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RETRACTS OF MONOUNARY ALGEBRAS CORRESPONDING TO GROUPOIDS

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(*Communicated by Sylvia Pulmannová*)

ABSTRACT. In [NOVOTNÝ, M.: *Construction of all homomorphisms of groupoids*, Czechoslovak Math. J. **46(121)** (1996), 141–153] is defined the monounary algebra $\mathrm{un}(G, \circ)$ corresponding to a groupoid $(G, \circ)$. The aim of this paper is to prove that each monounary algebra is up to isomorphism a retract of $\mathrm{un}(G, \circ)$ for some groupoid $(G, \circ)$.

Introduction

The importance of the notion of retract in several areas of mathematics is well known and is commonly appreciated. There are dozens of papers dealing with retracts of algebraic structures, we quote only some of them ([9]–[11], [16]–[18]).

Monounary algebras play a significant role in the study of algebraic and relational structures, especially in the case of finite structures (cf., e.g., Jónsson [8], Skornjakov [15], Chvalina [2]). Further, there exists a close connection between monounary algebras and some types of automata (cf. e.g., Bartol [1], Salij [14]).

M. Novotný [13] proved that all homomorphisms of groupoids can be constructed by means of homomorphisms of monounary algebras. In this construction he defined and investigated the notion of a monounary algebra denoted by $\mathrm{un}(G, \circ)$, which corresponds to a groupoid $(G, \circ)$.

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In [12], cyclic monounary algebras of the form $\mathrm{un}(G, \circ)$ were studied.

Given a monounary algebra (A, f) and the groupoid $(A, *_f)$ with $x *_f y = f(y)$, one can easily verify that (A, f) is both a subalgebra and a homomorphic image of $\mathrm{un}(A, *_f)$ (for the first part, just take the injection $i\colon y \to (y, f(y))$ and for the second part take the second projection $\pi_2\colon (x, y) \to y$). But in this case, (A, f) need not be a retract of $\mathrm{un}(A, *_f)$. (For the throughout definitions cf. Section 1 below.)

The aim of the present paper is to prove that each monounary algebra is up to isomorphism a retract of some $\mathrm{un}(G, \circ)$ for a groupoid $(G, \circ)$.

On the other hand, there exists a proper class of monounary algebras which are not isomorphic to any $\mathrm{un}(G, \circ)$.

Retracts of monounary algebras were investigated by the author [3]–[7].

1. Preliminaries

We recall some basic definitions.

A *monounary algebra* is a pair (A, f), where A is a non-empty set and f is a unary operation on A.

Let (A, f) be a monounary algebra. For $a \in A$ we put $f^0(a) = a$, and by induction, $f^n(a) = f(f^{n-1}(a))$ for each $n \in \mathbb{N}$.

A monounary algebra (A, f) is said to be *connected* if for each $x, y \in A$ there are $m, n \in \mathbb{N} \cup \{0\}$ such that $f^m(x) = f^n(y)$.

A maximal connected subalgebra (B, f) of (A, f) is called a *connected component* of (A, f); we will say also that B is a connected component of (A, f).

An element $a \in A$ is *cyclic* if $f^n(a) = a$ for some $n \in \mathbb{N}$. Let (B, f) be a connected component of (A, f). If each element of B is cyclic, then B is a *cycle* of (A, f).

Let (A, F) be an algebra. A subalgebra (B, F) of (A, F) is a *retract* of (A, F) if there is an endomorphism φ of (A, F) such that φ is a mapping of A onto B and $\varphi(b) = b$ for each $b \in B$; in this case φ is said to be a *retraction endomorphism*.

Let $(G, \circ)$ be a groupoid. A monounary algebra $\mathrm{un}(G, \circ)$ corresponding to $(G, \circ)$ is defined as follows: $\mathrm{un}(G, \circ) = (G \times G, g)$, where g is a unary operation on $G \times G$ such that if $(x, y) \in G \times G$, then $g((x, y)) = (y, x \circ y)$.

2. Underlying set of the groupoid $(G, \circ)$

In what follows, let (A, f) be a monounary algebra.

As we already announced above, our aim is to construct a groupoid $(G, \circ)$ such that (A, f) is isomorphic to a retract of $\mathrm{un}(G, \circ)$. In the present section we construct the underlying set G of the groupoid under consideration; the operation $\circ$ will be dealt with in Section 3.

