(A set A is directed upwards if, given two elements a, b of A, there is an element c of A that $a \leq c$ and $b \leq c$.)* In order to prove *SLLN* for such spaces (they need not to have o-units, e.g. s, c_0) we require the existence of a set of linear functionals with a special property.

Theorem 2.3. *Let X be a locally directed normed linear space with a set of σ-homomorphisms dense in the set of all lattice homomorphisms. If a sequence f_n obeys SLLN with respect to the norm, then f_n obeys SLLN with respect to the ordering.*

Proof. Since f_n obeys *SLLN* with respect to the norm, there is a set C of probability 1 such that for each $\omega \in C \|S_n(\omega)\| \to 0$. Let ω denote an element of this set such that inf sup $|S_n(\omega)| = z > 0$. By hypothesis, there is a σ-homomorphism T separating z and 0, i.e. $Tz = c > 0$. We have then

$$\|T\| \text{ inf sup } \|S_n(\omega)\| \geqslant \text{inf sup } |TS_n(\omega)| = c > 0 \Rightarrow$$

$$\text{inf sup } \|S_n(\omega)\| \geqslant \frac{c}{\|T\|} > 0,$$

a contradiction.

A natural example of the space to which the previous theorem is applicable is c_0.

Definitions 2.3. *Random variables f and g are said to be weakly orthogonal if $E\ (Tf \cdot Tg) = 0$ for each $T \in X^*$.*

Corollary 2.5. *Let X be a locally directed normed linear space with a countable set of σ-homomorphisms dense in the set of all lattice homomorphisms. Let f_n be a sequence of weakly orthogonal random variables such that $\sum \frac{E(Tf_n)^2}{n^2}$ converges for each $T \in X^*$. Then f_n satisfies SLLN.*

Proof. Since random variables are weakly orthogonal, *SLLN* holds for each σ-homomorphism. If there were an element $\omega \notin \cup\{\omega\,;\ TS_n(\omega) \nrightarrow 0\}$ such that inf sup $|S_n(\omega)| = z > 0$, a σ-homomorphism T would exist for which inf $T\langle z, \infty) >$ 0, a contradiction.

Theorem 2.4. *Let X be a locally directed space with a metric given by a system of M-seminorms $\{p\}$ (i.e. $p(x \vee y) = p(x) \vee p(y)$, x, $y \in P$). If a set of σ-homomorphisms dense in the set of all lattice homomorphisms exists, then any sequence f_n of random variables which obeys SLLN with respect to the metric satisfies SLLN with respect to the ordering.*

Proof. If for an element ω inf sup $|S_n(\omega)| = z > 0$, there is a σ-homomorphism T separating z and 0. There are a seminorm p and a real number c such that $|Tx| \leqslant cp(x)$ for each $x \in X$. If $Tz = d > 0$, then inf sup $p(S_n(\omega)) \geqslant \frac{d}{c} > 0$, a contradiction.

A known example is s.

III

In this chapter we consider spaces of the type A and B, respectively. The most interesting example of such spaces are L^p-spaces $(1<p<\infty)$. First we prove that Toeplitz' and Kronecker's Lemmas hold in such spaces.

Toeplitz's Lemma. *Let X be a space of the type $A(B)$. Then $x_n \xrightarrow{o} x$ implies $\frac{1}{n}\sum_1^n x_i \xrightarrow{o} x$.*

Proof. There are an increasing sequence y_n and a decreasing sequence z_n such that $y_n \leq x_n < z_n$ and $y_n \uparrow x \downarrow z_n$. Since $y_n \uparrow x$ implies $Ty_n \uparrow Tx$, we have $\frac{1}{n}\sum Ty_i \to Tx$. Hence we have $\frac{1}{n}\sum y_i \to x$, because X is of the type $A(B)$. Since $\frac{1}{n}\sum y_i$ is an increasing sequence and since $\frac{1}{n}\sum y_i \leq \frac{1}{n}\sum x_i \leq \frac{1}{n}\sum z_i$, the result follows.

Kronecker's Lemma. *Let X be of the type $A(B)$ Then the o-convergence of $\sum_1^\infty \frac{x_i}{i}$ implies that $\frac{1}{n}\sum_1^n x_i \xrightarrow{o} 0$.*

Proof. See [7], page 203.

Theorem 3.1. *Let X be a normed vector lattice of the type A with X^* separable in the weak topology. Let f_n be a sequence of random variables for which $\sum \frac{b\|f_n\|}{n}$ converges. Then f_n obeys SLLN with respect to the ordering.*

Proof. Owing to Kronecker's Lemma it is sufficient to show that

$$P\left\{\omega\,;\,\inf_k \sup_p \left|\sum_{k+1}^{k+p} \frac{f_i}{i}\right| = 0\right\} = 1\,.$$

In fact, it is enough to prove that for a fixed sequence z_n, $z_n \in X$, $z_n \downarrow 0$ we have

$$P\left\{\omega\,;\,\inf_k \sup_p \left|\sum_{k+1}^{k+p} \frac{f_i}{i}\right| \leq z_n\right\} = 1$$

for each n.