We will use the following notation. Let α be an ordinal and let a system of sets $\{B_\beta\}_{\beta<\alpha}$ be such that $B_\beta \subseteq B_\gamma$ for each $\beta \leq \gamma < \alpha$. Further assume that $\{\varphi_\beta\}_{\beta<\alpha}$ is a system of mappings $\varphi_\beta \colon B_\beta \to C$ for some set C such that if $\beta \leq \gamma < \alpha$, $b \in B_\beta$, then $\varphi_\gamma(b) = \varphi_\beta(b)$. By a union $\bigcup\limits_{\beta<\alpha} \varphi_\beta$ we understand the mapping $\varphi\colon \bigcup\limits_{\beta<\alpha} B_\beta \to C$ such that whenever $b \in B_\beta$, $\beta < \alpha$, then $\varphi(b) = \varphi_\beta(b)$.

First we are going to define by induction a set Λ of ordinal numbers.

For a set Γ of ordinals let Γ^+ be the smallest ordinal which is greater than any $\gamma \in \Gamma$.

Applying the Axiom of Choice we can suppose that the set A is well-ordered, i.e.,

$$A = \{a_\mu :\ \mu < \mu_0\}, \qquad \mu_0 \in \mathrm{Ord},$$

and also that the system of all connected components of (A, f) is well-ordered, i.e., (A, f) possesses the system $\{K_\iota\}_{\iota<\iota_0}$ of connected components, $\iota_0 \in \mathrm{Ord}$.

For each $\iota < \iota_0$ let x_ι be a fixed element of K_ι such that if K_ι contains a cycle, then x_ι is cyclic. Further we define certain subsets P^ι_n, $n \in \mathbb{N} \cup \{0\}$, of K_ι, which we call folds generated by x_ι; they are defined as follows:

$$P^\iota_0 = \{f^i(x_\iota) :\ i \in \mathbb{N} \cup \{0\}\}, \qquad P^\iota_1 = f^{-1}(P^\iota_0) - P^\iota_0,$$
$$P^\iota_{n+1} = f^{-1}(P^\iota_n) \qquad \text{for each} \quad n \in \mathbb{N}.$$

Now we will proceed by induction and define, for each ordinal $\eta < \mu_0$,

- a set $D_\eta \subseteq A$,
- a set $\Lambda_\eta \subset \mathrm{Ord}$,
- a mapping $\varphi_\eta \colon D_\eta \to \Lambda_\eta \times \Lambda_\eta$ such that
  - (∗1) if $\eta' \leq \eta'' < \mu_0$, then $D_{\eta'} \subseteq D_{\eta''}$, $\Lambda_{\eta'} \subseteq \Lambda_{\eta''}$,
  - (∗2) if $\eta' \leq \eta'' < \mu_0$, $d \in D_{\eta'}$, then $\varphi_{\eta'}(d) = \varphi_{\eta''}(d)$,
  - (∗3) if $\eta' < \mu_0$, $\lambda_1 \in \Lambda_{\eta'}$, then there are $\lambda_2 \in \Lambda_{\eta'}$, $d \in D_{\eta'}$ such that either $\varphi_{\eta'}(d) = (\lambda_1, \lambda_2)$ or $\varphi_{\eta'}(d) = (\lambda_2, \lambda_1)$,
  - (∗4) if $\eta' < \mu_0$, $d, e \in D_{\eta'}$, $\varphi_{\eta'}(d) = (\lambda_1, \lambda_2)$, $\varphi_{\eta'}(e) = (\lambda_1, \lambda_3)$, then $d = e$.

I. For $\eta = 0$ put $D_\eta = \emptyset$, $\Lambda_\eta = \emptyset$.

II. Let $\eta \in \mathrm{Ord}$, $\eta > 0$. Suppose that for all ordinals $\eta' < \eta$ sets $D_{\eta'}$, $\Lambda_{\eta'}$ and an injective mapping $\varphi_{\eta'} : D_{\eta'} \to \Lambda_{\eta'} \times \Lambda_{\eta'}$ are defined such that the conditions analogous to $(*1)$–$(*4)$ are valid, with the distinction that we take η instead of μ_0.

If $A \neq \bigcup\limits_{\eta'<\eta} D_{\eta'}$, then there is the smallest $\iota < \iota_0$ such that $K_\iota \nsubseteq \bigcup\limits_{\eta'<\eta} D_{\eta'}$ and there is the smallest $n \in \mathbb{N} \cup \{0\}$ such that $P_n^\iota \nsubseteq \bigcup\limits_{\eta'<\eta} D_{\eta'}$.

Denote $\beta = \Big(\bigcup\limits_{\eta'<\eta} \Lambda_\eta \Big)^+$.

a) Assume that $n = 0$ and $x_\iota \notin \bigcup\limits_{\eta'<\eta} D_{\eta'}$.

a1) If $f(x_\iota) = x_\iota$, then we set $D_\eta = \bigcup\limits_{\eta'<\eta} D_{\eta'} \cup \{x_\iota\}$, $\Lambda_\eta = \bigcup\limits_{\eta'<\eta} \Lambda_{\eta'} \cup \{\beta\}$ and

$$\varphi_\eta(a) = \begin{cases} \Big(\bigcup\limits_{\eta'<\eta} \varphi_{\eta'} \Big)(a) & \text{if } a \in \bigcup\limits_{\eta'<\eta} D_{\eta'}\,, \\ (\beta, \beta) & \text{if } a = x_\iota\,. \end{cases}$$

The induction assumption yields that $(*1)$–$(*4)$ are satisfied if we take η^+ instead of μ_0.

a2) If $f(x_\iota) \neq x_\iota$, then either x_ι belongs to a k-element cycle, $k > 1$, or all elements $f^i(x_\iota)$, $i \in \mathbb{N} \cup \{0\}$, are mutually distinct. We put $D_\eta = \bigcup\limits_{\eta'<\eta} D_{\eta'} \cup P_0^\iota$. In the first case $\Lambda_\eta = \bigcup\limits_{\eta'<\eta} \Lambda_{\eta'} \cup \big\{\beta, \beta+1, \dots, \beta+(k-1)\big\}$ and φ_η is an extension of $\bigcup\limits_{\eta'<\eta} \varphi_{\eta'}$ such that

$$\varphi_\eta\big(f^i(x_\iota)\big) = \begin{cases} (\beta+i, \beta+i+1) & \text{if } i = 0, \dots, k-1\,, \\ (\beta+k, \beta) & \text{if } i = k\,. \end{cases}$$

In the second case we set $\Lambda_\eta = \bigcup\limits_{\eta'<\eta} \Lambda_{\eta'} \cup \{\beta+n :\ n < \omega\}$ and φ_η is an extension of $\bigcup\limits_{\eta'<\eta} \varphi_{\eta'}$ such that

$$\varphi_\eta\big(f^i(x_\iota)\big) = (\beta+i, \beta+i+1) \qquad \text{for each} \quad i < \omega\,.$$

Also in this case $(*1)$–$(*4)$ are satisfied (with η^+ substituted for μ_0).

b) Assume that $x_\iota \in \bigcup\limits_{\eta'<\eta} D_{\eta'}$. In view of a1) and a2) also $P_0^\iota \subseteq \bigcup\limits_{\eta'<\eta} D_{\eta'}$, thus $n > 0$. There is the smallest element $y \in P_n^\iota - \bigcup\limits_{\eta'<\eta} D_{\eta'}$. Then $f(y) \in P_{n-1}^\iota \subseteq \bigcup\limits_{\eta'<\eta} D_{\eta'}$, i.e., there are $\eta' < \eta$ and $\alpha_1, \alpha_2 \in \Lambda_{\eta'}$ such that $\varphi_{\eta'}\big(f(y)\big) = (\alpha_1, \alpha_2)$.

We put

$$D_\eta = \bigcup_{\eta'<\eta} D_{\eta'} \cup \{y\}, \qquad \Lambda_\eta = \bigcup_{\eta'<\eta} \Lambda_{\eta'} \cup \{\beta\}.$$

Further, let φ_η be an extension of $\bigcup_{\eta'<\eta} \varphi_{\eta'}$ such that $\varphi_\eta(y) = (\beta, \alpha_1)$.

There exists $\eta_0 \leqq \mu_0$ such that $A = \bigcup_{\eta'<\eta_0} D_{\eta'}$. Thus for each η with $\eta_0 \leqq \eta < \mu_0$ we put $D_\eta = D_{\eta_0}$, $\Lambda_\eta = \Lambda_{\eta_0}$, $\varphi_\eta = \varphi_{\eta_0}$.