Let $z>0$ be given. The set

$$\left\{\omega\,;\,\inf_k \sup_p \left|\sum_{k+1}^{k+p} \frac{f_i}{i}\right| < z\right\}$$

is measurable because it may be written in the form

$$\left\{\omega\,;\,\inf_k \sup_p \left\{\sup_{1\leq i<p} h_i\right\} \leq z\right\}, \quad h_i = \left|\sum_{k+1}^{k+p} \frac{f_i}{i}\right|$$

and the measurability of the latter set follows from the relation

$$\left\{\omega ; \inf_k \sup_p \left\{\sup_{1\le i\le p} h_i\right\}\le z\right\}=\bigcap_T \left\{\omega ; \inf_k \sup_p T\,\{\sup h_i\}\le Tz\right\}$$

and because $T\,\{\sup h_i\}$ are random variables with values in R. We have

$$P\left\{\omega ; \inf_k \sup_p \left|\sum_{k+1}^{k+p}\frac{f_i}{i}\right|\le z\right\}\ge P\left\{\omega ; \inf \sup \sum_{k+1}^{k+p}\frac{|f_i|}{i}\le z\right\}\ge$$

$$P\bigcap_{T\uparrow a_n\downarrow 0}\left\{\omega ; \inf \sup \sum_{k+1}^{k+p}\frac{T|f_i|}{i}\le a_n\right\}\ge P\left\{\omega ; \inf \sup \sum_{k+1}^{k+p}\frac{T|f_i|}{i}\le a\right\}$$

$-d=1-d$ for every $d>0$, because the sum of continuous linear functionals is a continuous linear functional and because of the validity of the theorem on the diagonal sequence in R. ((a_n) means a sequence of real numbers.)

Theorem 3.2. *Let X be an AL-space (i.e. a normed Banach lattice in which $\|x+y\| = \|x\|+\|y\|$, $x, y\in P$) with X^* separable in the weak topology. If for a sequence f_n of random variables the series $\sum \frac{bf_i}{i}$ o-converges, then f_n obeys SLLN.*

Proof.

$$P\left\{\omega ; \inf_k \sup_p \left|\sum_{k+1}^{k+p}\frac{f_i}{i}\right|\le z\right\}\ge P\left\{\omega ; \inf \sup \sum \frac{T|f_i|}{i}\le a\right\}-$$

$$-d\ge P\left\{\omega ; \inf \sup \sum_{k+1}^{k+p}\frac{\|f_i\|}{i}\le \frac{a}{\|T\|}\right\}-d=$$

$$=P\left\{\omega ; \inf \sup \left\|\sum \frac{|f_i|}{i}\right\|\le \frac{a}{\|T\|}\right\}-d=1-d$$

since in spaces ordered by a normal cone the o-convergence of $\sum \frac{bf_i}{i}$ implies its convergence for each $T\in X^*$, which consequently implies the convergence of $\sum \frac{bf_i}{i}$ with respect to the norm. (See [6], page 113.)

Theorem 3.3. *Let X be a locally convex space of the type A which is a vector lattice with X^* separable in the weak topology. Let f_n be a sequence of integrable random variables such that $\sum \frac{E|f_i|}{i}$ $w(X, X^*)$-converges. Then f_n obeys SLLN.*

Proof. We have

$$P\left\{\omega ; \inf_k \sup_p \left|\sum_{k+1}^{k+p}\frac{f_i}{i}\right|\le z\right\}\ge P\left\{\omega ; \inf_k \sup_p \sum_{k+1}^{k+p}\frac{T|f_i|}{i}\le a\right\}-d$$

for each $d > 0$. Since $\sum \frac{ET|f_i|}{i} - \sum \frac{TE|f_i|}{i}$ and since, by Beppo—Levi heorem, $\sum \frac{T|f_i|}{i}$ converges a.e. provided that $\sum \frac{ET|f_i|}{i}$ converges, the result follows.

In the second part of this chapter the case of a space without topology will be discussed. Our final aim is to prove *SLLN* for regular spaces, i. . for vector lattices with the diagonal property for o-convergence.

Theorem 3.4. *Let X be a Dedekind σ-complete vector lattice of th type B, X^+ being separable with respect to $w(X^+, X)$-topology. If f_n is a sequen e of i able random variables such that the series $\sum \frac{E|f_i|}{i}$ $w(X, X^+)$-converges, then f_n obeys SLLN.*

Proof. Analogous to that of theorem 3.3

Corollary 3.1. *Let X be a Dedekind σ-complete regular space such that X^+ separates points of X and is separable in $w(X^+, X)$. Let f_n be a sequence of integrable random variables such that $\sum \frac{E|f_i|}{i}$ $w(X, X^+)$-conver es. Then SLLN holds.*

Proof. If a vector lattice has the diagonal property, then every o-bounded linear functional is o-continuous.

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УСИЛЕННЫЙ ЗАКОН БОЛЬШИХ ЧИСЕЛ ДЛЯ ПОРЯДКОВО СХОДЯЩИХСЯ СЛУЧАЙНЫХ ВЕЛИЧИН

Растислав Потоцкий

Резюме

В работе изучаются случайные величины с значениями в некоторой векторной решетке. Первая часть содержит необходимые для дальнейшего исследования понятия и результаты. Во второй части доказывается усиленный закон больших чисел для случая векторной решетки с порядковой единицей. В последней части приводится усиленный закон больших чисел для пространств, в которых справедлива лемма Кронекера.