NOTATION 2.1. Now we have $A = \bigcup_{\eta<\mu_0} D_\eta$. Put

$$\Lambda = \bigcup_{\eta<\mu_0} \Lambda_\eta, \quad \varphi = \bigcup_{\eta<\mu_0} \varphi_\eta,$$
$$G = \Lambda \cup \{\Lambda^+\},$$
$$\Omega = \varphi(A).$$

3. Operation $\circ$ of the groupoid $(G, \circ)$

Using 2.1, in this section a binary operation $\circ$ on G will be defined.

First we define $\alpha * \beta$ for $(\alpha, \beta) \in \Omega$ as follows. Let $(\alpha, \beta) \in \Omega$. There is $x \in A$ with $\varphi(x) = (\alpha, \beta)$. The definition of φ implies that $\varphi(f(x)) = (\beta, \gamma)$ for some $\gamma \in \Lambda$; put $\alpha * \beta = \gamma$.

LEMMA 3.1. *Let $\Box$ be a binary operation on G such that if $(\alpha, \beta) \in \Omega$, then $\alpha \Box \beta = \alpha * \beta$. Further let* $\operatorname{un}(G, \Box) = (G \times G, h)$. *Then Ω is closed with respect to h.*

Proof. Let $(\alpha, \beta) \in \Omega$. Then $h((\alpha, \beta)) = (\beta, \alpha \Box \beta) = (\beta, \alpha * \beta) \in \Omega$. $\square$

LEMMA 3.2. *Let the assumption of* 3.1 *hold. Then φ is an isomorphism of (A, f) onto (Ω, h).*

Proof. By 2.1, the mapping φ is surjective. From the construction in Section 2 it follows that φ is injective.

Let $x \in A$, $\varphi(x) = (\alpha, \beta) \in \Omega$. Then $\varphi(f(x)) = (\beta, \gamma)$ and $\gamma = \alpha * \beta$, which yields

$$\varphi(f(x)) = (\beta, \gamma) = (\beta, \alpha * \beta) = (\beta, \alpha \Box \beta) = h((\alpha, \beta)) = h(\varphi(x)).$$

Thus φ is an isomorphism of (A, f) onto (Ω, h). $\square$

Now we are going to define the operation $\circ$ on G. In A there exist (not necessarily distinct) elements a, a', a'', a''' such that $f(a''') = a''$, $f(a'') = a'$, $f(a') = a$; we take fixed elements with this property. Then there are ordinals $\delta, \tau, \tau', \tau'', \tau''' \in \Lambda$ such that

$$\begin{aligned} \varphi(a) &= (\tau, \delta)\,, \qquad & \varphi(a'') &= (\tau'', \tau')\,, \\ \varphi(a') &= (\tau', \tau)\,, \qquad & \varphi(a''') &= (\tau''', \tau'')\,. \end{aligned} \tag{1}$$

By the definition of $*$ we obtain

$$\tau''' * \tau'' = \tau', \qquad \tau'' * \tau' = \tau\,, \qquad \tau' * \tau = \delta\,. \tag{2}$$

Further denote $\lambda = \Lambda^+$; notice that $\lambda \notin \Lambda$, thus we have

$$(\alpha, \lambda) \notin \Omega \qquad \text{for any} \quad \alpha \in \Lambda\,. \tag{†}$$

NOTATION 3.3. Let $\circ$ be a binary operation on G defined as follows:

$$\alpha \circ \beta = \begin{cases} \alpha * \beta & \text{if } (\alpha, \beta) \in \Omega\,, \\ \delta & \text{if } \alpha = \lambda\,,\ \beta = \tau\,, \\ \tau & \text{if } \beta = \lambda\,, \\ \lambda & \text{otherwise.} \end{cases}$$

Put $(B, g) = \mathrm{un}(G, \circ)$.

In view of (†), $\alpha \circ \beta$ is correctly defined.

LEMMA 3.4. *(Ω, g) is a retract of (B, g).*

Proof. Let us define a retraction endomorphism $h\colon B \to \Omega$. For $(\alpha, \beta) \in B = G \times G$ we define

$$h\big((\alpha, \beta)\big) = \begin{cases} (\alpha, \beta) & \text{if } (\alpha, \beta) \in \Omega\,, \\ (\tau', \tau) & \text{if } \alpha = \lambda\,,\ \beta = \tau\,, \\ (\tau'', \tau') & \text{if } \beta = \lambda\,, \\ (\tau''', \tau'') & \text{otherwise.} \end{cases}$$

The mapping is correctly defined according to (†).

Let $(\alpha, \beta) \in \Omega$. Then $g\big((\alpha, \beta)\big) \in \Omega$ in view of 3.1, thus

$$h\Big(g\big((\alpha, \beta)\big)\Big) = g\big((\alpha, \beta)\big) = g\Big(h\big((\alpha, \beta)\big)\Big)\,.$$

For $(\alpha, \beta) = (\lambda, \tau)$ we obtain

$$\begin{aligned} h\Big(g\big((\alpha, \beta)\big)\Big) &= h\big((\beta, \alpha \circ \beta)\big) = h\big((\tau, \delta)\big) = (\tau, \delta) \\ &= (\tau, \tau' * \tau) = (\tau, \tau' \circ \tau) = g\big((\tau', \tau)\big) = g\Big(h\big((\alpha, \beta)\big)\Big)\,. \end{aligned}$$

Let $(\alpha,\beta)\in B$, $\beta=\lambda$. Then

$$\begin{aligned} h\big(g((\alpha,\beta))\big) &= h((\beta,\alpha\circ\beta)) = h((\lambda,\tau)) = (\tau',\tau) \\ &= (\tau',\tau''*\tau') = (\tau',\tau''\circ\tau') = g((\tau'',\tau')) = g\big(h((\alpha,\beta))\big). \end{aligned}$$

Finally, consider the remaining case for (α,β). Then

$$\begin{aligned} h\big(g((\alpha,\beta))\big) &= h((\beta,\alpha\circ\beta)) = h((\beta,\lambda)) = (\tau'',\tau') \\ &= (\tau'',\tau'''*\tau'') = (\tau'',\tau'''\circ\tau'') = g((\tau''',\tau'')) = g\big(h((\alpha,\beta))\big). \end{aligned}$$

Therefore h is a retraction endomorphism onto (Ω,g), thus (Ω,g) is a retract of (B,g). □

THEOREM 3.5. *Let (A,f) be a monounary algebra. There exists a groupoid $(G,\circ)$ such that (A,f) is isomorphic to a retract of the monounary algebra $\mathrm{un}(G,\circ)$ corresponding to the groupoid $(G,\circ)$.*

Proof. The assertion follows from 3.2 and 3.4. □

We conclude by giving an example which shows that there exists a proper class of monounary algebras which are not isomorphic to any $\mathrm{un}(G,\circ)$ for a groupoid $(G,\circ)$.

EXAMPLE 3.6. Let (A,f) be a monounary algebra such that $|A|>1$ and there is an $a\in A$ with $f(x)=a$ for each $x\in A$. We will show that $(A,f)\not\cong\mathrm{un}(G,\circ)$ for any groupoid $(G,\circ)$.

Suppose that there are a groupoid $(G,\circ)$ and an isomorphism φ of (A,f) onto $\mathrm{un}(G,\circ)=(G\times G,g)$. Denote $\varphi(a)=(a_1,a_2)$. Then

$$\begin{aligned} (a_1,a_2) &= \varphi(a) = \varphi\big(f(a)\big) = g\big(\varphi(a)\big) \\ &= g\big((a_1,a_2)\big) = (a_2,a_1\circ a_2), \end{aligned}$$

which implies $a_1=a_2=a_1\circ a_2$. If $b\in A-\{a\}$, $\varphi(b)=(b_1,b_2)$, then

$$\begin{aligned} (a_1,a_2) &= \varphi(a) = \varphi\big(f(b)\big) = g\big(\varphi(b)\big) \\ &= g\big((b_1,b_2)\big) = (b_2,b_1\circ b_2), \end{aligned}$$

thus $a_1=b_2$. Therefore

$$\varphi(A)\subseteq\big\{(x,a_1):\ x\in G\big\}.$$

Since $|A|>1$, we obtain that $\varphi(A)\neq G\times G$, which is a contradiction.

We have constructed (A,f) for each cardinality $|A|>1$, therefore there is a proper class of (A,f) with $(A,f)\not\cong\mathrm{un}(G,\circ)$ for any groupoid $(G,\circ)$.

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